
ANALYSIS OF PROBLEM-SOLVING DIFFICULTIES WITH QUADRATIC EQUATIONS AMONG SENIOR SECONDARY SCHOOLS STUDENTS IN ZARIA, NIGERIA

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ABSTRACT

The study analyzes the problem-solving difficulties in quadratic equation among senior secondary school students in Zaria, Nigeria, using the Jackson-Ashmore model. A total of 126 Senior Secondary 2 mathematics students randomly selected from three private schools in Zaria with a mean age of 17 constituted the sample size for the study. The Mathematics achievement tests (MAT), Mathematics competence test (MCT) and Problem-solving Test in quadratic equation (PSTQ) were used for the study. The students were classified as high achievers and achievers using the categorization test score designed by the investigator. Data were analyzed using facility values (FV), mean, simple percentages (%) and chi-square statistics. The findings from the study showed that the students performed poorly in mathematical problems involving quadratic equations. The students had difficulties not only in defining a problem goal and recall of appropriate mathematical information, but also with the correct strategies required to reason through and solve the problem. On the basis of this study it is recommended among others that the teacher as facilitator of the teaching-learning process should encourage the learners to concentrate on one point at a time and proceed stepwise in a logical manner to reduce attendant difficulty and improve critical reasoning. Also, by giving learners enough opportunities to do problem-solving exercises in quadratic equation will go a long way in assisting them to clarify a problem goal.

INTRODUCTION

The ability to solve complex mathematical problems is regarded as one of the key competencies in science, technology and mathematics education (Onu, 2003). Until recently,

research on problem-solving technique focused on the relationship between analytical and complex problem solving (Scherer and Tiemann, 2012). Anything is more critical is the ability to inculcate in beginners the appreciation of basic mathematical

concept taking into account the structure of problem-solving process and metacognitive aspects. Li (2011) suggested that in order to gain insight to individual's problem solving technique, the investigator must find a method that would allow for an observable sequence of process to be examined. Jackson (1975) and Ashmore (1979) have formulated problem-solving models based on the instructional strategies of the teachers. The problem-solving models suggest that a problem solver must be able to:

- a. define a problem
- b. collect information related to the solution of the problem
- c. reason through the problem state to the solution state and
- d. Check and evaluate the solution to a problem.

A major advantage of problem-solving is that it bridges the gap between a problem and a solution using information and reasoning (Kotsopoulos, 2007). In addition, it allows the learner to think more critically (Zaslavsky, 1997). Unfortunately, the response to recognition of the importance of problem-solving experiences in teaching has not kept pace with expected advances in educational practice. Investigation into the problem-solving behaviour of students revealed that the difficulty students face in problem-solving depends on a

number of factors such as : failure to direct attention to appropriate information on problem statement, cognitive ability or recall of relevant mathematical knowledge from memory, students socio-economic background and outright lack of adequate knowledge as well as method of instruction used (Henbee,1992; Mittag & Taylor,2001; Omwihiren,2004; Zheug,2007;Picciott,2008 and Zakaria,2010). Jackson- Ashmore model is step by step problem solving strategies that is based on the instructional strategy of the teacher. The summary of the problems indicate that the problem solver must be logical in defining the problem goal to the evaluation to the solution of the problems.

It is in the light of the above and other difficulties faced by students that the present study was initiated to investigate the problem-solving strategies of students with a view of finding out the difficulties they encounter with problems involving quadratic equation.

PURPOSE OF THE STUDY

The main purpose of the study was to analyze the extent to which individual students' academic ability could be used to rationalize the difficulties students have with quadratic equation. Specifically, the study sought to:

1. Determine the extent to which individual student's ability affects their performance

- in solving problems involving quadratic equation.
2. Find out the difficulties encountered by students of different academic ability in solving problems involving quadratic equation.

RESEARCH QUESTION

The main research question posed for this study is to

what extent will individual students' problem solving ability influence their performance in problems involving quadratic equation?

HYPOTHESES

The following hypotheses were tested at 0.05 levels of significance.

1. There is no significant difference between students' problem solving ability and their performance in problems involving quadratic equation.
2. There is no significant difference in difficulties faced by students' of different academic ability in solving problems involving quadratic equation.

METHODOLOGY

RESEARCH DESIGN AND POPULATION

Descriptive survey design was adopted. The population for the study consists of all senior secondary schools in Zaria, Nigeria. Zaria and its environs is made up of over twenty senior secondary schools with a population of over seven thousand (7000) students. The government funded schools unlike the privately owned ones are non-coeducational. Apart from having qualified teachers, the students offer mathematics as a major subject. The private schools were therefore considered appropriate for the study and it is from this population that the study sample was drawn.

SAMPLE AND SAMPLING TECHNIQUE

The study was conducted using 126 senior secondary (SS2) students drawn from three private secondary schools in Zaria metropolis in Nigeria. Prior to the commencement of the study, a face and Pearson's product moment content validated pre-test MAT and MCT was used to assess the students' ability for the study under investigation. The students obtained 50% and above in both MAT and MCT. They were therefore considered to be students with sufficient basic knowledge required to solve problems involving quadratic equation. For the purpose of this study, they were classified as high achievers and achievers based on a categorized

scale designed by the investigator. 64 students were classified as high achievers while 62 students were classified as achievers. The mean age of the students was 17years.

INSTRUMENTATION

Three types of instruments applied for the research were: Mathematics Achievement Test (MAT), Mathematics Competence Test (MCT) and Problem-Solving Test in Quadratic equation (PSTQ) were used in the study:

The MAT was a 25 multiple-choice objective test in algebra used to test students' background knowledge in manipulating equation and other related topics. The MCT was also a multiple-choice objective test in mathematics topics related

to quadratic equation. The MAT and MCT were used to test students' competency in mathematics. The reliability coefficient as computed from Pearson's product moment correlation for MAT and MCT were $r= 0.90$ and $r= 0.95$ respectively. These values suggest that the test were reliable and as such would test what it was designed for. In addition, content and face validity of the instrument was carried by the investigator who is a specialist in mathematics and other specialists in cognate field compared to the overall objective of the curriculum. The primary objective of the MAT and MCT items was to categorize the students into high achievers (A) for those that scored 65% and above and achievers (B) for those that scored 50-64%. This is shown in Table I.

Table 1: Distribution of Students According to Different Achievement Levels

School	High Achievers (A)	Achievers (B)	Total
X	20	22	42
Y	20	19	39
Z	22	23	45
Total	62	64	126

The problem-solving test in quadratic equation (PSTQ) consisted of five numerical problems in quadratic equation designed to assess students' problem-solving strategies with the topics. The objective of the PSTQ is two-fold: first to

compare the students' ability with their overall performance and second to identify the type of difficulties encountered by the students when scored within the framework of the Jackson (1975) and Ashmore (1979) model.

DATA ANALYSIS

The data collected from the study were analyzed through statistical facility values (F.V)*, percentages (%) mean and chi-square.

***Facility Value** of an item is the proportion of the number of examinees who got the answers right in a group to the number of examinees who attempted the questions. It also refers to the percentage of

individuals who attempted an item correctly. Facility value of an item is a variant and not stable. It may vary from group to group for the same item (Haladyna & Downing, 1989). Facility value is actually decimal expression of the sentence which is used to express score of students for a research of this nation.

RESULTS AND DISCUSSION

Table 2: Distribution of Number of Attempted and Corrected Solutions of High Achievers (X) and Achievers (Y) in Quadratic equation

PSTM	High Achievers(A)(N=62)			Achievers(B)(N=64)		
	No of Students Attempt	Correct Answer	FV	No of Students Attempt	Correct Answer	FV
1. Problems involving factorization (with coefficient of $x^2 = 1$)	77	21	27.3	75	19	25.3
2. Problems involving factorization (with coefficient of $x^2 > 1$)	6	18	29.0	68	20	29.4
3. Problems involving factorization (with coefficient of $x^2 < 1$)	77	32	41.6	74	30	40.5
4. Problems involving completing the square method	62	21	33.9	58	17	29.3
5. Problems involving use of formula for quadratics	77	21	27.3	75	19	25.3
Total =	338	112	33.1	337	108	32.1

Table 3: Problem-Solving difficulties of High Achievers (A) and Achievers (B) in different stages of the model for quadratic equation.

5	PSTQ 1		PSTQ 2		PSTQ 3		PSTQ 4		PSTQ	
	A	B	A	B	A	B	A	B	A	B
	NS %	NS %	NS %	NS %	NS %	NS %	NS %	NS %	NS %	NS %
a. Difficulty in defining problems	10 16.6	8 14.0	9 16.7	8 15.4	10 17.9	8 15.1	12 18.8	11 18.3	9 15.0	10 16.1
b. Difficulty in selecting information from Data	12 19.7	11 19.2	10 18.5	10 19.2	7 12.5	7 13.2	10 15.6	11 18.3	10 16.7	10 16.1
c. Difficulty in selecting information from memory	11 18.0	10 17.5	8 14.8	8 15.4	9 16.1	8 15.1	12 18.8	10 16.7	12 20.0	11 17.7
d. Difficulty in reasoning	8 13.1	7 12.3	9 16.7	10 19.2	9 16.1	10 18.9	9 14.1	9 15.0	9 15.0	9 14.5
e. Errors in Computation	10 16.4	10 17.5	8 14.8	7 13.5	11 19.6	10 18.9	11 17.2	10 16.7	10 16.7	11 17.7
TOTAL =	61 100	57 100	54 100	52 100	56 100	53 100	64 100	60 100	60 100	60 100
A=High Achievers, B=Achievers NS=No of Scripts Total = No. that failed the item in Table 2										

The performance of the student in the problem-solving test in quadratic equation (PSTQ) is shown in table 2. Altogether, a total of 675 attempted solutions to the five problems were analyzed. These were made up of the 338 from high achievers(X) and 337 from achievers. The number correct solutions for each problem revealed that 33.1% of the attempted solutions of the high achievers(X) were correct while the achievers(Y) got 32.1% of the correct solutions. Thus a total of 220 correct solutions were obtained. The mean scores of the groups were calculated from the total

scores of correct solutions in table 2 and analysis from chi-square statistics ($\chi^2_{obs} = 0.046$) shows that there was no significant difference ($P < 0.05$) between the mean scores of students of different groups in quantitative problem-solving in quadratic equation. The result shows that solving problems in mathematics particularly those problems involving quadratic equation that require critical manipulation do pose some difficulty to students. The findings from present study tend to suggest that cognitive or academic ability have no direct influence on students' achievement with problems

that are content based such as the ones used in this study. Earlier investigators suggest that problem-solving requires the use of skills not related to those needed for cognitive test and cognitive style has more to do with the ways by which knowledge is acquired rather than the content or subject matter itself (Kotsopoulos, 2007; Simons, 2009; Sterling, 2010 and Henderson, 2013). The finding from the present study is consistent with previous study.

The distribution of problem-solving difficulties encountered by students expressed in percentages(Table 3) at the different stages of the model indicates that the high achievers(A) and achievers(B) encounters difficulty with problems in quadratic equation as a non-significant close correlation exist between groups A and B respectively.

The findings from the study suggest that:

- i. Students seem to lack focus on the goal of the problem. The implication of this is that they will not be in a position to direct their attention to the appropriate information provided from the problem statement or from memory. This is probably what has been responsible for the difficulty encountered in identifying all information provided in the problem. For example in PSTQ 4 students were told to solve the equation $x^2 + x - 4 = 0$ It was found out that

18.8% of X and Y had difficulty in defining the problem and so could neither select information from data nor from memory. Thus most of them started calculating impulsively without determining the coefficient of x^2 and x which is the requisite mathematical knowledge required to solve the problem by the completing the square method. Consequently, they failed to reason through the problem to the solution correctly. Jegede (2007) suggested that adopting the Ashmore 4 stage model of solving quadratic equations and continuous exposure to calculations will enhance students' problem solving skills in chemistry and reduce difficulty encountered. Katz and Barton (2006) encouraged group work as it enables the learner solve problems that they would be unlikely to solve alone. In addition it offers students an environment in which they can learn and be motivated to learn as much as they can.

- ii. The ability to recall the relevant mathematical knowledge in the form of writing formula for quadratic equation was a problem for the students. In PSTQ 2 which required the use of factors to

determine the roots of the quadratic with coefficient of x^2 greater than 1, the students found it difficult. About 14.8% of A and 15.4% of B had difficulties in recalling this information from memory. Studies conducted by Polya (1957) and Mayer (2003) noted that Learning how to solve problems in mathematics is to know what to look for. Mathematics problems often require established procedures and knowing what and when to apply them. Buchanan(1987) noted that in identifying procedures, a problem solver have to be familiar with the problem situation and be able to collect the appropriate information, identify a strategy or strategies and use the strategy appropriately. In this case, there was no observable difference in difficulties encountered by students of different academic ability in solving problems involving quadratic equations.

CONCLUSION AND RECOMMENDATION

The finding of this study revealed major difficulties encountered by students in solving problems involving quadratic equation. It further shows that generally, students irrespective of different cognitive ability, had difficulty in defining the problem goal and recalling appropriate

information from memory which are the key ingredients of the Jackson-Ashmore problem-solving model.

In view of the findings above, it is therefore recommended that:

1. Teacher as facilitator of the teaching-learning process should encourage the learners to concentrate on one point at a time and proceed stepwise in a logical manner to reduce attendant difficulty and improve critical reasoning.
2. Teacher should make problem-solving lessons exciting by encouraging group work with frequent board demonstrations so as to demystify the difficulty encountered in problems involving quadratic equations.
3. By giving learners enough opportunities to do problem-solving exercises in quadratic equation will go a long way in assisting them to clarify a problem goal and increasing their reasoning skills.
4. Memory aids containing key equations relating to quadratic equation problem topics should be provided by the school as part of instructional materials to reduce the difficulty encountered in solving quadratic equations.

5. This study should be repeated with other similar mathematically based concepts so as to compare findings.

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