



## A Next Generation Matrix Method for the Basic Reproduction Number of Substance Abuse in Commercial Drivers' Population with Prosecution

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### ABSTRACT

This study focused on Next Generation Matrix Method for Reproductive numbers in modelling substance abuse. In this research work, a deterministic mathematical model of substance abuse using the Jailing approach is constructed from the compartmentalised model. Building on the insights from epidemiology, our model entails controlling the spread of abusers' ideologies in society. We introduce a simple compartmental model suitable to describe the abusers' group. The population is divided into five compartments, denoting the Susceptible, substance Users, Abusers, Prosecuted and Rehabilitated commercial drivers' populations respectively. The motive is basically to examine whether the substance abusers population will persist or on the other hand cease to exist. Following Generation Matrix method and algorithm for calculating the basic reproduction number were explicitly enumerated. Both the Next Generation Matrix and digraph reduction Method were evaluated and the results were found to be similar. These approaches could however also be practical for larger systems in the derivations for

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### INTRODUCTION

Generally, cases of driving under the influence of stimulants and charging substances have been directly associated with road carnage. Commercial vehicles could be deadly for both the driver and other occupants when driver is under the influence of some substance. The phenomenon of drug driving are usually kept in check by the advanced and developed nations. These are usually enforced by strict drug use evaluation and monitoring mechanisms. Unfortunately, number of developing nations have a very high number of drivers that keeps abusing several substances with impunity.

Joah *et al.* (2018) inferred that majority of commercial drivers in developing nations such as Nigeria use drugs as enhancing substance or stimulants to keep awake when driving long distances and relieve them of fatigue. The highest cases of road traffic incidences and accident caused deaths are mostly found in the African continent. There are laws against driving under

the influence of drugs, but the effectiveness and enforcement of the laws in combating this phenomenon are questionable.

A study by Carl *et al.* (2001) on Epidemic increases in Cocaine and Opiate use by trauma center patients revealed that marijuana is the common illegal substance found in impaired drivers, victims of crashed accidents and serious injured drivers. Moreover, some of other substances are Opium, Amphetamine, Cocaine, Alcohol, Ayahausca, Central Nervous System Depressants, DMT, GHB, Hallucinogens, Heroin, Inhalants, Netamine, Khat, Kratim, LSD, MDMA (Ecstasy/Mocky), Mescaline (Peyote), Methamphetamine, Over - The - Counter Medicines (Dextromethorphan, Loperamide), PCP, Prescription Opioids, Prescription Stimulants, Psilocybin, Rohypnol (Flunitrazepam), Salvia, Steroids (Anabolic), Synthetic Cathinones (Bat salts), Tobacco; NIDA(2013).

In Nigeria, the most common types of abused substances by local drivers as cited by



Haladu (2003) are categorized as follows:-

1. **Stimulants:** These are substances that directly act and stimulate the central nervous system. Users at the initial stage experience pleasant effects such as energy increase. The major source of these comes from caffeine substance.
2. **Hallucinogens:** These are drugs that alter the sensory processing unit in the brain. Thus, producing distorted perception, feeling of anxiety and euphoria, sadness and inner joy, they normally come from marijuana, LSD etc.
3. **Narcotics:** These drugs relieve pains, induce sleeping and they are addictive. They are found in Heroin, Codeine, Opium etc.
4. **Sedatives:** These drugs are among the most widely used and abused. This is largely due to the belief that they relieve stress and anxiety, and some of them induce sleep, ease tension, cause relaxation or help users to forget their problems. They are sourced from Diazepam, Alcohol, Promotazine, Chloroform.
5. **Miscellaneous:** This is a group of volatile solvents or inhalants that provide euphoria, emotional disinhibition and perpetual distortion of thought to the user. The main sources are glues, spot removers, tube repair, perfumes, chemicals etc.
6. **Tranquilizers:** They are believed to produce calmness without bringing drowsiness, they are chiefly derived from Librium, Diazepam etc.

#### Definition of Terms

**Mathematical Model:** Is a representation in mathematical terms of the behaviour of real devices and objects (Gutfraind,2009).

**Next Generation Matrix:** Is the method use to drive  $R_0$ , for a compartmental model of the spread of infectious diseases (Diekman *et al.* (1990) and Van den Driessche and Watmough, 2002).

**Homogeneous population:** Is the one where all individuals can be regarded as the same type in each compartment.

**Drug/Substance:** Any substance that enters the human body and can change either the function or structure of the human organism. This broad definition includes practically all foreign materials—even food, vitamins, plants, snake venom, air pollution, and pesticides.

**Substance Abuse:** (also refer to drug abuse)"substance abuse" in this work, will refer to the abuse of all the addictive drugs, dangerous drugs and abused chemicals. The term will be used interchangeably with the term "drug abuse," although it could be argued that these terms are slightly different in meaning. The terms "drug dependent," "dependence," and "addiction" may also be used interchangeably, although these terms do also have slightly different meanings. (UNSC, in its resolution 1566, 2004)

**Road Carnage:** The savage and excessive killing of many people via road.

**Withdrawal by Prosecution:** Symptoms that can occur after long-term use of a drug is reduced or stopped, perhaps through Jailing; these symptoms occur if tolerance to a substance has occurred, and vary according to substance. Withdrawal symptoms can include negative emotions such as stress, anxiety, or depression, as well as physical effects such as nausea, vomiting, muscle aches, and cramping, among others. Withdrawal symptoms often lead a person to use the substance again (National Institute of Drug Abuse,2018) .

**Jail:** A correctional institution used to detain persons who are in the lawful custody of the government(either accused persons awaiting trial or convicted persons serving a sentence) .

**Recovery:** A process of change through which commercial drivers with substance use disorders improve their health and wellness, live self-directed lives, and strive to reach their full potential.



## LITERATURE REVIEW

A study conducted by World Bank collection of development indicators placed smoking prevalence among adult male in Nigeria at 17.4% and alcohol consumption study conducted by the World Health Organisation (WHO) placed 56% of Nigerians over the age of 15 consume alcohol. The study also reveals that 50% of Nigerians in urban area consume alcohol.

A study of high schools in Northern Nigerian; Lucchetti et al. (2014), states that substance abuse ranged between 1.1 - 3.5% with a male to female ration of substance abuse of 3:1.3% smoked cigarettes, benzodiazepines abused by 3.5% and solvent by 1.5%. Abuse of cocaine and heroine was 1.1% and 1.3% respectively.

Ajibade et al. (2017) investigated the rationale and implications of psychoactive substances use among commercial road transport workers in Lokoja, Nigeria, using cross-sectional survey research design. Their study population comprised of trailer drivers, taxi drivers, tipper drivers and long distance bus drivers registered in Lokoja. However, due to variation in the population of each of the study groups, they allotted a quota of forty five (45) participants to each of the groups (totalling one hundred and eighty, 180) in order to prevent over-representation of one group over the other. Selection of respondents at garage/park level was however done using purposive sampling technique. Their data were collected using semi-structured questionnaire and statistically analysed using frequencies, percentage, and weighted means. Results of their study showed that majority of the respondents used psychoactive substances to overcome depression, stay awake while driving, overcome daily problems and for pleasure. These were however found to sometimes result to alteration in the moods and emotional state of the respondents, road traffic accidents, hangover and further involvement in substances usage. In view of these findings, they recommended that Government should intensify campaign through print and electronic media as well as by organising seminar and workshop for members of commercial road transport workers not only in the

study area but also in Nigeria generally, on the risks (social, psychological and health) of psychoactive substance usage.

Dahlgren M.K et al. (2020) observed that growing numbers of individuals are exploring the use of cannabis for medical or recreational purposes, and the proportion of cannabis-positive drivers involved in fatal crashes increased from 8% in 2013 to 17% in 2014, raising concerns about the impact of cannabis use on driving. Result also showed that cannabis users demonstrated impaired driving relative to HC participants with increased accidents, speed, and lateral movement, and reduced rule-following. Their work concluded that Chronic, heavy, recreational cannabis use was associated with worse driving performance in non-intoxicated drivers, and earlier onset of use was associated with greater impairment and that results may however be related to other factors associated with early exposure such as increased impulsivity.

According to Shuaib et al. (2016), they found that the recent rise in the number of road traffic accidents by a report of the Ghana National Road Safety Commission in 2012, calls for a review of drug driving. Their study was conducted to assess the use of drugs by commercial drivers in Ghana and to determine the social factors that contribute to drug driving. Drivers of different vehicles like cargo trucks and passenger buses were selected from commercial bus and cargo terminals and a total of 300 drivers were selected for their study. The determinant factor(s) were selected into the fitted logistic regression model, a five 5% statistical significance level required for a variable to stay in a model. Approximately, 34% of the respondents admitted using drug when driving and seventy 70% learned how to drive from unapproved driving institutions. They deduced that there are significant association between drug driving and Educational status, Distance traveled, Time (hours) used in driving by commercial drivers in Ghana.

The work by Xiaohan et al. (2019) inferred that drug courts solve problems, namely, reducing drug use and recidivism. Legal professionals and scholars have recognized that there are multiple internal and external constraints that could undermine the drug court operation. At



this point, the mechanism(s) by which threaten factors affect drug court program fidelity has been neither theoretically modeled nor empirically tested. They found that using a national sample and the Structural Regression Analysis (SRA), collaboration and judicial decision making are the most important factors for maintaining drug court program fidelity. This is because collaboration and judicial decision making can mediate the constraints and challenges from many quarters, including the lack of information sharing, evaluation and treatment.

Lasebikan *et al.* (2012) reported the widespread use of psychoactive substances in their study at a local government in a south-western state of Nigeria. They observed that new trend of local alcohol beverage generally called 'sepe' tended to have replaced older ones such as palm wine. All substances of abuse were freely available and openly displayed at motor parks except for cocaine and narcotics. There was poor law provision and enforcement of laws prohibiting sale and use around motor parks or while driving. Their study showed the feasibility and value of conducting rapid assessments among commercial vehicle drivers in Nigeria. One outcome of the study was the development of a guide on rapid assessment of alcohol and other substance use assessment and a measure of brief intervention among them. Presentation of these findings should contribute to increased awareness and improved response from the government.

Subasish *et al.* (2020) highlighted the landscape of marijuana regulation which is changing now rapidly. Several countries have already legalized marijuana in some form, and marijuana for recreational purposes will soon become legal for a quarter of the U.S. population. The potential increase in traffic fatalities due to impaired drivers remains a prominent debate among policymakers and in the media. One recent report stated that the fraction of fatal crashes in which at least one driver tested positive for marijuana has increased nationwide by an average of 10 percent from 2013 to 2016. In Louisiana, crashes involving marijuana have increased by 195% from 2010 to 2016. The study collected seven years of marijuana impaired crash data from Louisiana to identify the key association

factors and their patterns. Their research team identified the hidden association patterns of key attributes from the complex crash dataset using the cluster correspondence analysis and they found several key risk patterns including female drivers at the intersection involved in multiple vehicle right angle crashes, multiple vehicle head-on crashes at two lane roadways with no separation, multi-vehicle rear-end crashes at two-lane roadways with separation, careless single-vehicle crashes, single-vehicle crashes during dark with no streetlights, and open country interstate crashes. These findings may however be useful in the coordination of regional and local behavioral safety efforts to lessen the occurrence and injury level of marijuana impaired traffic crashes.

George and Patrick (2019) explained the role of physical and psychological health problems in drug use treatment process. Less worries and compliance with normal driving ethics should usually not necessitate addiction to a substance before every wheel drive.

A work by Umar *et al.* (2017) studied that the use of illicit substances among commercial bus drivers is not only increasing but strangely predisposing the drivers and their passengers to health hazards. There is however paucity of empirical data on the factors associated with this increase. Their study was conducted to explore the determinants of substance abuse among commercial bus drivers in Kano Metropolis, Kano state, Nigeria. Methodology: A descriptive cross sectional design was used for the study where by 196 respondents were selected through a multi-stage cluster sampling technique. Findings from the study showed that eight out of every ten 81.1% of the respondents has ever abused a substance. The desires to relax/sleep after a hard days job 84.8%, work hard 48%, relieve stress 81%, relieve anxiety 66.5% and pleasure 72% are the major factors associated with the abuse of substances by the respondents. The most commonly abused substances by the respondents include solution 93.3%, coffee 85.2%, Tramadol 80.6%, local stimulant tea (Gadagi) 78.1%, cola-nut 66.3% and tobacco 65%. It was deduced that the major determinants of substance abuse among

commercial bus drivers are the desires to work hard, relieve of stress and anxiety.

Research findings by Rebecca (2011) examined the social and historical significance of coerced drug treatment within the criminal justice system. Drug courts, the most prominent example of her approach, served as a case study to explore how seemingly contradictory perspectives on substance use—therapeutic and punitive—are merged to justify increased criminal justice oversight of defendants in the name of facilitating recovery. Drawing on an analysis of drug court organizational documents and interviews with key advocates, she examined the punitive, therapeutic, and medical knowledge drug court advocates draw on and construct to justify an increased role for the courts in solving the problem of addiction, and links these theories historically to broader discussions about the causes of crime and the courts' role in solving social problems. Overall, she established connection on how scientific theories are fused with moral considerations in the name of an “enlightened” criminal justice approach to complex social problems.

Although a sociological approach by Olaniyi D.T(2020) examined the relationships between substance abuse and criminal behaviours among the commercial motor drivers in Surulere LGA of Lagos State and relates it to national development. His findings of the two hypotheses tested revealed that there is no significant relationships between abuse of alcohol by the commercial vehicle drivers and their involvement in road accidents which may result to loss of lives of many that should contribute meaningfully to the national development, but there is significant relationship between abuse of marijuana by the commercial vehicle drivers and their involvement in traffic offence. The study then recommends among others that Lagos State government must place strict ban on the sales of psychoactive substances in public places especially in the public motor parks. Other works with modelling approach can be found in (Peter, et. al., 2019), (Peter, et. al., 2020), (Peter, O. J., Yusuf, A. et. al., 2021), (Peter, et. al., 2020), (Adebisi, et. al., 2018), other applications of numerical methods can be found in (Peter et. al.,

2021), (Ishola et. al., 2022), (Uwahren, 2021), (Abioye, et al., 2018), (Abioye, et al., 2020), (Ayoade et. al., 2020). (Oyedepo, et al., 2018) and (Oyedepo, et al., 2021).

This work serves as an extension and an improvement to the existing work by the inclusion of a prosecution compartment and thereby leading to jailing before the abuser gets rehabilitation. A different mathematical concept is also employed (see Fig. 2).

In the next section, the model of substance abuse as a disease will be developed and analyzed with the effect of prosecution facility in the compartment.

### METHODOLOGY

This work will divide the commercial drivers into five compartments depending on their substance use status, determine the common abused substances by the commercial drivers and their supposed assumption behind them, formulation of the mathematical model of substance abuse with control strategies, use the Next Generation Matrix and Differential Transform Method to solve the model.

The sum population of the entire drivers,  $N(t)$ , takes form;

$$N(t) = S(t) + D(t) + A(t) + J(t) + R(t)$$

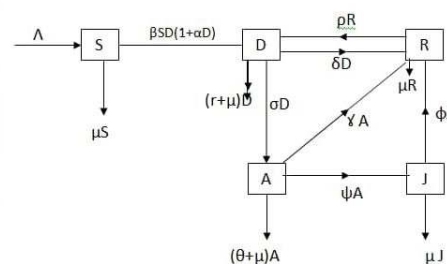


Figure 1: The Flow Diagram of Model 1

$$\frac{dS(t)}{dt} = \Lambda - \beta S(1 + \alpha D) - \mu S$$



$$\frac{dD(t)}{dt} = \beta S(1 + \alpha D) - (\sigma + \delta + r + \mu)D + \rho R$$

$$\frac{dA(t)}{dt} = \sigma D - (\theta + \psi + \gamma + \mu)A$$

$$\frac{dJ(t)}{dt} = \psi A - (\phi + \mu)J$$

$$\frac{dR(t)}{dt} = \gamma A + \delta D + \phi J - (\rho + \mu)R$$

### The Substance Invariant Region

The population size  $N$  can be determined by

The solution of the system remains positive at any point in time if the initial values of all the variables are positive.

#### Theorem 1

Consider  $\Omega = \{S(t), D(t), R(t), J(t), A(t) \in \mathbb{R}^5, S(0) > 0, D(0) > 0, R(0) > 0, J(0) > 0, A(0) > 0\}$ ,

then, the solution of  $\{S(t), D(t), R(t), J(t), A(t)\}$  are positive for  $t \geq 0$ .

#### Proof :

Boundedness refers to the region in which solutions of the model or system is uniformly bounded in the proper subset  $\Omega \subset \mathbb{R}_+^5$ .

Looking at the total population at any time (t):

$$N(t) = S(t) + D(t) + R(t) + J(t) + A(t),$$

$$\frac{dN(t)}{dt} = \Lambda - \mu S,$$

$$\begin{aligned} \frac{dN}{dt} &= \Lambda - \beta S(1 + \alpha D) - \mu S + \beta S(1 + \alpha D) - (\sigma + \delta + r + \mu)D + \rho R \\ &+ \sigma D - (\theta + \psi + \gamma + \mu)A + \psi A - (\phi + \mu)J + \gamma A + \delta D + \phi J - (\rho + \mu)R \\ &= \Lambda - \mu S - rD - \theta A - \mu A \mu J - \mu R \end{aligned}$$

$$\frac{dN(t)}{dt} = \Lambda - \mu S - (r + \mu)D - (\theta + \mu)A - \mu(J + R).$$

There exists no substance addiction, jailing and recovery in the absence of drug. Thus  $D = 0, J = 0, R = 0, A = 0$ .

The equation thus becomes

$$\frac{dN}{dt} = \Lambda - \mu S.$$

If the total population  $N$  is equal to the number of susceptible  $S$ , it implies that  $N = S$ , such that,

$$\frac{dN}{dt} = \Lambda - \mu N.$$

On integrating both sides of the equation

$$\int \frac{dN}{\Lambda - \mu N} \leq \int dt$$

$$-\frac{\ln(\Lambda - \mu N)}{\mu} \leq t + C$$

$$\ln(\Lambda - \mu N) \leq t + C$$

$$\ln(\Lambda - \mu N) \geq -\mu(t + C)$$

$$(\Lambda - \mu N) \geq e^{-\mu(t+C)}$$

$$(\Lambda - \mu N) \geq e^{-\mu t} + e^{-\mu C}$$

where  $e^{-\mu C} = K$ . Then  $\Lambda - \mu N \geq K e^{-\mu t}$ . Utilizing the condition at  $N(0) = S(0)$ ,  $\Lambda - \mu N(0) = K$ .

Therefore,  $\Lambda - \mu N = (\Lambda - \mu N(0))e^{-\mu(t)}$ . Further simplification yields

$N \leq \frac{\Lambda}{\mu} - \frac{(\Lambda - \mu N(0))}{\mu} e^{-\mu(t)}$ . As time tends to infinite, i.e.  $t \rightarrow \infty$ , then the side of population  $N \rightarrow \frac{\Lambda}{\mu}$ .

Deductively,  $0 \leq N \leq \frac{\Lambda}{\mu}$  and  $N(t) \leq \frac{\Lambda}{\mu}$ . Hence,

$$\Omega = \{(S, D, A, J, R) \in \mathbb{R}_+^5 : S + D + A + J + R \leq \frac{\Lambda}{\mu}\}.$$

### Existence and Positivity of Solution

The below results guaranteed by the substance abuse model governed in equation (1) is well posed in a feasible region  $\Omega$

**Lemma 1.2** Suppose the initial conditions be

$$\{S(0) > 0, D(0) > 0, A(0) > 0, J(0) > 0, R(0) > 0\} \in \Omega$$

Then the solution set  $\{S, D, A, J, R\}(t)$  of the model system is positive  $\forall t > 0$ , employing similar methods in (Gao and Hethcote, 2006)

**Proof:** The first equation of the model (1) gives

$$\frac{dS}{dt} = \Lambda - [\mu + \beta(1 + \alpha D)]S$$

$$\geq -[\mu + \beta(1 + \alpha D)]S$$

$$\frac{dS}{dt} \geq -[\mu + \beta(1 + \alpha D)]S$$

Applying the variable separable approach



$$\frac{dS}{S} \geq -[\mu + \beta(1 + \alpha D)]dt$$

Applying integrals to both sides

$$\int \frac{dS}{S} \geq - \int [\mu + \beta(1 + \alpha D)]dt$$

$$\ln S(t) \geq -[\mu + \beta(1 +$$

$\alpha D)]t + C$

$$S(t) \geq K \exp(-[\mu + \beta(1 + \alpha D)]t)$$

At time  $t = 0$  and applying the initial conditions yield

$$S(0) \geq 0$$

In a likewise manner

$$D(t), \quad A(t), \quad J(t) \text{ and } R(t) > 0 \quad \forall t \geq 0$$

### Basic Reproduction Number of Substance Abuse $R_0$

This is the average number of secondary cases per infectious case in a population made up of both susceptible and non-susceptible hosts. Here, we term it expected number of new substance abusers that arose by initial abuser in a susceptible population. Thus,  $R_0$  is seen as a Threshold term to determine the condition for abusers outbreak

### Substance Abuse Basic Reproduction Number

Here, we will use the concepts of Next Generation Matrix (Shaibu et al, 2018) to establish the linear stability of the disease-free equilibrium. The basic reproduction number is the number of secondary infections produced by one infected person in a completely susceptible population.

The reproduction number combines the biology of infections with the social and behavioral factors causing contact rates (Van den Driessche P., 2017).

The basic reproduction number  $R_0$  is defined as the spectral radius of the next generation matrix.

**Definition:** The spectral radius of a matrix  $X$  is defined as the maximum of the absolute values of the matrix

$$i.e. \rho(X) = \sup\{|\lambda|: \lambda \in \rho(X)\}$$

with  $\rho(X)$  representing the set of eigenvalues of the matrix  $X$ .

### Substance Abuse Basic Reproduction Number by Next Generation Matrix

**Theorem 1.2** Li (2002) stated that suppose  $F$  a non zero non negative matrix and  $V$  a non-singular  $M$ -matrix such that  $F - V$  is irreducible.

Then

$$R_0 = \rho(FV^{-1})$$

and  $R_0$  is the reciprocal of the smallest positive root  $x$  of the polynomial equation **Proof:** From the assumptions,

$$FV^{-1}$$

is a non-zero non-negative matrix, and by Li et al.(2002) and Perron-Frobenius theory the principal submatrix corresponding to the non-zero rows of  $F$  is irreducible.

Thus

$$\lambda = \rho(FV^{-1}) > 0$$

is the largest positive root of the polynomial equation

$$\det(\lambda I - FV^{-1}) = 0$$

Since  $V$  is non-singular and  $\lambda > 0$ , the polynomial equation is equivalent to

$$\det(F\lambda^{-1} - V) = 0$$

(where  $V - F\lambda^{-1}$  is a singular  $M$ -matrix) and  $\lambda^{-1}$  is the smallest positive root. The proof suffices from the definition of

$$R_0 = \rho(FV^{-1})$$

**Next Generation matrix:** This is the method used to derive the basic reproduction number, for a compartmental model of the spread of substance abuse.

Deriving  $R_0$  with the next generation matrix (Van den Driessche, 2017) is sometimes difficult whenever the dimension of the model is increasing. An option of solution to this problem is employing the graph-theoretic method (de-Camino-Beck et al, 2009) to derive the reproduction number  $R_0$  in a simpler form.

Computing  $R_0$ , we first assume  $f_i(x)$  the rate of appearance of new abusers in compartment  $i$ , and  $v_i(x)$  the net reducing rate of substance abusers in compartment  $i$  due to substance abusers flow inside system of abusers compartments.

Supposing



$F = (f_1, \dots, f_{2n})$  and  
 $V = (v_1, \dots, v_{2n})$ ,  
 Let partition the derivative  
 $TF(E_0)$   
 and  $TV(E_0)$  as

$$TF(E_0) = \begin{pmatrix} F & 0 \\ 0 & 0 \end{pmatrix}, TV(E_0) = \begin{pmatrix} V & 0 \\ J_3 & J_4 \end{pmatrix}$$

with

$$F = \begin{pmatrix} \frac{\partial F_1}{\partial D} & \frac{\partial F_1}{\partial A} & \frac{\partial F_1}{\partial J} \\ \frac{\partial F_2}{\partial D} & \frac{\partial F_2}{\partial A} & \frac{\partial F_2}{\partial J} \\ \frac{\partial F_3}{\partial D} & \frac{\partial F_3}{\partial A} & \frac{\partial F_3}{\partial J} \end{pmatrix} (x_0)$$

$$V = \begin{pmatrix} \frac{\partial V_1}{\partial D} & \frac{\partial V_1}{\partial A} & \frac{\partial V_1}{\partial J} \\ \frac{\partial V_2}{\partial D} & \frac{\partial V_2}{\partial A} & \frac{\partial V_2}{\partial J} \\ \frac{\partial V_3}{\partial D} & \frac{\partial V_3}{\partial A} & \frac{\partial V_3}{\partial J} \end{pmatrix} (x_0)$$

Suppose  $X = (S, D, A, J, R)^T$ , thus  $F$  and  $V$  can be derived from the model equation; also with its Jacobian matrices at substance-free equilibrium.

$$\text{Thus } R_0 = \Gamma(FV^{-1})$$

with  $\Gamma$ , the spectral radius

Using the Next Generation Matrix, we can derive the threshold value for substance abuse.

We recall that set of D.Es for Model 1 is;

$$\frac{dS}{dt} = \Lambda - \beta S(1 + \alpha D) - \mu S$$

$$\frac{dD}{dt} = \beta S(1 + \alpha D) - (\sigma + \delta + r + \mu)D + \rho R$$

$$\frac{dA}{dt} = \sigma D - (\theta + \Psi + \Upsilon + \mu)A$$

$$\frac{dJ}{dt} = \Psi A - (\phi + \mu)J$$

$$\frac{dR}{dt} = \Upsilon A + \delta D + \phi J - (\rho + \mu)R$$

Setting the Jacobian matrix of the system of differential equations, we have

$$F(t) = \begin{bmatrix} \beta S(1 + \alpha) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$V(t) = \begin{bmatrix} (r + \mu + \sigma + \delta) & 0 & 0 \\ -\sigma & (\gamma + \psi + \phi + \mu) & 0 \\ 0 & -\psi & (\theta + \mu) \end{bmatrix}$$

$$F(t) = \begin{bmatrix} \beta S(1 + \alpha) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

suppose  $Z = \frac{1}{v}$  i.e  $V - 1$ , thus

$$Z = \begin{bmatrix} (r + \mu + \sigma + \delta)^{-1} & 0 & 0 \\ \frac{\sigma}{(r + \mu + \sigma + \delta)(r + \psi + \phi + \mu)} & (r + \psi + \phi + \mu)^{-1} & 0 \\ \frac{\sigma \psi}{(r + \mu + \sigma + \delta)(r + \psi + \phi + \mu)(\theta + \mu)} & \frac{\psi}{(r + \psi + \phi + \mu)(\theta + \mu)} & (\theta + \mu)^{-1} \end{bmatrix} \quad (3.1)$$

$$F \cdot V^{-1} = \begin{bmatrix} \frac{\beta S(1 + \alpha)}{r + \mu + \sigma + \delta} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_0 = \Gamma(F \cdot V^{-1})$$

$$R_0 = \frac{\beta S(1 + \alpha)}{r + \mu + \sigma + \delta} \quad (3.2)$$

Thus;

$$R_0 = \Gamma(F \cdot V^{-1}) = \frac{\beta S(1 + \alpha)}{r + \mu + \sigma + \delta}$$

We deduce that; the substance-free equilibrium  $E_0 \left( \frac{\Lambda}{\mu}, 0, 0, 0, 0 \right)$  associated with the mode set of D.Es is locally asymptotically stable if

$$R_0 < 1$$

i.e. substance abusers get apprehended to prosecution or rehabilitation.

$E_0$  becomes unstable iff

$$R_0 > 1$$

i.e. the act of abuse of substance is prevalent.

### Reduction on Digraph associated with

$$F\lambda^{-1} - V$$

A directed acyclic graph is a directed graph with no 1 directed cycles. i.e. it consists of vertices and edges, with each edge directed from one vertex to another, such that following those directions will never form a closed loop.



A graph is a DAG iff it can be topologically ordered, by arranging the vertices as a linear ordering that is consistent with all edge directions.

Example of a DAG

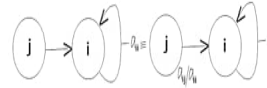


Figure 3: Eliminating of arcs through a trivial node

According to Tomas de Comino et al(2009)

**Rule 1** (Creating a trivial mode): To reduce the loop

$$-a_{ii} < 0 \text{ to } -1$$

at mode  $i$ , every arc entering  $i$  has weighted divided by  $a_{ii}$  (see diagram fig 1)

Keep in mind that; there may be a loop at  $j$  in diagram fig \* and that after reduction mode  $i$  has a loop with weight  $-1$ . This operation is equivalent to dividing row  $i$  of  $\det(F\lambda^{-1} - V)$  by  $a_{ii} > 0$ , which multiplies the value of the determinant by  $\frac{1}{a_{ii}}$ .

**Rule 2: (Eliminating of arcs through a trivial mode)**

For a trivial node  $i$  on a path

$$j \rightarrow i \rightarrow k$$

the two arcs are replaced by  $j \rightarrow k$  with weight equal to the product of weights on arc  $j \rightarrow i$  and  $i \rightarrow k$ . Weights on multiple arcs  $j \rightarrow k$  are added. If there are no more paths through the trivial mode  $i$ , then it can be overlooked. See Fig 2 where node  $i$  is not shown in the third digraph.

Here, if  $j$  is equal to  $k$ , then weights are added to the loop. This operation is equivalent to the elementary Gaussian elimination that adds  $a_{ki}$  times row  $i$  and  $k$ , thus reducing the  $(k, i)$  entry to 0. This operations leaves the determinant unchanged.

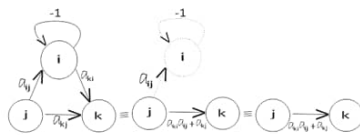


Figure 2: Creating a trivial node

**Computing Reproduction number  $R_0$  from the digraph associated with  $F\lambda^{-1} - V$ ; Algorithm**

Employing the Theorem in  $R_0$  and the two digraph reduction rules, we now give a symbolic computation method to calculate  $R_0$ .

For a connected dynamical system, consider the matrix

$$F\lambda^{-1} - V.$$

Suppose this matrix is irreducible, then draw the associated digraph with arc  $j \rightarrow i$  iff the  $(i, j)$  entry of the matrix is non-zero. If  $j = i$  and the  $(i, i)$  is  $-1$ , then  $i$  is assumed a trivial node.

1. In the digraph choose a node  $i$  with a loop.
2. Rule 1 to make node  $i$  trivial.
3. Recalculate all paths going through node  $i$  using Rule 2. Node  $i$  can then be disregarded.
4. Further steps 1,2,3 until just one node remains
5. Set the weight of this loop to zero giving an equation for  $\lambda$ . Then by the Theorem,  $R_0$  is the reciprocal of the smallest positive root of this equation.

**Ps:** In implementation nodes other than  $i$  can also be made trivial at step 2, but node  $i$  must be trivial before step 3 is performed. This algorithm is equivalent to symbolic Gaussian elimination on

$$\det(F\lambda^{-1} - V) = 0$$

if the digraph is not strongly connected, then the algorithm can be applied to each strongly connected component.

Applying the step of the algorithm and thus solving the system of the model D.E as that of next generation matrix, yields



$$F\lambda^{-1} = \begin{bmatrix} \beta\lambda^{-1}S(1 + \alpha) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and}$$

$$V = \begin{bmatrix} (r + \mu + \sigma + \delta) & 0 & 0 \\ -\sigma & (r + \Psi + \phi + \mu) & 0 \\ 0 & -\Psi & (\theta + \mu) \end{bmatrix} (x_0)$$

Thus  $F\lambda^{-1} - V$  becomes

$$\begin{bmatrix} a\lambda^{-1} - b & 0 & 0 \\ \sigma & -c & 0 \\ 0 & \Psi & -d \end{bmatrix}$$

## RESULTS AND DISCUSSION

This research centres on a new approach of deriving the basic reproduction number  $R_0$ , known as the graph-theoretic method for modelling substance abuse. This approach is equivalent to Gaussian elimination and since the rules of digraph reduction method does not take the path of direct calculation of matrix inverse or determinants, this approach can be practical for larger systems.

## CONCLUSION

In this research, the next generation matrix method was employed to compute the threshold number for the use and abuse of substance,  $R_0$ . The use and abuse of substances would continue to spread among commercial drivers if the substance abuse number is less than one and the abuse of substances would die out of the system if the abuse number is greater than one (1).

The result of this approach was employed to compare with that of graph reduction rules. We infer that there is no significant difference in either results.

In a more developing studies, an optimal control approach to the spread of substance abusers population is examined using apprehension, prosecution and/or jailing facilities as a strategy for control.

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