



Uncertainty Distribution Models for Key Input Parameters for Design and Analysis of Flexible Pavement in Nigeria

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ABSTRACT

The road transportation sector is cardinal to accessing the growth and socio-economic development of any country. This is because it contributes directly and immensely to support other sectors of the economy to function properly. Flexible pavement is a multi-layer system of which its design is mostly based on empiricism, assuming a deterministic process with final output as pavement thickness. Each layer thickness is designed and placed in a manner that the wheel loading does not become excessive for the subgrade to bear. Various studies have revealed uncertainties due to variations of thickness and layer modulus which could result in premature failure of the pavement. This variability could be as a result of poor design practice, inadequate quality control, complex traffic loading, complex material properties, lack of fit of design model as well as subgrade characteristic. To address these variabilities many highway agencies were forced into probabilistic design approach to accommodate uncertainties. Of crucial importance to this type of design is the characterization of material properties and establishment of appropriate probability distribution models. This study focused on the development of probability distribution models of key input pavement parameter to the design and analysis of flexible pavement. Data on the geometry and material properties were collected from Pavement Evaluation Unit (PEU), Kaduna and Federal Ministry of Power Works and Housing (FMPWH), Abuja. The probability distribution models of the variation of layer thicknesses, asphalt stiffness, base stiffness and subgrade stiffness were developed using Easy fit 5.5 statistical packages. The results of the studies revealed that the best distribution model for the layer thicknesses as well as stiffness considered in the study is the normal distribution.

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INTRODUCTION

The large stochastic/random elements in pavement deterioration caused by immeasurable and unpredictable factors (Harvey, 2012) introduces a measure of

variability and uncertainty in pavement performance and may increase the cost of maintenance and rehabilitation over time. Analysis of data obtained by in situ core and ground penetration radar indicates spatial

variation in pavement thickness layer. Likewise, the variation of layer modulus (Maji et al., 2016).

Uncertainty in engineering analysis and design is commonly defined as knowledge incompleteness due to inherent deficiencies in acquired knowledge.

It can be used to characterize the state of a system as being unsettled or in doubt, such as the uncertainty of the outcome (Ayyub and Klir, 2006). Because uncertainty is an important dimension in the analysis of risks, the nature and ways of dealing with these uncertainties have been of interest to the highway community for a long time.

Probability is the mechanism employed to manage the uncertainty that underlies all real-world data and phenomena. It assists in gauging the belief and to quantify the lack of certitude that is inherent in the process that generates the data been analyzed (Martinez and Martinez, 2002). For example, to adequately simulate a real system, one needs to understand the probability distributions that correctly model the underlying processes.

Uncertainty of probabilistic input consists of specifying the types of probability distributions of the variables under consideration as well as the parameters of the distributions (Oberguggengerger and Fellin, 2004). An adequate understanding of the influence of input parameter variability on the output of engineering computations requires that the uncertainty itself be captured in mathematical terms (Oberguggengerger, 2004). In the specific area of pavement engineering, techniques such as probabilistic analyses, general stochastic analysis and random fields have been used to explore the impact of material variability and uncertainty quantification in pavement design and performance (Castillo and Caro, 2014).

Quantification and analyzing variability of pavement materials and

design inputs are, therefore fundamentals in developing a probabilistic-based design that evaluates reliability (Dalla Valle and Thom, 2016). An important aspect to this approach is the appropriateness or fitness of distribution models to the substantive situation and/or the data sample of interest and this is the solution the test of goodness-of-fit (GOF) seek to proffer.

The test is very relevant in risk analysis and is used to check how close the data set distribution is to the hypothesized theoretical distribution (Babashamsi et al., 2016). The simplest case of a GOF test is a statistical null hypothesis of a completely specified distribution against all possible alternative distributions (Anderson, 1995). Thus, the outcome of the GOF test is used to statistically verify an assumed theoretical distribution used in design and analysis or to disprove it. Fitting each random variable into the best probability distribution model is key to performance prediction and ultimately management decisions on any road pavement under considerations.

According to Kaura, (2015), in the assignment of distribution models to a random variable, two or more distribution may appear to be plausible probability distribution models. In that case, the test of goodness-of-fit (GOF) can be used to delineate the relative degree of validity of the different distributions.

Goodness-of-Fit tests is a procedure to determine whether a sample n observation, x_1, \dots, x_n of certain distribution is considered as a sample from a given specified distribution (Anderson, 2011). Calculating statistics of Goodness-of-Fit helps to rank the fitted distributions according to the quality of fit over the raw data (Mehranian, and Pakgohar, 2014). The ultimate ends for performing this GOF analysis is to obtain information which helps take best-informed decisions under uncertainty conditions using best-fitted distribution. This distribution is a model of

the stochastic process, which is faced in the real world.

Material variability can be described in statistical terms such as mean, standard deviation together with its probability density distribution (Dalla Valle, 2005). A useful dimensionless way of expressing the variability of a material's property is to use the ratio of the standard deviation over the mean, known as the coefficient of variation (COV). Knowledge of the coefficient of variation of each design input is extremely important to more accurately estimate their influence on the predicted pavement life (Dalla Valle and Thom, 2016).

As earlier stated, the sources of variability in pavement engineering have been of interest to researchers for a very long time. Researchers have carried out studies in the field of reliability analysis on pavement and have concluded on parameters that are crucial to pavement performance. Since these parameters are key, they must be treated as random variables in a probabilistic non-arbitrary manner to account for variabilities that are associated with them to avoid the premature failures and to maintain the pavements at a satisfactory level service.

Chou, (1990) carried out a study on reliability design procedure for airport flexible pavements. He concluded that the performance of convectional flexible pavement is sensitive, in descending order to variable of gear load, the thickness of granular base, subgrade modulus and thickness and modulus of bituminous concrete surface course respectively for subgrade strain-failure criterion, and to variation of gear load, thickness and modulus of asphalt concrete, thickness and modulus of granular base and subgrade modulus for the bituminous strain-failure criteria.

Timm et al., (2000) and Dalla Valle and Thom, (2016), submitted that the

parameters with great influence on the variability of predicted fatigue performance without considering variable loads are; thickness and stiffness of asphalt concrete layer. Also, for rutting, the parameters of great influence are; base thickness, asphalt concrete thickness, and stiffness of subgrade. However, when the effect of variable load was considered, the output variability for fatigue and rutting was doubled. This is an indication that traffic is a single most important parameter in predicting fatigue and rutting life of the pavement.

Dalla Valle and Thom, (2016), researched the reliability of pavement design. The research revealed that the parameters with the greatest influence on the variability of predicted fatigue performance are the asphalt stiffness modulus and thickness. The parameters with the greatest influence on the variability of predicted deformation performance are the granular sub-base thickness, the asphalt thickness and the subgrade stiffness.

Maji et al., (2016) developed a probabilistic approach for asphaltic overlay design by considering variability in input parameter. A sensitivity analysis of pavement layer moduli performed in this study revealed that the asphaltic overlay thickness is sensitive to the resilient modulus of base layer for fatigue and subgrade layer for rutting.

Prozzi et al., (2005), carried out a study on the simulation-based reliability of pavement structures using empirical-mechanistic models. The design of the structure in question is based on the AASHTO design approach. A detailed examination of the failed pavements showed that parameters of great influence on the pavement performance are the surface asphalt thickness and model error.

From the foregoing, subgrade, base and asphalt concrete surfacing moduli

as well as their respective thicknesses (excluding that of subgrade) have been identified as key input parameters that have great influence on fatigue and deformation/rutting lives of pavement. Therefore, they are considered as random variables in this study and fitted to obtain the probably best distribution models.

Though many reliability studies have been carried out in Nigeria, most of these studies rely solely on distribution models specified by authors abroad based on their peculiarities. This might not necessarily fit into the Nigerian situation. Thus, this study attempt to determine uncertainty, appropriate distribution models based on Nigerian conditions, for pavement design and analysis.

METHODOLOGY

The data on the geometry and material properties of flexible pavement were collected from the pavement evaluation unit (PEU), Kaduna and Federal

Ministry of Power Works and Housing (FMPWH), Abuja. Pavement input parameters used for the study are extracted from Region 1 of the trunk road study of Nigeria (Claros et al.,1986). Specifically, master test section MTS, 1 of Claros et al., (1986) based on the information presented in Nigeria overlay design procedure was used. The MTS is located on road A236. The A236 highway runs from the A2 highway at Zaria, Kaduna State to the A3 highway at Jos the capital of Plateau State. (Zaria - Pambegua - Saminaka – Plateau).

Road A236 is a trunk route with two lanes in opposite directions. The road has a total length of 170 km, and width of 7.3m (FMPWH, 2016). The road still has similar features as described from the previous study of Claros et al., (1986) and there are no current studies to advance the existing data.

The input parameters of interest are as shown in Figure 1 below.

Table1: Input Parameters for Road A236 (FMPWH, 2016).

Layer Materials	Layer Thickness (mm)	Layer Modulus (N/mm ²)	Poison Ratio
Asphalt concrete AC	63.50	6205.28	0.35
Granular base GB	139.70	620.53	0.20
Granular sub-base GSB	71.12	310.26	0.35
Granular Subgrade GS	7620	179.26	0.40

The statistics of the design input parameters used in this study are presented in Table 2.0 below

Table 2: Statistics of the design input parameters

Property	Mean (μ)	Coefficient of Variation	Reference	COV Used (%)	Standard Deviation	Assumed Distribution
Asphalt concrete thickness T_{AC}	63.50	0.03 – 0.12	Timm,et al., (2000),	7	4.45	Normal
Granular base thickness T_{GB}	139.70	0.10 – 0.15	Noureldin et al., (1994)	12	16.76	Normal

Property	Mean (μ)	Coefficient of Variation	Reference	COV Used (%)	Standard Deviation	Assumed Distribution
Granular sub-base thickness T_{GSB}	71.12	0.10 – 0.20	Noureldin et al., (1994)	15	10.67	Normal
Granular Subgrade thickness T_{GS}	7620	-	-	-	-	Deterministic
Asphalt concrete modulus E_{AC}	6205.28	0.10 – 0.40	Timm,et al., (2000)	25	1551.32	Normal
Granular base modulus E_{GB}	620.53	0.5 – 0.60	Timm,et al., (1999)	30	186.16	Normal
Granular sub-base modulus E_{GSB}	310.26	0.10 – 0.30	Noureldin et al., (1994)	15	46.54	Normal
Granular Subgrade modulus E_{GS}	179.26	0.20 – 0.45	Noureldin et al., (1994)	35	62.74	Normal

Generation of the Random Variables

Generation of random numbers for modelling purposes is widely used and well known in the world of science (Dalla Valle, 2005). The random number generation tool in Easyfit 5.5 statistical computer package (2011) developed by Mathwave was used to generate 100,000 random numbers for each key input parameter in line with previous studies by (Dalla Valle and Thom, 2016). This simulation is only possible if the mean and coefficient of variation are known. Another factor of consideration is probability distribution; the normal distribution was used in the random number generation as have been used by several authors because of its relative simplicity.

Though the lognormal distribution is said to be best describe construction

variables (Babashamsi et al., 2016), a study by Li, (1997) as cited by Dalla Valle,(2005) revealed that difference between the results from assumed normal and lognormal distributions in simulation exercises is negligible. It is important to note that no two random number generation is the same, but they must statistically agree at all times. Thus, best fit models are obtained devoid of the assumed theoretical model used.

2.2 Distributions Fitting and Tests of Goodness-of-Fit (GOF)

In assigning of distribution models to each random variable, the Kolmogorov-Smirnov (K-S) test of goodness-of-fit was used. It was chosen because of its relative advantage. It is a very relevant test in risk analysis used to check how close the data

set distribution is to the hypothesized theoretical distribution (Babashamsi et al., 2016). The test involves a comparison between the experimental cumulative frequency and an assumed theoretical distribution. If a large difference exists, then the assumed distribution is rejected (Lytton and Zollinger, 1993).

Five distribution models usually encountered in engineering applications were tested in this study for goodness-of-fit for each of the considered key input parameters. The distribution models are; Normal, Lognormal, Gumbel (extreme type I), Weibull and Frechet. This was implemented using in Easyfit 5.5 statistical computer package (2011) developed by Mathwave.

RESULTS AND DISCUSSIONS

In this section, the data presented in Tables 1.0 and 2.0 for the pavement material properties in the previous section; asphalt concrete thickness, and base thickness, as well as asphalt concrete modulus, base modulus, and subbase modulus, and subgrade modulus were used as baseline data for the simulation and distribution fitting.

Statistical Characteristics of Layer thickness Variability

Pavement performance can vary significantly due to variability in layer thickness, which is mainly due to the construction process and quality control procedures put in place. Although a unique value has been assigned as design thickness, the actual (as-built) thickness is not constant (Dalla Valle and Thom, 2016). The pavement layer thickness is expected to have a certain probability distribution with a higher density around the target mean thickness.

Five commonly encountered distribution models in engineering pavement prediction; Normal, Lognormal, Gumbel (extreme type I), Weibull and Frechet were considered in this study and the results obtained are presented in the following section.

Asphalt Concrete Thickness T_{AC}

Having established control criteria for rutting and fatigue crack in pavement performance, and statistical parameter (mean, standard deviation and distribution model) of the asphalt concrete thickness, artificial data set were generated using easyfit statistical package. Table 3.0 shows the statistical parameters of the asphalt concrete thickness for all distribution models considered.

Table 3.0: Statistical parameters of asphalt concrete thickness

S/No	Distribution Name	Distribution Parameters
1	Frechet	$\mu=17.425$ $\sigma=61.294$
2	Gumbel Max	$\mu=3.4657$ $\sigma=61.515$
3	Lognormal	$\mu=0.07041$ $\sigma=4.1488$
5	Normal	$\mu=4.4449$ $\sigma=63.515$
5	Weibull	$\mu=17.852$ $\sigma=65.44$

Five distributions model were considered in the analysis; namely; Normal, Lognormal, Gumbel (extreme type I), Weibull and Frechet. The normal distribution is the reference model upon which all other distribution models are based. Distributions parameter for normal and lognormal are mean (μ) and standard

deviation (σ) while that for Frechet and Weibull are shape (α) and scale (β).

The Kolmogorov Smirnov test of goodness of fit was used to determine the suitable distribution model for asphalt concrete thickness. This was implemented using the Easyfit statistical package. The results of the goodness-of-fit are presented in Table 4.0.

Table 4.0: Result of Kolmogorov Smirnov Test of Goodness of Fit for Asphalt Concrete Thickness

S/No	Distribution Model	$\alpha = 0.2$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$	KS Statistics	Rank
		0.00339	0.00387	0.00429	0.0048	0.00515		
1	Normal	No	No	No	No	No	0.00204	1
2	Lognormal	Yes	Yes	Yes	Yes	Yes	0.01552	2
3	Weibull	Yes	Yes	Yes	Yes	Yes	0.0557	3
4	Gumbel Max	Yes	Yes	Yes	Yes	Yes	0.07243	4
5	Frechet	Yes	Yes	Yes	Yes	Yes	0.0857	5

The test was performed at 0.2, 0.1, 0.05, 0.02 and 0.01 level of significance (that is $\alpha = 0.2, 0.1, 0.05, 0.02$ and 0.01). A hypothesis was put forward that each of the five distribution models; Normal, Lognormal, Gumbel (extreme type I), Weibull and Frechet can be used to model asphalt concrete thickness.

From Table 4.0, the critical values for the Kolmogorov Smirnov test statistics for normal distribution at all the confidence levels considered were all greater than KS statistics obtained from the test. Therefore, the hypothesis that normal distribution can be used to model asphalt concrete thickness is accepted. Also, it is clear from the table that the critical values of KS at all confidence level is less KS statistics for the lognormal, Weibull, Gumbel max and Frechet. Therefore, these distributions models are not considered acceptable for

asphalt concrete thickness. Also considering the KS statistic for all the distribution models, the normal distribution has the lowest (ranked 1) which implies it is the best fit for asphalt concrete.

From the foregoing, it can be concluded that the probability distribution model for asphalt concrete thickness for Nigerian site considered is normal. This is in line with the findings of Timm et al., (1999); Timm et al., (2000); Dalla Valle and Thom, (2016); Retherford, and McDonald, (2010).

Granular Base Thickness T_{GB}

As explained in the previous section with the asphalt concrete, the same procedure was followed to arrive at similar results in the granular base layer. The statistical parameter (mean, standard deviation and distribution model) of the granular base is presented in Table 5.0.

Table 5.0: Statistical parameters of granular base thickness

S/No	Distribution Name	Distribution Parameters
1	Frechet	$\mu=9.9325$ $\sigma=130.83$
2	Gumbel Max	$\mu=13.081$ $\sigma=132.14$
3	Lognormal	$\mu=0.12225$ $\sigma=4.932$
4	Normal	$\mu=16.777$ $\sigma=139.69$
5	Weibull	$\mu=10.348$ $\sigma=146.61$

The Kolmogorov-Smirnov test of goodness of fit was used to determine the suitable distribution model for granular

base and the result is as presented in Table 6.0.

Table 6.0: Result of Kolmogorov Smirnov Test of Goodness of Fit for Granular Base Thickness

S/No	Distribution Model	$\alpha = 0.2$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$	KS Statistics	Rank
		0.00339	0.00387	0.00429	0.0048	0.00515		
1	Normal	No	No	No	No	No	0.00247	1
2	Lognormal	Yes	Yes	Yes	Yes	Yes	0.002489	2
3	Weibull	Yes	Yes	Yes	Yes	Yes	0.0464	3
4	Gumbel Max	Yes	Yes	Yes	Yes	Yes	0.07098	4
5	Frechet	Yes	Yes	Yes	Yes	Yes	0.0955	5

Table 6.0 above shows the result of the Kolmogorov-Smirnov test of goodness of fit for granular base thickness. The test was performed at 0.2, 0.1, 0.05, 0.02 and 0.01 level of significance (that is $\alpha = 0.2, 0.1, 0.05, 0.02$ and 0.01). A hypothesis was put forward that each of the five distribution models; normal, lognormal, Weibull, Gumbel max and Frechet can be used to model granular base thickness.

From the table, for normal and lognormal distributions, the critical values for the Kolmogorov Smirnov test statistics at all the confidence levels considered were all greater than KS statistics obtained from the test. Therefore, the hypothesis that normal and lognormal distribution can be used to model granular base thickness is accepted. Also, it is clear from the table that the critical values of KS at all confidence

level is less KS statistics for the Weibull, Gumbel max and Frechet. Therefore, these distributions models are not considered acceptable for asphalt concrete thickness.

Also considering the KS statistic for all the distribution models, the normal distribution has the lowest, followed by lognormal. This implies that the normal distribution model is the best fit for granular base thickness then followed by lognormal.

From the foregoing, it can be concluded that the probability distribution model for granular base thickness for Nigerian site considered is normal (ranked 1), then lognormal (ranked 2). This is in line with the findings of Timm et al., (1999); Timm et al., (2000); Dalla Valle and Thom, (2016); Retherford, and McDonald, (2010).

Granular Sub-base Thickness T_{GSB}

As explained in the previous section with the granular base, the procedure was followed to arrive at similar results in the granular sub-base layer. The

statistical parameter (mean, standard deviation and distribution model) of the granular sub-base is presented in Table 7.0 below.

Table 7.0: Statistical Parameters of Granular Sub-base Thickness

S/No	Distribution Name	Distribution Parameters
1	Frechet	$\mu=7.757$ $\sigma=65.249$
2	Gumbel Max	$\mu=8.3275$ $\sigma=66.31$
3	Lognormal	$\mu=0.15493$ $\sigma=4.2526$
4	Normal	$\mu=10.68$ $\sigma=71.117$
5	Weibull	$\mu=8.2038$ $\sigma=75.412$

The Kolmogorov Smirnov test of goodness of fit was used to determine the

suitable distribution model for granular base as presented in Table 8.0 below.

Table 8.0: Results of Kolmogorov-Smirnov Test of Goodness of Fit for Granular sub-base Thickness

S/No	Distribution Model	$\alpha = 0.2$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$	KS Statistics	Rank
		0.00339	0.00387	0.00429	0.0048	0.00515		
1	Normal	No	No	No	No	No	0.00238	1
2	Lognormal	Yes	Yes	Yes	Yes	Yes	0.03348	2
3	Weibull	Yes	Yes	Yes	Yes	Yes	0.03814	3
4	Gumbel Max	Yes	Yes	Yes	Yes	Yes	0.07272	4
5	Frechet	Yes	Yes	Yes	Yes	Yes	0.10431	5

The test was performed at 0.2, 0.1, 0.05, 0.02 and 0.01 level of significance (that is $\alpha = 0.2, 0.1, 0.05, 0.02$ and 0.01). A hypothesis was put forward that each of the five distribution models; normal, lognormal, Weibull, Gumbel max and Frechet can be used to model granular sub-base thickness.

From the table, for normal distribution, the critical values for the Kolmogorov Smirnov test statistics at all the confidence levels considered were all greater than KS statistics obtained from the test. Therefore, the hypothesis that normal distribution can be used to model sub-base

thickness is accepted. At the confidence level of 0.2, the Kolmogorov Smirnov test statistics for lognormal distribution is greater than that of KS statistic. Thus, the lognormal distribution is acceptable for granular sub-base, but only at a confidence level of 0.2.

Also, it is clear from the table that the critical values of KS at all confidence level is less KS statistics for the Weibull, Gumbel max and Frechet. Therefore, these distributions models are not considered acceptable for granular sub-base thickness. Also considering the KS statistic for all the

distribution models, the normal distribution has the lowest (ranked 1) which implies it is the best fit for asphalt concrete. From the foregoing, it can be concluded that the best probability distribution model for granular sub-base thickness for Nigerian site considered is normal.

Statistical Characteristics of Layer Stiffness

The stiffness of asphalt mixtures is of paramount importance in determining how well a pavement performs and is fundamental to the analysis of pavement response to traffic loading. Subgrade stiffness is another key input parameter to analytical pavement design. The scope of the design, for example in terms deformation life, is to ensure that there is only limited deformation of the

subgrade at the end of the design life due to stresses induced by traffic loads to the subgrade (through the road pavement). Subgrade stiffness affects not only the level of stresses in the subgrade but also the levels of stresses generated in all the overlying pavement layers (Dalla Valle and Thom, 2016).

Asphalt Concrete Modulus E_{AC}

The statistical parameter (mean, standard deviation and distribution model) of the asphalt concrete modulus is presented in Table 9.0.

The statistical parameters (mean, standard deviation and distribution model) of asphalt concrete modulus for all the five distribution models are as shown in Table 9.0.

Table 9.0: Statistical parameters of Asphalt Concrete Modulus

S/No	Distribution Name	Distribution Parameters
1	Frechet	$\mu=20895.0$ $\sigma=3.2341E+7$ $\gamma=-3.2335E+7$
2	Gumbel Max	$\mu=1204.8$ $\sigma=5513.2$
3	Lognormal	$\mu=0.04227$ $\sigma=10.51$ $\gamma=-30508.0$
4	Normal	$\mu=1545.2$ $\sigma=6208.7$
5	Weibull	$\mu=4.8526$ $\sigma=7402.9$ $\gamma=-586.46$

The Kolmogorov-Smirnov test of goodness of fit was used to determine the suitable distribution model for asphalt concrete modulus as presented in Table 10.0.

Table 10.0: Result of Kolmogorov Smirnov Test of Goodness of Fit for Asphalt concrete modulus

S/No	Distribution Model	$\alpha = 0.2$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$	KS Statistics	Rank
		0.00339	0.00387	0.00429	0.0048	0.00515		
1	Normal	No	No	No	No	No	0.00149	1
2	Lognormal	Yes	Yes	Yes	Yes	Yes	0.0097	2
3	Weibull	Yes	Yes	Yes	Yes	Yes	0.02182	3
4	Gumbel Max	Yes	Yes	Yes	Yes	Yes	0.07154	5
5	Frechet	Yes	Yes	Yes	Yes	Yes	0.06143	4

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The test was performed at 0.2, 0.1, 0.05, 0.02 and 0.01 level of significance (that is $\alpha = 0.2, 0.1, 0.05, 0.02$ and 0.01). A hypothesis was put forward that each of the five distribution models; normal, lognormal, Weibull, Gumbel max and Frechet can be used to model asphalt concrete modulus. From the table, for normal distribution, the critical values for the Kolmogorov Smirnov test statistics at all the confidence levels considered were all greater than KS statistics obtained from the test. Therefore, the hypothesis that normal distribution can be used to model asphalt concrete modulus is accepted.

Also, it is clear from the table that the critical values of KS at all confidence level is less KS statistics for the Lognormal, Weibull, Gumbel max and Frechet distributions. Therefore, these distributions

models are not considered acceptable for asphalt concrete modulus. Also considering the KS statistic for all the distribution models, the normal distribution has the lowest (ranked 1) which implies it is the best fit for asphalt concrete.

From the foregoing, it can be concluded that the best probability distribution model for asphalt concrete modulus for Nigerian site considered is normal. This is in line with the findings of Noureldin et al., (1994); Retherford, and McDonald, (2010).

Granular Base Modulus E_{GB}

The statistical parameter (mean, standard deviation and distribution model) of the granular base modulus is presented in Table 11.0.

Table 11.0: Statistical parameters of Granular base Modulus

S/No	Distribution Name	Distribution Parameters
1	Frechet	$\mu=3.5311$ $\sigma=709.31$ $\gamma=-211.48$
2	Gumbel Max	$\mu=145.73$ $\sigma=536.03$
3	Lognormal	$\mu=0.03949$ $\sigma=8.4652$ $\gamma=-4129.4$
4	Normal	$\mu=186.91$ $\sigma=620.15$
5	Weibull	$\mu=4.5057$ $\sigma=835.78$ $\gamma=-143.61$

The Kolmogorov Smirnov test of goodness of fit was used to determine the

suitable distribution model for granular base modulus as presented in Table 12.0.

Table 12.0: Result of Kolmogorov Smirnov Test of Goodness of Fit for Granular base Modulus

S/No	Distribution Model	$\alpha = 0.2$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$	KS Statistics	Rank
		0.00339	0.00387	0.00429	0.0048	0.00515		
1	Normal	No	No	No	No	No	0.00176	1
2	Lognormal	Yes	Yes	Yes	Yes	Yes	0.00632	2
3	Weibull	Yes	Yes	Yes	Yes	Yes	0.01869	3
4	Gumbel Max	Yes	Yes	Yes	Yes	Yes	0.07116	4
5	Frechet	Yes	Yes	Yes	Yes	Yes	0.12273	5

The test was performed at 0.2, 0.1, 0.05, 0.02 and 0.01 level of significance (that is $\alpha = 0.2, 0.1, 0.05, 0.02$ and 0.01). A hypothesis was put forward that each of the five distribution models; Normal, Lognormal, Weibull, Gumbel max and Frechet can be used to model granular base modulus. From the table, for normal distribution, the critical values for the Kolmogorov Smirnov test statistics at all the confidence levels considered were all greater than KS statistics obtained from the test. Therefore, the hypothesis that normal distribution can be used to model granular base modulus is accepted.

Also, it is clear from the table that the critical values of KS at all confidence level is less KS statistics for the Lognormal, Weibull, Gumbel max and Frechet

distributions. Therefore, these distributions models are not considered acceptable for asphalt concrete modulus. Also considering the KS statistic for all the distribution models, the normal distribution has the lowest (ranked 1) which implies it is the best fit for granular base modulus.

From the foregoing, it can be concluded that the best probability distribution model for granular base modulus for Nigerian site considered is normal. This is in line with the findings of Noureldin et al., (1994).

Granular Sub-base Modulus E_{GSB}

The statistical parameter (mean, standard deviation and distribution model) of the granular sub-base modulus is presented in Table 13.0 below.

Table 13.0: Statistical parameters of Granular Sub-base Modulus

S/No	Distribution Name	Distribution Parameters
1	Frechet	$\mu=7.6928$ $\sigma=284.32$
2	Gumbel Max	$\mu=36.489$ $\sigma=289.06$
3	Lognormal	$\mu=0.15597$ $\sigma=5.7251$
4	Normal	$\mu=46.799$ $\sigma=310.12$
5	Weibull	$\mu=8.1551$ $\sigma=328.94$

The Kolmogorov Smirnov test of goodness of fit was used to determine the suitable distribution model for granular

subbase modulus and the result is presented in Table 14.0 below.

Table 14.0: Result of Kolmogorov-Smirnov Test of Goodness of Fit for Granular Sub-base Modulus

S/No	Distribution Model	$\alpha = 0.2$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$	KS Statistics	Rank
		0.00339	0.00387	0.00429	0.0048	0.00515		
1	Normal	No	No	No	No	No	0.0233	1
2	Lognormal	Yes	Yes	Yes	Yes	Yes	0.0335	2
3	Weibull	Yes	Yes	Yes	Yes	Yes	0.03778	3
4	Gumbel Max	Yes	Yes	Yes	Yes	Yes	0.07246	4
5	Frechet	Yes	Yes	Yes	Yes	Yes	0.10439	5

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The test was performed at 0.2, 0.1, 0.05, 0.02 and 0.01 level of significance (that is $\alpha = 0.2, 0.1, 0.05, 0.02$ and 0.01). A hypothesis was put forward that each of the five distribution models; Normal, Lognormal, Weibull, Gumbel max and Frechet can be used to model granular sub-base modulus.

From the table, Kolmogorov Smirnov test statistics at all the confidence levels considered were all less than KS statistics obtained from the test for all the five distributions but a comparison of the KS statistics of all distribution models revealed that the normal distribution had the least KS statistics values, followed by lognormal, Weibull, Gumbel max and Frechet in ascending order. Thus, the best distribution

model for granular sub-base modulus is normal (ranked 1), followed by lognormal (ranked 2), then Weibull (ranked 3), Gumbel max (ranked 4) then finally Frechet (ranked 5) in descending order.

From the foregoing, it can be concluded that the best probability distribution model for granular sub-base modulus for Nigerian site considered is normal. This is in line with the findings of Noureldin et al., (1994)

Granular Subgrade Modulus E_{SG}

The statistical parameter (mean, standard deviation and distribution model) of the granular subgrade modulus is presented in Table 15.0 below.

Table 15.0: Statistical parameters of Granular Sub-base Modulus

S/No	Distribution Name	Distribution Parameters
1	Frechet	$\mu=3.4152$ $\sigma=235.4$ $\lambda=-97.839$
2	Gumbel Max	$\mu=48.908$ $\sigma=150.66$
3	Lognormal	$\mu=0.03724$ $\sigma=7.4266$ $\lambda=-1502.2$
4	Normal	$\mu=62.727$ $\sigma=178.89$
5	Weibull	$\mu=4.5814$ $\sigma=284.58$ $\lambda=-81.421$

The Kolmogorov Smirnov test of goodness of fit was used to determine the suitable distribution model for granular

subgrade and the result is presented in Table 16.0.

Table 16: Result of Kolmogorov-Smirnov Test of Goodness of Fit for Granular Subgrade Modulus

S/No	Distribution Model	$\alpha = 0.2$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$	KS Statistics	Rank
		0.00339	0.00387	0.00429	0.0048	0.00515		
1	Normal	No	No	No	No	No	0.00224	1
2	Lognormal	Yes	Yes	Yes	Yes	Yes	0.00646	2
3	Weibull	Yes	Yes	Yes	Yes	Yes	0.01963	3
4	Gumbel Max	Yes	Yes	Yes	Yes	Yes	0.07047	4
5	Frechet	Yes	Yes	Yes	Yes	Yes	0.12818	5

The test was performed at 0.2, 0.1, 0.05, 0.02 and 0.01 level of significance (that is $\alpha = 0.2, 0.1, 0.05, 0.02$ and 0.01). A hypothesis was put forward that each of the five distribution models; Normal, Lognormal, Weibull, Gumbel max and Frechet can be used to model granular subgrade modulus.

From the table, for normal distribution, the critical values for the Kolmogorov Smirnov test statistics at all the confidence levels considered were all greater than KS statistics obtained from the test. Therefore, the hypothesis that normal distribution can be used to model granular subgrade modulus is accepted. For the Lognormal, Weibull, Gumbel max and Frechet distributions, the Kolmogorov-Smirnov statistics at all confidence level was greater than that of the KS statistics leading to the rejection of these distributions models as the best fit.

A comparison of the KS statistics revealed that the normal distribution had the least values, followed by Lognormal, Weibull, Gumbel max and Frechet in ascending order. Thus, the best distribution model for granular sub-base modulus is normal (ranked 1), followed by lognormal (ranked 2), then Weibull (ranked 3), Gumbel max (ranked 4) then finally Frechet (ranked 5) in descending order.

From the foregoing, it can be concluded that the best probability distribution model for granular subgrade modulus for Nigerian site considered is normal. This is in line with the findings of Retherford and McDonald, (2010).

CONCLUSION

From the results of the study, the following conclusions can be made

- i. The critical design input parameter that has a great influence on the performance flexible pavement is; asphalt concrete, sub-base, and base thicknesses as well as modulus of asphalt concrete, granular base, granular sub-base and subgrade.

- ii. The distribution models that best fit the layer thicknesses, as well as their modulus (stiffness) for probabilistic assessment of pavement performance, are the normal distribution followed by the lognormal.

The findings of this study are valuable in providing designers and researchers with the best distribution models for flexible pavement components thickness and stiffness for purpose of probabilistic design and assessment of flexible pavement in Nigerian situations.

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