



Active Suspension Control System Using H-Infinity Mixed Sensitivity Technique With Automatic Weights Selection

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ABSTRACT

Weight selection is critical in the determination of the sensitivity function (which defines performance in terms of disturbance rejection) and the complementary sensitivity function (which specifies the set-point tracking of the system). The selection of weight in conventional H-infinity control is rigorous and requires high designer's experience. This paper presents the development of a low cost and efficient robust controller for the active suspension system using H-infinity (H_∞) mixed sensitivity with automatic weights selection. In this work, a higher level designer's specification for the active suspension of a half car is mapped into a multi-objective optimization problem; the Multi-Objective Particle Swarm Optimization (MOPSO) algorithm is deployed to solve the well-posed control optimization problem in order to achieve the stated design specification. The formulation of the control problem involves the use of weighing functions with tunable parameters. Mixed sensitivity technique is used to calculate the controller gain such that it stabilizes the system and minimizes the infinite norm of the system. Analyses of the robust stability and robust performance were carried out after simulation and the results indicate a good robust stability and performance. For the ride comfort, the developed controller outperforms conventional H-infinity using Algebraic Riccati Equation (ARE) and H_2 controller using LMI in terms of the rise time and settling time. The computation time (a reflection of cost of design) is reduced by reduction of order of the controller which justifies the ease of its implementation.

ARTICLE INFO

Article History

Received: January, 2022

Received in revised form: February, 2022

Accepted: February, 2022

Published online: June, 2022

KEYWORDS

Active Suspension, Mixed Sensitivity Technique, H-Infinity, Weight Selection, Controller order reduction

INTRODUCTION

Suspension systems are the systems that provide relative motion between wheels and the vehicle. These systems are basically combinations of mass, springs and dampers. The performance specifications of the system are determined by the spring constants and the damping coefficients. Suspension systems are categorized into passive suspension systems and active suspension systems. The system in which the spring constant and the damping coefficient are constant with respect to certain design specifications is called passive suspension system. They can only give good comfort and road holding at certain speeds and frequencies [1]. In

reality, variation in road profile and drivers' behaviour cannot be contained in a passive suspension system. Thus, this drives the need for an active suspension system in which the spring and damping coefficients varies in real time depending on the road profile and drivers' condition. Active suspension systems make use of actuators in the system that apply external force to the system. Though active suspension systems have addressed the challenges of passive system effectively, it suffers certain problems ranging from inability to handle uncertain dynamics and its responsiveness to disturbances if not properly controlled (especially using classical controllers).

Therefore, this necessitate the use of robust controllers.

Robust controllers are concerned with the control of uncertain plants having uncertain dynamics which are subjected to disturbances [2-3]. Even in uncertain environments, these controllers seek to achieve desired performance [4]. Over the years, many research works have explored robust controllers among which is H infinity control. One of the reasons for its wide application is that it allows the performance specifications to be expressed in terms of weighing functions. The rigour involves in the conventional trial-and-error approach of tuning the weighting functions makes the H infinity control inferior to other controllers. Hence, there is need to encapsulate automatic selection in designs in order to reduce computation time while meeting some high-level design specifications. Furthermore, another worrying challenge to researchers is the high cost of implementation of H infinity controllers. This calls for the need to design a cost effective controller.

Therefore, this paper presents the development of a low cost and efficient robust controller for the active suspension system using H-infinity (H_∞) mixed sensitivity with automatic weights selection.

The remainder of this paper is organized as follows: Section 2 briefly presents an overview of optimization algorithms and opposition-based

learning application. Section 3 explains the standard simulated Kalman filter algorithm, the concept of opposition-based learning and the proposed enhance version of SKF. Section 4 provides the experimental settings and discusses the experimental results. Section 5 concludes the paper.

REVIEW OF LITERATURE

In this section, the fundamental theories that are pivotal to this work are defined and the contributions of some authors in the subject of active suspension system & robust H_∞ controllers are discussed.

Definitions of Fundamental Concepts

Half car model

The model of the half car is adopted from the work of Suzuki *et al* in [5] as depicted in Figure 1. The half car model is made up of two different configurations of mass, spring and dampers. The physical constants having suffix f denote the front, whilst those with r denote the rear side of the car. θ represents the car body pitch angle.

The use of newton's equations of dynamic bodies was used to obtain the state equations and the output vectors of the system in [5].

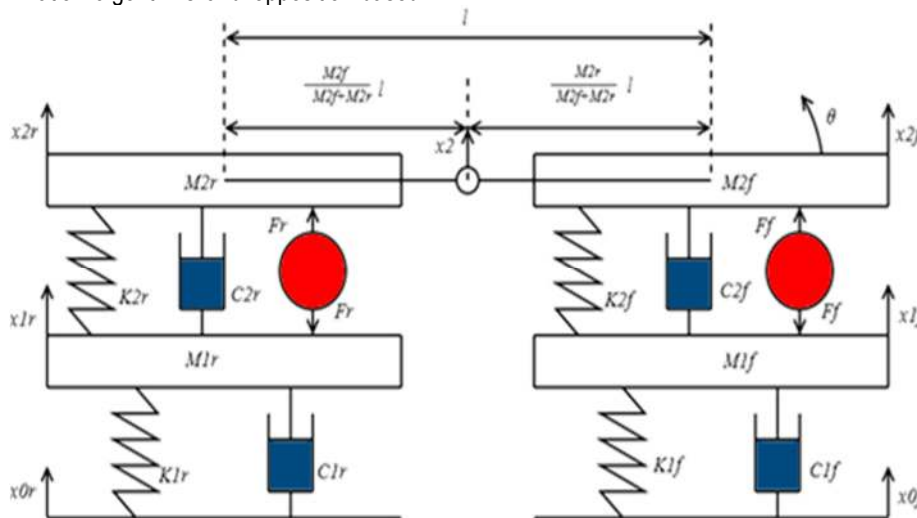


Figure 1: Half Car Model Diagram [5]

The state space representation given by Suzuki *et al* in [18]:

$$\dot{x}(t) = Ax(t) + B_1w(t) + B_2u(t)$$

$$z(t) = Cx(t) + Du(t)$$

$$(1) \quad A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ \frac{K_{1f}}{M_{1f}} & 0 & \frac{K_{2f}}{M_{1f}} & 0 & -\frac{C_{1f} + C_{2f}}{M_{1f}} & 0 & \frac{C_{2f}}{M_{1f}} & 0 \\ 0 & -\frac{K_{1r}}{M_{1r}} & 0 & -\frac{K_{2r}}{M_{1r}} & 0 & -\frac{C_{1r} + C_{2r}}{M_{1r}} & 0 & \frac{C_{2r}}{M_{1r}} \\ 0 & 0 & -\frac{K_{2f}}{M_{2f}} & 0 & \frac{C_{2f}}{M_{2f}} & 0 & -\frac{C_{2f}}{M_{2f}} & 0 \\ 0 & 0 & 0 & -\frac{K_{2r}}{M_{2r}} & 0 & \frac{C_{2r}}{M_{2r}} & 0 & -\frac{C_{2r}}{M_{2r}} \end{bmatrix} \quad (2)$$

$$(3) \quad B_1 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 0 & 0 \\ 0 & 0 \\ \frac{C_{1f}}{M_{1f}} & 0 \\ 0 & \frac{C_{1r}}{M_{1r}} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{1}{M_{1f}} & 0 \\ 0 & -\frac{1}{M_{1r}} \\ \frac{1}{M_{2f}} & 0 \\ 0 & \frac{1}{M_{2r}} \end{bmatrix}$$

$$(4) \quad C = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ -\frac{K_{1f}}{M_{1f}} & 0 & \frac{K_{2f}}{M_{1f}} & 0 & -\frac{C_{1f} + C_{2f}}{M_{1f}} & 0 & \frac{C_{2f}}{M_{1f}} & 0 \\ 0 & -\frac{K_{1r}}{M_{1r}} & 0 & -\frac{K_{2r}}{M_{1r}} & 0 & -\frac{C_{1r} + C_{2r}}{M_{1r}} & 0 & \frac{C_{2r}}{M_{1r}} \\ 0 & 0 & -\frac{K_{2f}}{M_{2f}} & 0 & \frac{C_{2f}}{M_{2f}} & 0 & -\frac{C_{2f}}{M_{2f}} & 0 \end{bmatrix}$$



$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -\frac{1}{M_{1f}} & 0 \\ 0 & -\frac{1}{M_{1r}} \\ \frac{1}{M_{2f}} & 0 \\ 0 & \frac{1}{M_{2r}} \end{bmatrix} \quad (5)$$

Robust Control

In robust control, the general problem is to determine a control law that will maintain the output constrained within the specified bounds regardless of the uncertainty [6]. Robust controllers stem from the limitations of optimal controllers to handle uncertainty.

Before the advent of robust controllers, optimal controllers appeared to meet performance specifications while ensuring system stability but they cannot provide guaranteed stability margins [7-8]. This led to the development of robust controllers that guarantee stability in system uncertainty. The comparison of robust and optimal controllers is as shown in Table 1:

Table 1: Comparison of Controllers [9]

Controller Synthesis	Merits	Demerits
H _∞ Synthesis	1. Concerned with system sensitivity	1. Higher mathematical understanding needed
H ₂ Synthesis	2. Direct controller synthesis 1. Concerned with system stability	1. A large number of iterations needed
LQG Synthesis	2. Direct controller synthesis 1. Utilizes noise information;	1. Stability not guaranteed; 2. Exact system model required 3. Large number of iterations
LQR Synthesis	1. Guaranteed robust stability,	1. Feedback is required of state vector 2. Requires an exact system model 3. Large number of iterations needed

Robust control in a system is achieved by minimizing the effect of uncertainty on a system within its specified frequency range [4]. Uncertainty can be model uncertainty, external disturbances and measurement noise [10]. The

general robust control two-port representation is discussed in [11].

For a given system, if a controller can be calculated such that it can stabilize a class of uncertain systems within all possible variations under the effect of uncertainty on the nominal



model, then that system is said to have robust stability [4].

Hence robust stability is therefore stated as: "A system is robustly stable with the controller $K(s)$ if and only if the complementary sensitivity function $T(s)$ for the nominal plant satisfy the condition." [6]

$$\begin{aligned} \|K(I + GK)^{-1} \Delta\|_{\infty} &< 1 \\ \|K(I + GK)^{-1}\|_{\infty} &< \frac{1}{\|\Delta\|_{\infty}} \end{aligned} \quad (7)$$

Hence, from equation (7), an optimization problem can be articulated for robust stability, that is to calculate a controller that will minimize

$$\|K(1 + GK)^{-1}\|_{\infty} \text{ expressed as:}$$

$$\min_{K \text{ stabilizing}} \|K(1 + GK)^{-1}\|_{\infty} \quad (8)$$

For a given system, if a controller can be calculated such that it will satisfy the performance

- i. **To achieve set point tracking** $\|(1 + GK)^{-1}\|_{\infty}$
- ii. To achieve disturbance rejection $\|GK(1 + GK)^{-1}\|_{\infty}$
- iii. To achieve noise rejection $\|-(1 + GK)^{-1} GK\|_{\infty}$
- iv. To minimize control effort $\|K(1 + GK)^{-1}\|_{\infty}$

Hence, the mathematical optimization problem is expressed as:

$$\min_{K \text{ stabilizing}} \|SW\|_{\infty} \quad (9)$$

Where W is a weighing function added for the system to achieve the desired specified specifications.

H^{∞} Control

H^{∞} control involves the design of controllers to achieve robust stabilization with guaranteed performance margin as opposed to linear quadratic Gaussian (LQG) controllers [12]. The

$$\|T(s)\ell_m(\omega)\|_{\infty} = \sup_{\omega} \|T(s)\ell_m(\omega)\| < 1$$

(6)

"if the perturbed systems have the same number of closed right-half plane poles as the nominal system"

This can also be simply rewritten as

requirement within all possible variations under the effect of uncertainty on the nominal model, then the system is said to have robust performance [4].

To achieve robust performance, various problems can be formulated, which is to calculate a stabilizing controller that minimizes the following:

control problem is expressed as an optimization problem which is solved by using a controller.

The H^{∞} control generally is finding controller $K(s)$, such that

1. The controller internally stabilizes the system
2. Some prescribed performance level, γ (gamma) is minimized such that:

$$\|T(j\omega)_{zw}\|_{\infty} < \gamma \quad (10)$$

In solving the problem posed in equation (10), two methods are used generally: The Riccati equations approach and the linear matrix inequality (LMI) approach [13].



The solution of $\|T(j\omega)_{zw}\|_{\infty} < \gamma$ exists under the necessary and sufficient conditions of Glover & Doyle given by [10].

$$\min_{K \text{ stabilizing}} \left\| \begin{array}{c} (1+GK)^{-1} \\ K(1+GK)^{-1} \end{array} \right\|_{\infty} \quad (11)$$

The objective is to find the controller that will stabilize the plant so that the system can achieve robust performance by attenuating disturbances at low frequency and minimizing high

$$\begin{aligned} A^T X + XA - XBB^T X + (1-\gamma^{-2})^{-1} C^T C &= 0 \\ AY + YA^T - YC^T CY &= 0 \end{aligned} \quad (12)$$

Then the controller K can be calculated as:

$$K = \left[\begin{array}{c|c} \frac{A - BB^T X - (1-\gamma^{-2})^{-1} ZYC^T C}{B^T X} & ZYC^T \\ \hline & 0 \end{array} \right] \quad (13)$$

Where,

$$Z = (I - \gamma^{-2} YX)^{-1} \quad (14)$$

In mixed sensitivity synthesis, both sensitivity functions are penalized with weighting

$$\left\| \begin{bmatrix} W_1 S \\ W_2 R \\ W_3 T \end{bmatrix} \right\|_{\infty} \text{ or } \left\| \begin{bmatrix} W_1 S \\ W_2 K S \\ W_3 T \end{bmatrix} \right\|_{\infty} \quad (15)$$

The functions S , R and T are shaped by the weighing functions W_1 , W_2 and W_3 respectively. W_2 penalizes R , where R is the control input, so as to minimize energy.

H_∞ Mixed Sensitivity Synthesis Approach

For practical application of robust controllers, it will be desired for the system to be robustly stable and has robust performance and a common approach to attaining this is using the mixed sensitivity synthesis [4]. The two optimization problems are combined as follows:

frequency control effort and to stabilize the system subjected to uncertainties at high frequencies.

The solution of this problem is found when there exist the solutions of the two following equations:

functions, as such it is called "mixed sensitivity". It assumes the general form [4]:

shapes the closed-loop system at lower frequencies $\omega < \omega_c$. W_2 shapes the closed-loop system above cutoff frequency $\omega > \omega_c$.

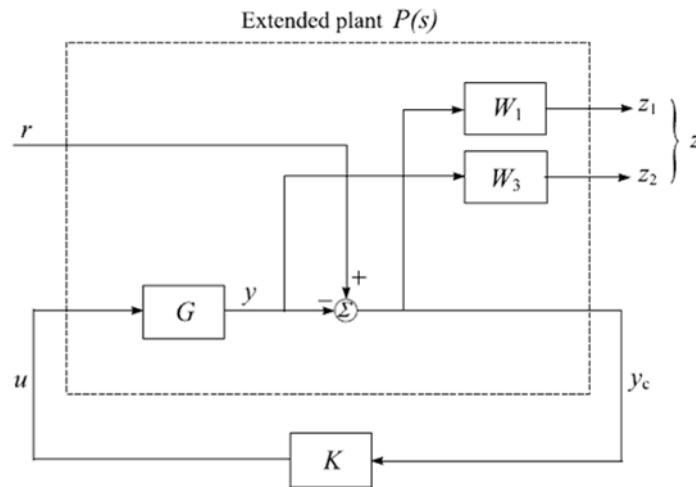


Figure 2: Representation of Mixed-Sensitivity Synthesis [14]

In selecting the weighing functions, the following conditions must be satisfied [20]:

1. The weights selected must be stable.
2. For the weighing function of S, for the minimization of S at low frequencies. The function must be of an integral action, i.e. given that

$$W_1 = \frac{as + b}{cs + d} \quad (16)$$

To achieve integral action,

$$a \ll 1$$

$$b \ll 1$$

3. For the weighing function of KS or R, it is desired that the gain K outside the system bandwidth is small. Hence this weighing function is suggested by Skogestad

$$W_2 = \frac{1 + \tau_1 s}{1 + \tau_2 s} \quad (17)$$

With the condition that

$$\tau_2 \ll \tau_1$$

4. The weighing function of T is selected for tracking requirements and noise attenuation.

Hence, the selection of these weighing functions to satisfy these criteria is thus a repetitive process and require the high-level

expertise of the automation engineer. This is an optimization problem where an objective function is to be minimized or maximized subject to some constraints.

One of the most important tools for solving these problems are meta-heuristics, this is because the objective function to be maximized or minimized can be subjected to many constraints, hence the relevance of designers' experience is reduced [13].

Multi Objective Particle Swarm Optimization

Multi Objective Particle Swarm Optimization (MOPSO) is the extended version of the Particle Swarm Optimization (PSO) used in solving multi-objective optimizations. PSO as developed by [16], is a metaheuristic technique that mimics the movements of birds in search of food. The PSO has gained much more acceptance due to its simplicity and low computational cost [17-18].

In PSO, during the entire run, particles are retained as members of the population; however, the position of each particle is updated based on the initial position and velocities. Due to the extensive application of the PSO, an extension for solving multi-objective problems where more than one objective function is involved was proposed as the MOPSO. The pseudo-code for the MOPSO is as shown [19].



```
Begin  
Initialize swarm  
Initialize leaders in an external archive  
Quality(leaders)  
g = 0  
While g < gmax  
    For each particle  
        Select leader  
        Update Position  
        Mutation  
        Evaluation  
        Update pbest  
    End For  
    Update leaders in the external archive  
    Quality(leaders)  
    g++  
End While  
Report results in the external archive  
End
```

Figure 3: Pseudocode for MOPSO

Many stopping criteria can be used such as the maximum number of iterations, number of fitness evaluations; convergence of the algorithm.

Review of Related Works

A Linear Quadratic Gaussian (LQG) controller was proposed in [14] for an active suspension system of a quarter car. Here, the body acceleration was used as a performance index that needed to be minimized in order to achieve an improved ride comfort. The control input was a solution of Reduced Ricatti Equation which is a feedback gain matrix. The superior performance of this work over the passive suspension systems and optimal controllers is revealed by the simulation results; albeit, the controller's effectiveness is relegated by systems with parametric perturbations. Ali and Noor in [20] designed a H^∞ controller for the uncertain pneumatic actuator having large variation of loads. In order to tune the parameters of the weighting function on a well formulated SISO-based optimization problem, Particle Swarm Optimization (PSO) was used. On simulation, this approach guarantees robust stability conditions of the system. However, it can only be deployed on SISO systems whereby the loop shaping was done on open loop transfer function; hence, this controller cannot give good robust performance.

A Proportional Integral Derivative (PID)-based technique was presented in [21] for active

suspension of a quarter car. Despite the introduction of a road bump as a step input, analysis was only done on time domain; more so, the paper did not consider the need for effective selection of PID parameters. The front and rear weights of wheels were considered as uncertainties in [5] in order to design a controller for the enhancement of comfort and body stability using state feedback. In this paper, the half car model was derived by using Newton's motion equation; thereafter, the H_2 controller is then designed on specified criteria and optimized using LMI optimization. The H_2 active robust controller outperforms the passive controller in robust stability upon simulation and analysis in both time and frequency domain. However, the robust performance is poor due to the existence of high disturbance attenuation level γ . More so, the resulting H_2 controller transfer function is too complex because of its high order which makes the cost and burden of its implementation to be high. Toda *et al* in [22] synthesized a controller for the half car active suspension system including both the vertical and pitch motion. For good ride performance, stability and comfort were both considered; more so, the uncertainty due to mass as a function of load variation in vehicle was imposed in order to introduce robustness. The formulated control problem was solved by using finite Linear Matrix Inequalities.



This approach guarantees robust stability and a fair degree of performance. However, the use of sinusoidal wave as road surface input impedance is unrealistic; also, as the order of the developed controller is of higher order, it is very difficult to implement this controller practically. In [17], a fuzzy logic controller for the quarter car semi active suspension system was designed, albeit, without considering the uncertainty imposed by variation of mass of car. Simulation of passive suspension was done when subjected to both step and sine input disturbances; the same simulation was carried using the quarter car model and the riding performances. In order to reduce the tradeoff between conflicting performances, fuzzy logic control is then used. In order to achieve the desired control performance, the different membership function and fuzzy rules are decided using expert's knowledge. The superior qualitative robustness of this approach over the passive suspension system is great; however, it cannot perform well at slightest variation in vehicle's mass. A state-feedback controller for the quarter car system towards different road profile at different heights was proposed in [23]. The aim of the design was to lessen the body acceleration with respect to steady state and transient characteristics due to the different road input disturbance. The controller was intended to achieve the desired stability and performance. However, the plausibility of this approach is hampered by the trial and error procedure that was used to obtain the state-feedback gains; these gain values often result to a controller that is not physically realizable. Furthermore, all the state variables were assumed to be available for direct measurement; this assumption is not real as actuator faults persist in reality. Payghan *et al* in [24] presented a controller for the Skyhook active suspension system by formulating a feedback control law using an exponentially variable damper to achieve the objective of minimizing the effect of road surface's irregularities on the vehicle; the body acceleration was minimized. The simulation results on SIMULINK reveals that the use of variable damper can only be efficient when the state variables are available for direct

measurement without noise; hence, the simplest errors will render this approach inefficient. Furthermore, a comparative analysis, on a quarter car suspension system, between the performance of the Linear Quadratic Regulator (LQR) and Model Predictive Controller (MPC) was made by Durmaz in [25].

The nominal plant is used while the respective controllers were used to minimize the performance criteria in turns; the results were investigated and compared under different characteristics. The LQR and the MPC give optimal performance; however, in the presence of plant model uncertainty, the predictive performance of the MPC algorithm becomes poor because it requires an accurate plant model to predict future horizons and generate an appropriate control signal for the plant. In [26-27], a Robust Model Predictive Controller (RMPC) was designed for an active suspension system model which has seven degrees of freedom. In order to control the electrohydraulic actuator, a fractional order PID controller was applied. MATLAB simulation results reveal that despite the efficiency of this approach, its implementation cost is high because MPC controllers require high processing power for its online computation.

The literatures above justify that most controllers will find it very hard to withstand high design specifications and will only gain mathematical feasibility without being physically realizable. Therefore, it is expedient that a model-order reduction technique be used for simple approximation of controllers' orders.

METHODOLOGY

In this section, the procedures that are used to achieve the H^∞ mixed sensitivity control with automatic weight selection for active suspension of a half car are discussed.

Building the Uncertain Model and checking the existence of control solution

The half car model for the active suspension system model is modelled with parametric variations as shown in Table 1. This uncertain system is due to the uncertainty in the front and rear mass of the system. The model is built using the containments of the Table 2 which

is mapped into an uncertain continuous-time state-space model which also displays the

information on inputs, outputs and states using the appropriate MATLAB function.

Table 2: Uncertain Parameters [18]

Variables	Description	Nominal Value	Unit	Uncertainty
M_{2f}	Front car body mass	2	Kg	Range [1.45-2.45]
M_{2r}	Rear car body mass	2	Kg	Range [1.45-2.45]

More so, the existence of control solution is investigated using the controllability and the observability test. The result indicate the system is both state controllable and state observable which gives us the freedom to design a controller to achieve stability and performance.

Simulink Model Representation of the System and Performance Specifications

The Simulink model of the system is as represented in Figure 4. The model represents a typical robust control representation with a random signal generator as the random disturbance input to the system. The Body Acceleration and the Pitch Acceleration were displayed on the scope for direct measurement.

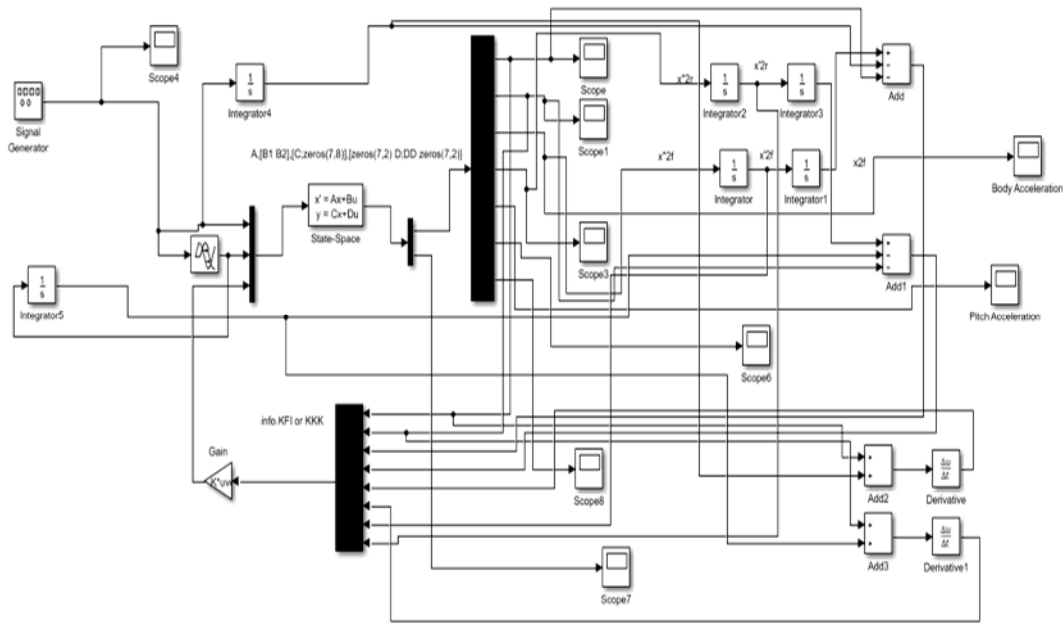


Figure 4: Simulink Representation of the System

In the frequency domain analysis, to model the road surface disturbance, a sinewave of

magnitude 0.002 V is used. Also, for the time domain analysis, a square wave of frequency 0.2



Hz and magnitude 0.02 V is applied. The length of the vehicle is 3m, the velocity of the vehicle also as 30km/h. A time delay of 0.36s is used.

In the modelling, the state variable vector $x(t)$ is

$$x(t) = \begin{bmatrix} x_{1f} - x_{0f} & x_{1r} - x_{0r} & x_{2f} - x_{1f} & x_{2r} - x_{1r} & \dot{x}_{1f} & \dot{x}_{1r} & \dot{x}_{2f} & \dot{x}_{2r} \end{bmatrix}^T \quad (18)$$

The direct measured output $z(t)$ is defined by:

$$z(t) = \begin{bmatrix} x_{1f} - x_{0f} & x_{1r} - x_{0r} & \ddot{x}_{2f} & \ddot{x}_{2r} & \ddot{\theta} & F_f & F_r \end{bmatrix}^T \quad (19)$$

It is desired to determine a controller K such that the perturbed closed-loop system remains stable and the performance objective is

satisfied for all such perturbation. The desired performance is as shown in Table 3.

Table 3: Desired System Performance Specifications

Settling Time	< 2 Seconds
Rise time	< 2 seconds

Robust Stability, Robust Performance and H-infinity Control

The test of robust stability is carried on the system to ascertain the extent at which the closed loop system will tolerate uncertainty without losing its stability. The robust stability test reveals the upper and lower bounds of the stability margin and the frequency at which instability occurs. The robust performance test shows the upper and the lower bounds of the performance margin. The tests on robust performance and robust stability are executed using the appropriate MATLAB command.

In H-Infinity control, it is desired to find a controller that satisfies the necessary and sufficient conditions given by Glover & Doyle [23]. It is desired to find a controller, such that $\|T(j\omega)_{zw}\|_{\infty} < \gamma$ and that the system is internally stable. In order to satisfy $\|T(j\omega)_{zw}\|_{\infty} < \gamma$, it is more practically sufficient to just find a controller such that the norm is less than a positive number, hence, the suboptimal solution. The Algebraic Riccati Equation is used for the H-Infinity controller design.

Weighting Functions and MOPSO

The weighing functions are selected for the Mixed Sensitivity synthesis to indicate the

desired performance of the system. They are selected such that they are stable and that they satisfy performance criteria. The weighing

functions respectively are $W_1 = \frac{as+b}{cs+d}$ such

that $a \ll 1$ and $b \ll 1$ and $W_2 = \frac{1+\tau_1s}{1+\tau_2s}$

with the condition that $\tau_2 < \tau_1$. The stability of the weighing functions is determined from their respective eigenvalues.

In a multi-objective optimization, it is desired to optimize two or more objective functions simultaneously. In order to solve the objective functions in equation (11), a multi-objective optimization technique like the MOPSO become very handy.

The following are the optimization parameters for the MOPSO: Maximum Number of Iterations=200, Population Size=200, Number of Variables=6, Repository Size=100, Inertia Weight=0.5, Inertia Weight Damping Rate=0.99, Personal Learning Coefficient=1, Global Learning Coefficient=2, Number of Grids per Dimension=7, Inflation Rate=0.1, Leader Selection Pressure=2, Deletion Selection Pressure=2, Mutation Rate=0.1.

Controller Order Reduction

The model order reduction is done using balancing techniques to compute the reduced-order approximation by minimizing the norm $\|G(s) - G_r(s)\|$. This is implemented using the balanced-reduction function in MATLAB.

SIMULATION AND RESULT

The uncertain system has uncertain continuous-time state-space model with 14 outputs, 4 inputs, 8 states. The step response of the uncertain system is shown in Figure 5 which depicts the respective structured uncertainties.

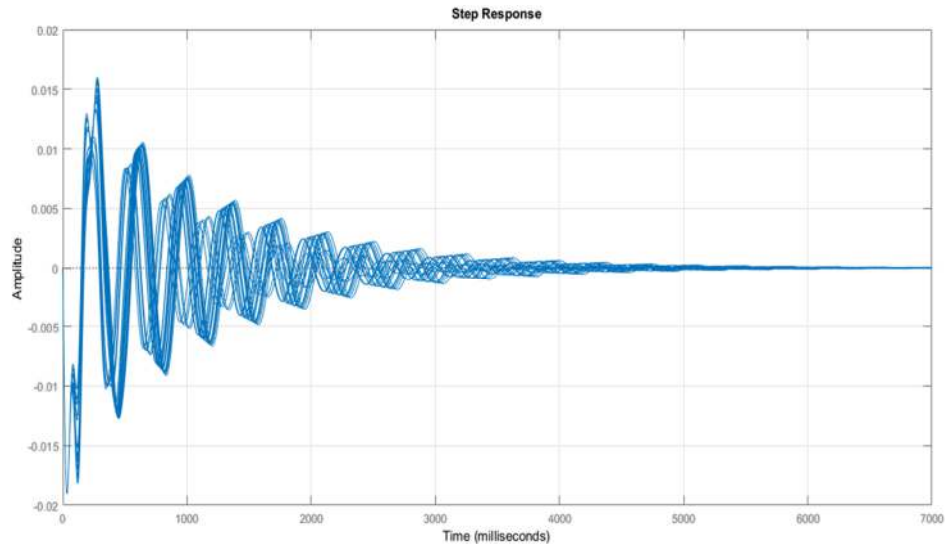


Figure 5: Response Plot of the Uncertain System with Uncertain Parameters

It is deduced from the plot that the system stabilizes at about 6000 seconds without a physical controller; hence, in the design of an active suspension system, there is more concern with performance than stability.

The result of optimal selection of parameters of the weighing functions using MOPSO are obtained on simulation with respect

to the predefined parameters that have been stated in section III. After the first iteration, the particles are spread over the search space; each particle represents a potential solution. As the iteration goes on, the algorithm search for the optimal result and the particles lie at the pareto front and form the optimal curve as depicted in Figure 6.

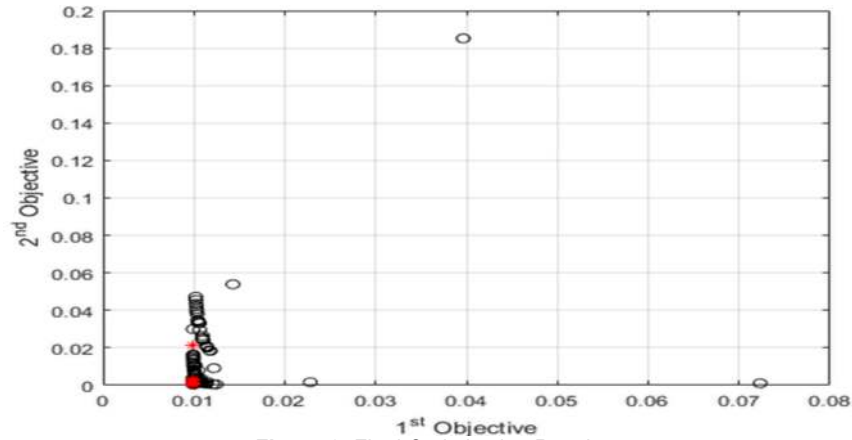


Figure 6: Final Optimization Results

The optimal values of the respective parameters of the weighing functions are the following:

$$a = 0.3821, b = 0.1000, c = 9.9887, d = 7.4506, \tau_1 = 4.9346 \text{ and } \tau_2 = 3.7631$$

Bu using the optimized weighing functions, the controller is obtained by solving the algebraic reduced Ricatti equations. The controller transfer function is obtained as thus:

$$-0.01914 s^{21} + 4441 s^{20} + 4.146e05 s^{19} + 3.204e07 s^{18} + 1.918e09 s^{17} + 5.007e10 s^{16} + 1.882e12 s^{15} + 2.822e13 s^{14} + 5.617e14 s^{13} + 5.218e15 s^{12} + 2.651e16 s^{11} + 8.515e16 s^{10} + 1.872e17 s^9 + 2.948e17 s^8 + 3.406e17 s^7 + 2.919e17 s^6 + 1.854e17 s^5 + 8.617e16 s^4 + 2.849e16 s^3 + 6.344e15 s^2 + 8.536e14 s + 5.241e13$$

$$s^{22} + 7.114e05 s^{21} + 1.239e11 s^{20} + 5.362e12 s^{19} + 4.733e14 s^{18} + 1.39e16 s^{17} + 4.763e17 s^{16} + 8.765e18 s^{15} + 8.771e19 s^{14} + 5.394e20 s^{13} + 2.225e21 s^{12} + 6.511e21 s^{11} + 1.403e22 s^{10} + 2.277e22 s^9 + 2.822e22 s^8 + 2.687e22 s^7 + 1.963e22 s^6 + 1.091e22 s^5 + 4.527e21 s^4 + 1.358e21 s^3 + 2.782e20 s^2 + 3.48e19 s + 2.005e18$$

The obtained transfer function is of order 22, and the singular value plot of the controller is as shown in Figure 7. It is evident that at lower frequencies, less than 1 rad/s, the gain is very high at about 8 dB and at higher frequencies, where noise exists, the gain of the controller falls

to about -48dB; the gain is set low in order to tolerate disturbance rejection. There is a bump at around 50 rad/s due to noise at high frequency; more so, the singular value plot of the controller looks like that of an integrator.

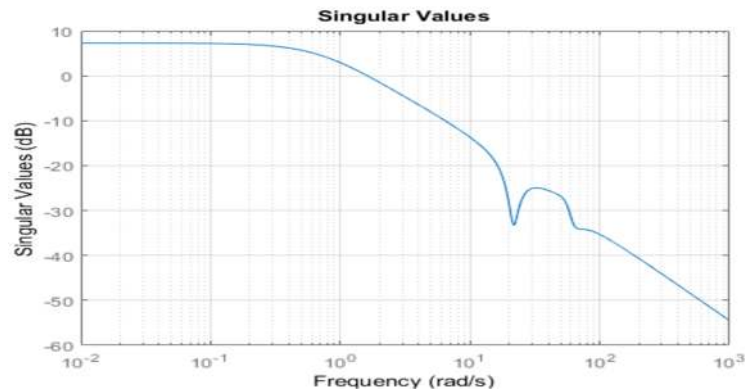


Figure 7: Singular Value Plot of the Controller

The value of the gamma for H-Infinity optimization results is 0.0392. It is desired that gamma should be low so as to enhance disturbance rejection.

The order of the developed controller is high, as such balance reduction method was employed to obtain different reduced orders of

controller until an order that meets the performance specification is obtained. The reduced controller order is 8; at this order, the settling time is 0.8643 seconds and the rise time is to 0.2263 seconds.

The reduced controller for order 8 after balanced reduction is;

$$-2.236e-13 s^8 + 1.195e-08 s^7 + 0.001966 s^6 + 0.03263 s^5 + 6.213 s^4 + 65.83 s^3 + 2066 s^2 - 0.273 s + 1.138e-08$$

$$s^8 + 1.514e05 s^7 + 6.409e06 s^6 + 7.253e08 s^5 + 1.694e10 s^4 + 5.653e11 s^3 + 6e12 s^2 + 9.308e13 s + 6.616e13$$

In order to ascertain the extent at which the closed loop system will tolerate uncertainty without losing its stability, a robust stability test was done by using appropriate MATLAB command. It can be concluded from the result that the uncertain system is robustly stable with respect to modeled uncertainty; It also follows that the system can tolerate up to about 283% of the parametric modelled uncertainty.

The result of test of robust performance of the controller reveals that the system achieved performance robustness to modeled uncertainty.

Simulations of the system with the developed controller were carried on MATLAB with performance analysis in time domain. The response for the body acceleration is shown in Figure 8. After the perturbation caused by the disturbance, even with the uncertainty, the effect of the controller can be seen as the system settles at 0.8643 seconds. This is the time it settles to within (2-5%) of the steady state error. However, approximations in MATLAB can affect this value when the amplitude is very small.

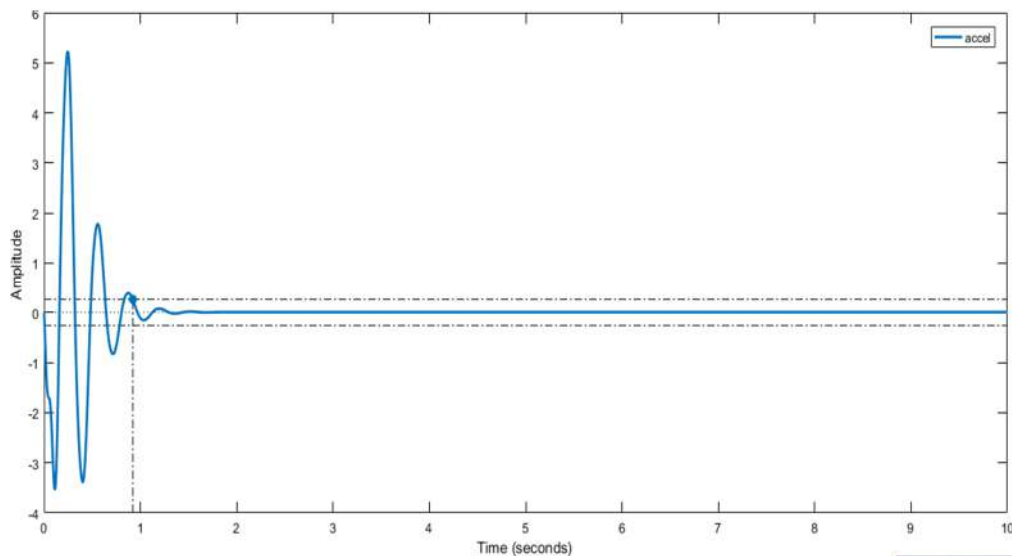


Figure 8: Time Response of Body Acceleration

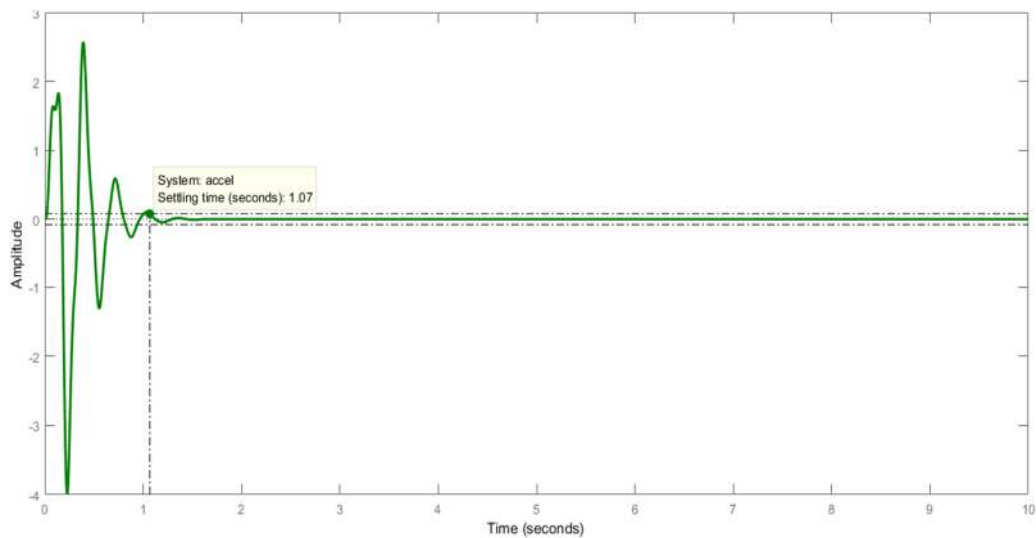


Figure 9: Time Response of Pitch Acceleration

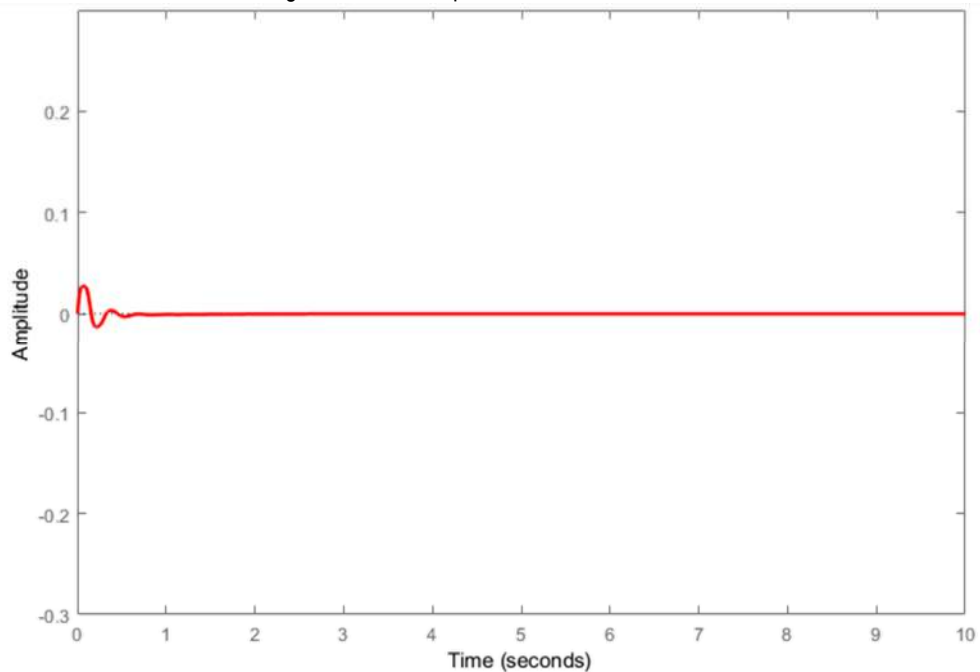


Figure 10: Time Response of Input Force

The pitch acceleration of the vehicle is also responsible for the ride comfort as it determines the vertical displacement as depicted in Figure 9. When the developed controller was

applied, it settled at 1.07 seconds. This falls within the specified performance requirement of the suspension system. The input force, as revealed in Figure 10, also settles fast at 0.9 seconds which



is a good measure of the driving stability by the actuator.

In order to ascertain a healthy performance analysis of the developed controller, a controller was designed using the conventional Algebraic Riccati Equations. A square wave, with the same amplitude as the step, is used as input. The value of gamma obtained is 44.6187.

To evaluate the ride comfort of the active suspension system, body acceleration and pitch acceleration are used. For the driving stability, the front vertical force is used.

The simulation results of the controller are shown in the Figure 11, Figure 12 and Figure 13.

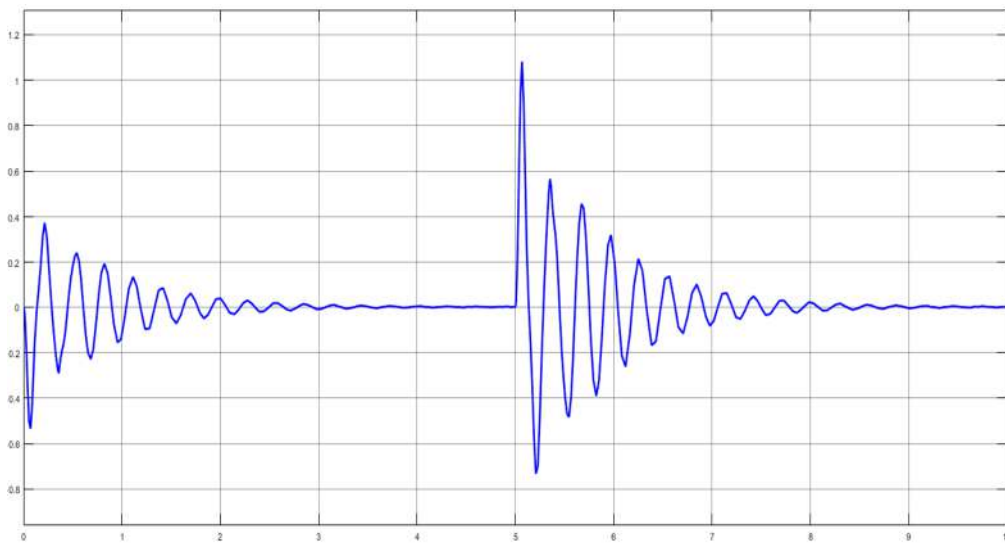


Figure 11: Time Response of Body Acceleration

At the start of the simulation, an input disturbance is injected into the system, in the form of a square wave to model the road surface disturbance, the body acceleration stabilizes at

about 5 seconds. Note that the frequency of the square wave is reduced from 0.2 to 0.1 in the simulation to make the settling time visible.

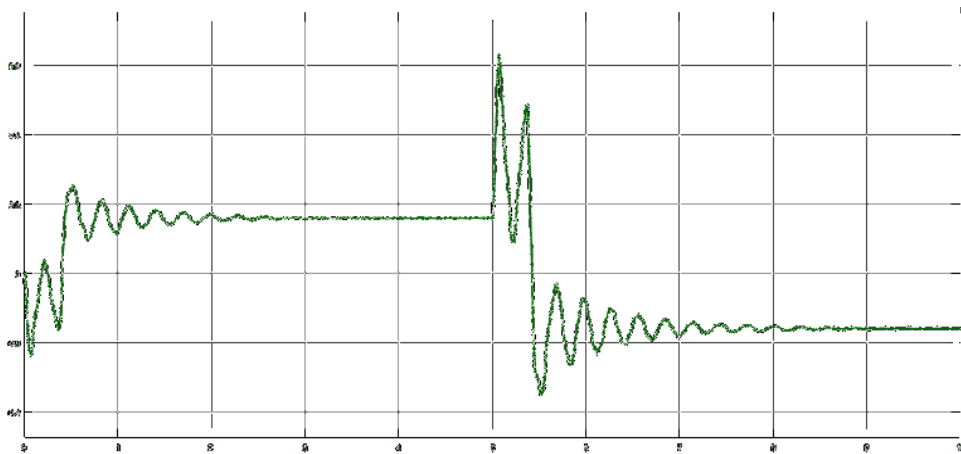


Figure 12: Time Response of Pitch Acceleration

The pitch acceleration to the disturbance settles at about 4.5 seconds. This implies that the pitch acceleration will stabilize after 4.5 seconds. The physical input force to the system stabilizes at about 1.1 seconds. This input force identifies the driving stability.

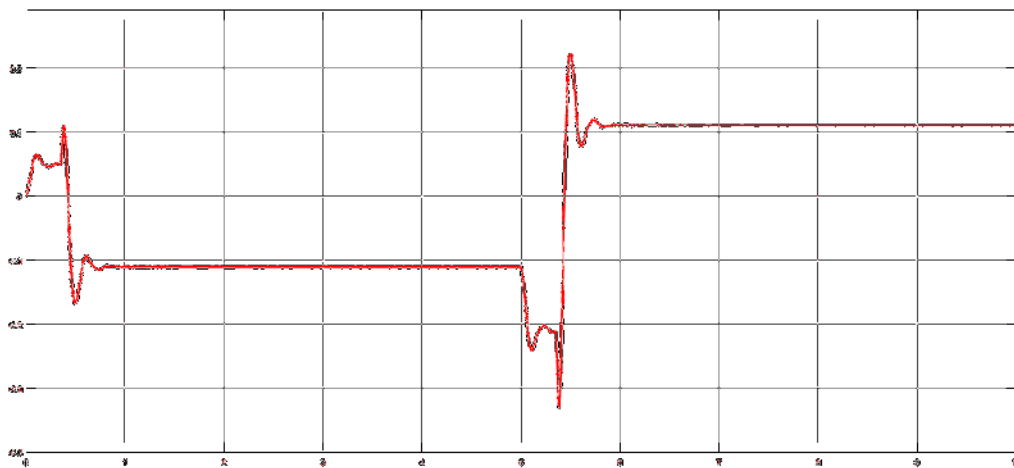


Figure 13: Time Response of Front Vertical Force

The results obtained from the H-Infinity control using Algebraic Riccati Equation can be summarized in Table 4.

Table 4: Summary of Results for H-Infinity Control Using ARE

	Body Acceleration	Pitch Acceleration	Input force
Rise time	0.26 seconds	0.55 seconds	0.35 seconds
Settling time	3.15 seconds	3.50 seconds	1.10 seconds

The result obtained with the use of the developed controller is compared with the results obtained using conventional H-Infinity controller and H₂ controller using LMI; this is shown in Table

5 and Table 6. It should be noted that the details of the procedures adopted for the H₂ controller using LMI are not discussed but are the same with the methods in [18].

Table 5: Controller Comparison Based On Settling Time (Seconds)

	Developed Controller	H-Infinity controller using ARE	H ₂ controller using LMI
Body acceleration	0.86	3.15	0.95
Pitch acceleration	1.07	3.50	1.10
Input force	0.82	1.10	0.95

Table 6: Controller Comparison Based On Rise Time (Seconds)



	Developed Controller	H-Infinity controller using ARE	H ₂ controller using LMI
Body acceleration	0.23	0.26	0.25
Pitch acceleration	0.24	0.55	0.45
Input force	0.15	0.35	0.20

From the results, it can be deduced that the developed controller gave better results in terms of both settling time and rise time with respect to the H-Infinity controller using ARE and H₂ controller using LMI.

In making the comparative analysis on the costs of the controllers, the time required by each controller to compute an input for the system is used. The bar chart in Figure 14 depicts the computation cost of each controller in terms of computation time.

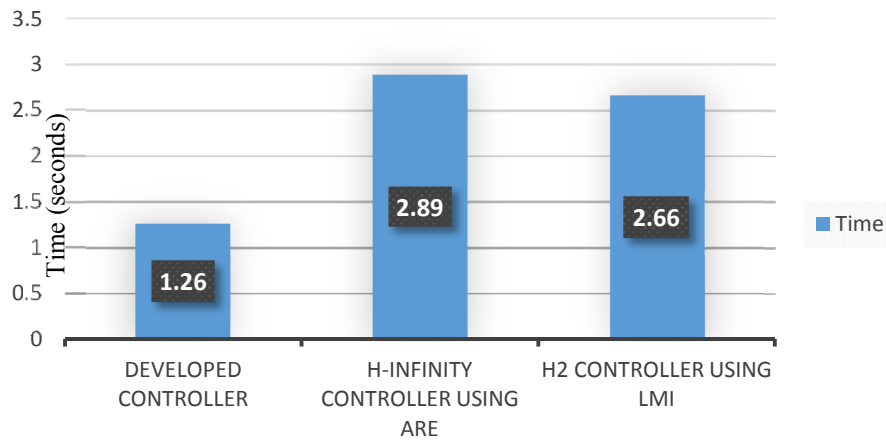


Figure 14: Bar Chart Showing the Relative Cost of Controllers

From the bar chart, the developed controller has the least computation time which implies the ease of its physical implementation with respect to cost; hence it is the cheapest of all the three controllers in terms of implementation cost.

A Graphical User Interface is designed for the proposed controller to allow the ease of computation in design; a screenshot of the application environment is shown in Figure 15.

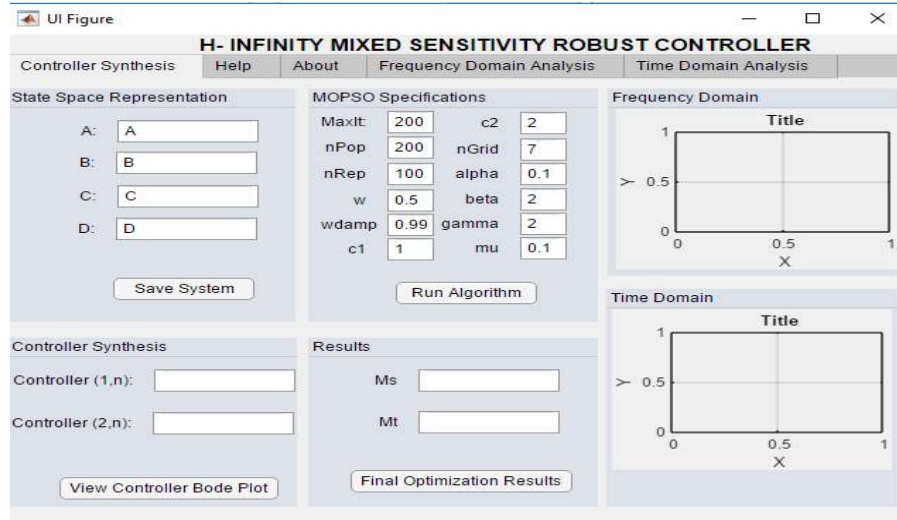


Figure 15: Screenshot of Application Environment

CONCLUSION

This work developed a low-cost H_{∞} mixed sensitivity controller for an active suspension of a half car model. The performance of this approach is dependent on the parameters of the weighing functions. In order to achieve an optimal selection of the parameters of the weighing functions, a well-posed design problem is mapped into a multi-objective optimization problem which adequately satisfies a design specification in terms of the rise time and settling time. MOPSO was used to solve the optimization problem. The body acceleration and pitch acceleration are used for the indication of the ride comfort whereas the input force signifies the driving stability. The values of the rise time and settling time in time domain analysis reveal that the developed controller meets the design specifications; more so, it guarantees a robust stability and robust performance. Comparative analysis between the performance of the developed controller and those of the H-Infinity controller using ARE and H_2 controller using LMI reveals that the developed controller outperforms both of them in terms of design specifications and cost of implementation. H-infinity control requires high mathematical understanding, hence a simple Graphical User Interface (GUI) was also designed to make the controller user friendly for industrial

use. It should be noted that this approach uses controller order reduction. MOPSO offers quick convergence; however, it cannot handle constraints effectively. Therefore, it is recommended that other multi-objective optimization algorithms can also be used for further investigation of performance of this approach.

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