



## Solution of an SIR Infectious Disease Model by Differential Transform Method

M. L. Olaosebikan<sup>1\*</sup>, A. A. Victor<sup>2</sup>, O. A. Uwaheren<sup>2</sup>, T. A. Ayoola<sup>1</sup>, M. O. Ajisope<sup>3</sup>

<sup>1</sup>Department of Mathematical Sciences Osun State University Osogbo, Osun State, Nigeria.

<sup>2</sup>Department of Mathematics, University of Ilorin, Kwara State, Nigeria

<sup>3</sup>Department of Mathematics, Federal University Oye-Ekiti, Oye-Ekiti, Nigeria

### ABSTRACT

A mathematical model on the transmission dynamics of infectious disease using the concept of differential equation was developed. Differential Transform Method (DTM) was employed to attempt the series solution of the model. The validity of the DTM in solving the model was established by classical fourth-order Runge-Kutta method implemented in Maple 18. The comparison between DTM solution and Runge-Kutta(RK4) were performed. The results obtained confirm the accuracy and potential of the DTM to cope with the analysis of modern epidemics.

### ARTICLE INFO

Article History

Received: June, 2020

Received in revised form: July, 2021

Accepted: October, 2021

Published online: November, 2021

### KEYWORDS

SIR Infectious Disease Model, Differential Transform Method

### INTRODUCTION

Infectious diseases are illnesses caused by organisms that are usually tiny in size, such as bacteria, viruses, fungi, or parasites, and are transmitted from one person to another, either directly or indirectly. People can also become infected after coming into contact with an infected animal that is carrying a pathogenic bacterium that can infect humans (Anderson and May 1991).

Infectious infections are a primary cause of death worldwide, especially in low-income nations and among children under the age of five. (Ghosh *et al.*, 2005, 2006; Naresh *et al.*, 2008, 2009; Peter *et al.*, 2018). The World Health Organization put two infectious diseases, lower respiratory infections and diarrheal diseases, in the top 10 causes of mortality globally in 2009 (WHO 2003). A multitude of pathogenic pathogens can cause both of these disorders. In recent years, deaths from the infectious diseases HIV/AIDS and tuberculosis have decreased considerably, and they no longer rank among the top ten causes of mortality worldwide. However, in low-income nations, these diseases remain a primary cause of death. Another infectious disease that is a leading cause of death in low-income nations is malaria. These three illnesses are caused by the same pathogenic agent. COVID-19, a newly discovered infectious disease caused by the virus SARS-CoV-2, became the

leading cause of death in 2020. COVID-19 was identified as the third top cause of death in the United States in 2020, according to data analyzed by the Centers for Disease Control and Prevention (CDC). COVID-19, a newly discovered infectious disease caused by the virus SARS-CoV-2, became the leading cause of death in 2020. COVID-19 was recognized as the third largest cause of death in the United States in 2020, behind heart disease and cancer, according to statistics analyzed by the Centers for Disease Control and Prevention (CDC). (Baba *et al.*, 2021, Abioye *et al.*, 2021, Peter *et al.*, 2021)

The aim of this paper is to present the application of Differential Transform Method to the proposed model and to verify the validity of the Differential Transform Method in solving the model using Maple 18's classical fourth-order Runge-Kutta method as a basis for comparison. We adopt the approach used by (Peter *et al.*, 2017, and Abioye *et al.*, 2018) for the solution of the model.

Differential Transform Method was first introduced (Zhou 1986) to study the initial value problems in electrical circuit for a computation in linear and non-linear system. Differential transform method (DTM) is proved to be an excellent tool to investigate analytical and numerical solutions of nonlinear ordinary differential equations. The concept of differential transformation is briefly presented and some well-

Corresponding author: Olaosebikan, M. L. ✉ [olaosebikanajani62@gmail.com](mailto:olaosebikanajani62@gmail.com) ✉ Department of Mathematical Sciences Osun State University, Oshogbo. © 2021, Faculty of Technology Education, ATBU Bauchi. All rights reserved

known properties of this DTM are rewritten in a more generalized forms. The Differential Transformation Method is one of the semi-analytical method commonly used for solving ordinary and partial differential equations in the forms of polynomials as approximations of exact solution. (Peter et al., 2017). Other semi-analytical methods includes: Homotopy Analysis Method (HAM), Homotopy Perturbation Method (HPM), Reduced Differential Transform Method (RDTM), Viarational Iterational Method (VIM), Parameter Expansion Method (PEM) .The methods mentioned above have been used as a tools to approximate linear and non-linear problems in Physics and Engineering.

The main advantages of DTM over other techniques are, its provides desired accuracy and reliability to the series solution with remarkable convergence rate, it requires less size computational work and yield high accuracy. ( Peter et al., 2018, Oyedepo et al., 2021)

### MODEL FORMULATION

The compartment used in this model consist of three classes, namely; Susceptible class  $S(t)$ : these are individuals who are susceptible to the disease or are prove to the disease. Infected  $I(t)$ : these are individuals who are infected with the disease and are capable of transmitting the disease to these in the susceptible class. Recovered class  $R(t)$ : these are who have recovered from the disease. Recruitment rate to susceptible class is either by immigration or by birth at a rate  $(\pi)$ . Susceptible individuals get infected through interaction with the infected individuals at a rate  $(\beta)$ ,  $\gamma$  represent rate of recovery from infection. In each of the classes, individual can die naturally at a rate  $(\mu)$ , the disease induced death rate is given as  $(\delta)$ , the above illustration can be represented by the system of differential-equations given below:

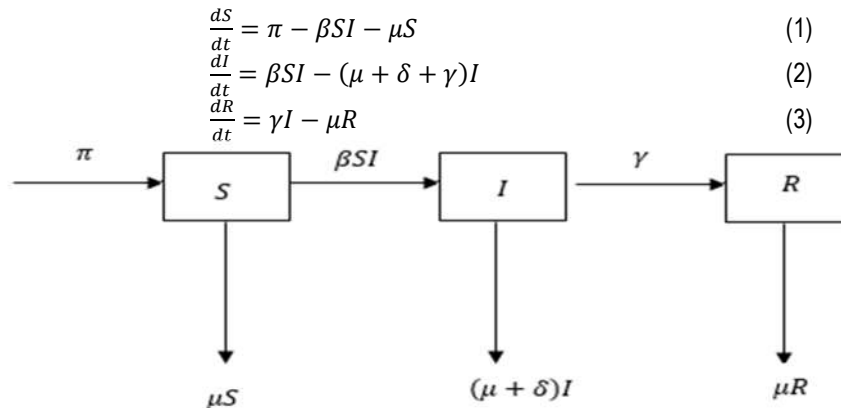


Figure 1: The model's flow diagram

Table 1: Description of Parameters/Variables

Variable	Description
$S(t)$	Susceptible class at time t
$I(t)$	Infected individual at time t
$R(t)$	Recovered individuals at time t
Parameter	Description
$\pi$	Recruitment rate
$\beta$	Force of infection
$\mu$	Natural death rate
$\delta$	Disease induced death rate
$\gamma$	Recovered rate



### DIFFERENTIAL TRANSFORM METHOD

The process involved in DTM is as follows: Given an arbitrary function of  $x$ , suppose

$y(x)$  is a non-linear function of  $x$ , then  $y(x)$  can be expanded in a Taylor series about a point  $x = 0$  as

$$y(x) = \sum_{k=0}^{\infty} x^k \frac{1}{k!} \left[ \frac{d^k}{dx^k} y(x) \right]_{x=0}$$

Thus, the differential Transform of  $y(x)$  is given as:

$$Y(k) = \frac{1}{k!} \left[ \frac{d^k}{dx^k} y(x) \right]_{x=0}$$

and the inverse differential Transform is given as

$$y(x) = \sum_{k=0}^{\infty} Y(k) x^k$$

Table 3 gives some operational properties of DTM. In the table,  $c(x)$  and  $d(x)$  are arbitrary functions with transforms  $C(k)$  and

$D(k)$  are the transformed functions respectively.

Table 2: Basic operation properties of the DTM

S/No	Original Function	Transformed Function
1	$y(x) = c(x) \pm d(x)$	$Y(k) = C(k) \pm D(k)$
2	$y(x) = \alpha c(x)$	$Y(k) = \alpha C(k)$ , $\alpha$ is a constant
3	$y(x) = \frac{dc(x)}{dx}$	$Y(k) = (k+1)C(k+1)$
4	$y(x) = \frac{d^2 c(x)}{dx^2}$	$Y(k) = (k+1)(k+2)C(k+2)$
5	$y(x) = \frac{d^n c(x)}{dx^n}$	$Y(k) = (k+1)(k+2) \cdots (k+n)C(k+n)$
6	$y(x) = 1$	$Y(k) = \delta(k)$
7	$y(x) = x$	$Y(k) = \delta(k-1)$ , $\delta$ is the Kronecker delta
8	$y(x) = e^{\lambda x}$	$Y(k) = \frac{\lambda^k}{k!}$
9	$y(x) = c(x)d(x)$	$Y(k) = \sum_{n=0}^k D(n)C(k-n)$
10	$y(x) = (1+x)^n$	$Y(k) = \frac{n(n-1)(n-2) \cdots (n-k+1)}{k!}$

#### Solution of the Model by DTM

In this section, we apply the steps involved in differential transform method to model (1) as follows: Using the operational properties (1),

(2) and (3) in Table 1 and applying them to the system of differential equations in (1) we obtain the following system of transformed equations

Corresponding author: Olaosebikan, M. L. ✉ [olaosebikanajani62@gmail.com](mailto:olaosebikanajani62@gmail.com) ✉ Department of Mathematical Sciences Osun State University, Oshogbo. © 2021, Faculty of Technology Education, ATBU Bauchi. All rights reserved



below,

$$S(k+1) = \frac{1}{k+1} [\pi b(k,0) - \sum_{l=0}^k S(l)(\beta I(k-l)) - \mu S(k)]$$

$$I(k+1) = \frac{1}{k+1} [\sum_{l=0}^k S(l)(\beta I(k-l)) - K_1 I(k)]$$

$$R(k+1) = \frac{1}{k+1} [\gamma I(k) - \mu R(k)]$$

Where:  $K_1 = (\mu + \delta + \gamma)$

Subject to the initial conditions  
 $S(0) = 60$ , Using the initial conditions and the

parameter values in the table and with the help of Maple 18, obtain the following expansion up to 5<sup>th</sup> order.

Hence,

$$S(t) = \sum_{n=0}^k S(k)t^k = 80 - 331.260t - 541.6405400t^2 + 2763.559466t^3 + 11147.24610t^4 - 21503.61468t^5 + \dots$$

$$I(t) = \sum_{n=0}^k I(k)t^k = 40 + 306.920t + 514.9785800t^2 - 2794.054479t^3 - 11016.93851t^4 + 21907.54068t^5 + \dots$$

$$R(t) = \sum_{n=0}^k R(k)t^k = 20 - 2.440t + 1.707840000t^2 + 1.635757507t^3 - 7.043205590t^4 - 21.83384998t^5 + \dots$$

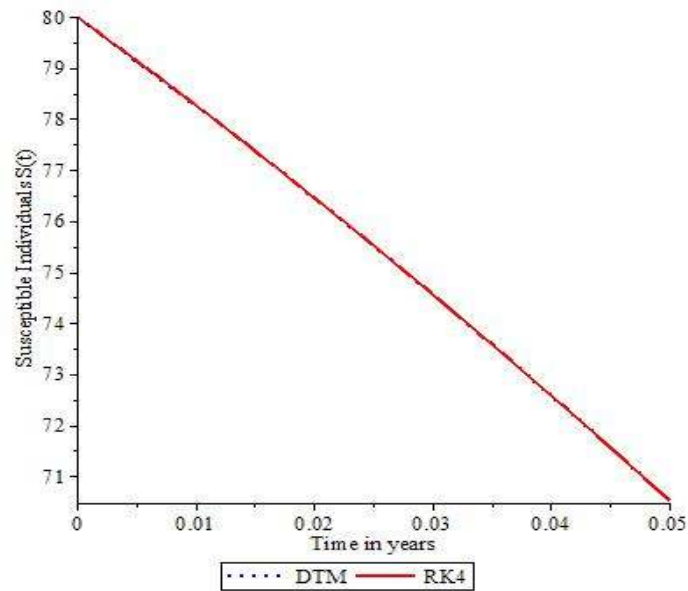
### NUMERICAL SIMULATION AND GRAPHICAL ILLUSTRATION OF MODEL

In this section, we present the numerical simulation which demonstrate the analytical results for the proposed model. This is achieved by using the set of parameter values given in the Table 2. We considered the following initial conditions for the different compartments.

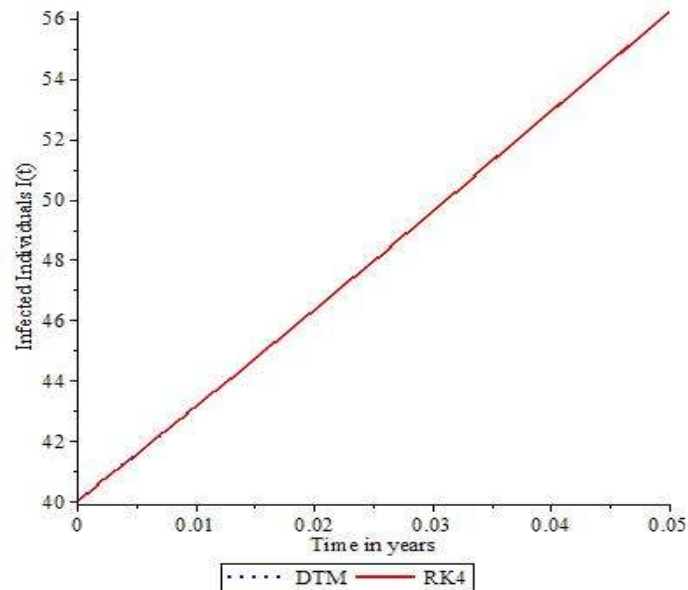
$S(0) = 80$ ,  $I(0) = 40$ ,  $R(0) = 20$ . The DTM is demonstrated against Maple 18's fourth order Runge-Kutta procedure for the solution of the model. Fig (1) to (3) shows the combined plots of the solutions of  $S(t)$ ,  $I(t)$  and  $R(t)$  by DTM and RK4

**Table 2:** Parameters values for model

Parameter	Value	Source
$\pi$	10	Lauria <i>et al.</i> ,(2009)
$\beta$	0.10	Peter, <i>et al.</i> ,(20017)
$\mu$	0.142	Mushayabasa, (2014)
$\delta$	0.75	Nthiiri, (2016)
$\gamma$	0.01	Mushayabasa, (2014)



**Fig1:** Solution of Susceptible Population by DTM and RK4



**Fig 2:** Solution of Infected Population by DTM and RK4

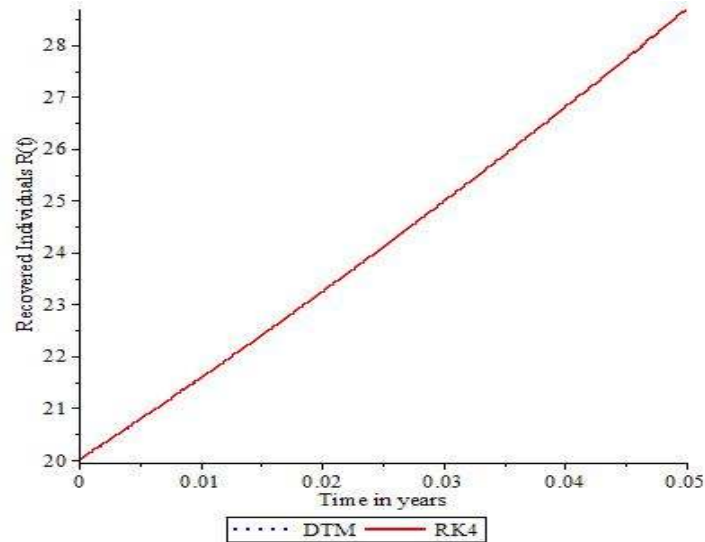


Fig 3: Solution of Recovered Population by DTM and RK4

#### DISCUSSION OF RESULTS FOR DIFFERENTIAL TRANSFORM METHOD

The solutions obtained by using Differential Transform Method with given initial conditions compared favourably with the solution obtained by using classical fourth-order Runge-Kutta method. The solutions of the two methods follows the same pattern and behaviour. This shows that Differential Transform Method is suitable and efficient to conduct the analysis of typhoid models.

#### CONCLUSION

We present a deterministic models on the transmission dynamics of infectious disease. We Differential Transform Method (DTM) is employed to attempt the series solution of the model. Numerical simulations were carried out to compare the results obtained by Differential Transform Method with the result of classical fourth-order Runge-Kutta method. The results of the simulations were displayed graphically. The results obtained confirm the accuracy and potential of the DTM to cope with the analysis of modern epidemics.

#### REFERENCES

Abioye A. I., Peter O. J., Ogunseye H. A., Oguntolu F. A., Oshinubi K., Ibrahim A. A., Khan I. (2021) *Mathematical*

*Model of COVID-19 in Nigeria with Optimal Control. Results in Physics.* 24, 104598.

<https://doi.org/10.1016/j.rinp.2021.104598>

Abioye, A. I., Ibrahim, M. O., Peter, O. J., Amadiogwu, S. and Oguntolu, F. A. (2018). *Differential Transform Method for Solving Mathematical Model of SEIR and SEI Spread of Malaria. International Journal of Science: Basic and Applied Research (IJSBAR)* 40(1):197-219

Adebisi A. F., Peter O. J., Ayoola T. A., Oguntolu F. A., Ishola C. Y. (2018). *Approximate Solution of Typhoid Fever Model by Variation Iteration Method ATBU, Journal of Science, Technology & Education (JOSTE);* 6 (3), 254- 265

Anderson, R. M. & May, R. M. (1991). *Infectious diseases of humans: Dynamics and control. Jama the Journal of the American Medical Association,* 268(23), 33–81.

Ayoade, A. A; Peter, O. J; Amadiogwu S; Abioye A. I; Victor A. A; Adebisi A. F. (2018). *Solution of Cholera Disease Model by Parameter Expansion Method. The Pacific Journal of Science and Technology.* 19(2): 36-50



- Baba I. A, Yusuf A., Nisar KS, Abdel-Atye AH, Nofal F. T. (2021) A Mathematical model to assess the imposition of lockdown during COVID-19 pandemic. *Results Phys.* 20:103716.
- Ghosh, M., P. Chandra, P. Sinha, and J. Shukla. (2005). "Modeling the Spread of Bacterial Disease: Effect of Service Providers from an Environmentally Degraded Region". *Appl. Math. Comp.* 160:615-647.
- Ghosh, M., P. Chandra, P. Sinha, and J. Shukla. (2006). "Modeling the Spread of Bacterial Infectious Disease with Environmental Effect in a Logistically Growing Human Population". *Nonlinear Analysis: RWA.* 7(3):341-363.
- Lauria, D. T., Maskery, T., Poulos, C., Whittington, D. (2009) An optimization model for reducing typhoid cases in developing countries without increasing public spending, *Vaccine, JVAC-8805* <http://dx.doi.org/10.1016/j.vaccine.2008.12.032>
- Mushayabasa, S, C. P. Bhunu, C. P & Ngarakana-Gwasira. E.T (2013) Mathematical analysis of a typhoid model with carriers, direct and indirect disease transmission. *IJMSEA*, 7(1): 79-90.
- Naresh, R., S. Pandey, and A. K. Misra. (2008). "Analysis of a Vaccination Model for Carrier Dependent Infectious Diseases with Environmental Effects". *Nonlinear Analysis: Modelling and Control.* 13(3): 331-350
- Nthiiri. J. K., Lawi G. O., Akinyi, C.O., Oganga, D. O., Muriuki, W. C Musyoka, M. J Otieno, P. O., L. Koech, L. (2016). Mathematical modelling of typhoid fever disease incorporating protection against infection. *British journal of mathematics and computer science*, 14(1): 1-10.
- Oyedepo, T., Adebisi, A.F. Tayo, R. M. Adedeji, J. A. Peter, O. J. (2021) Perturbed least squares technique for solving volterra fractional integro-differential equations based on constructed orthogonal polynomials. *Journal of Mathematical and Computer Science.* 11, 203- 218
- Peter, O. J., Shaikh A. S., Ibrahim M. O., Nisar K. S., Baleanu D., Khan I., and Abioye A. I.. (2021). Analysis and Dynamics of Fractional Order Mathematical Model of COVID-19 in Nigeria Using Atangana-Baleanu Operator. *Computers, Materials and Continua.* 66(2), 1823-1848
- Peter, O. J and Awoniran, A.F. (2018). Homotopy perturbation method for solving sir infectious disease model by incorporating vaccination. *Pac. J. Sci. Technol.* 19(1), 133-140
- Peter, O. J and Ibrahim, M. O. Application of Differential Transform Method in Solving a Typhoid Fever Model. *International Journal of Mathematical analysis and Optimization: Theory and Applications.* 1(1), 250-260. (2017).
- Peter, O. J., Abayomi, A. A., Adebisi, A. F., Ayoola, T. A. and Bitrus, S. (2018) Solutions of the SIRB Cholera Model Using Homotopy Analysis Method. *Futo Journal Series (FUTOJNLS)* 4(2), 168 – 176.
- Peter, O.J., O.B. Akinduko, F.A. Oguntolu, and C.Y. Ishola. (2018). "Mathematical Model for the Control of Infectious Disease". *J. Appl. Sci. Environ. Manage.* 22(4): 447 – 451.
- World Health Organization (WHO/V and B/03.7). (2003). Background document: The diagnosis, treatment and prevention of Typhoid fever. Assessed on 20/11/2016 from <http://www.who.int/iris/typhoid/media/world-typhoid-report-2016/en/>
- Zhou. J. K. "Differential Transformation and its Application for Electrical Circuits" Huazhong University Press China, (1986).