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PERFORMANCE OF FREQUENCY DOMAIN METHODS IN THE ANALYSIS OF MULTICOMPONENT TRANSIENTS WHEN THE NOISE IS VERY HIGH

By

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### ABSTRACT

*Transient multiexponential signals occur in different areas of applied science and methods for their analysis continue to evolve. In addition to time-domain techniques, frequency-domain techniques have been reported. This paper presents performance comparison of selected frequency-domain techniques when the Signal to Noise Ratio (SNR) is very low. The techniques are Minimum norm, Autoregressive moving average (ARMA) and Multiple Signal classification (MUSIC). It is concluded that at very low SNR, minimum norm is the best in terms of accuracy and stability even though its estimation variance is very high. It is followed by MUSIC and ARMA in that order.*

### INTRODUCTION

Parameter estimation of transient multiexponential signals has been a major area of research over the past several decades. Transient multiexponential signals are naturally found in many physical phenomena. Examples are temperature modulation of metal oxide semiconductor (MOS) sensors (Gutierrez-Osuna, Gutierrez-Galvez and Powar, 2003),

$$S(t) = \sum_{i=1}^M A_i \exp(-\gamma_i t) + n(t) \quad 0 < t < \infty \quad (1)$$

where  $y(t)$  is uncorrelated white Gaussian noise with variance  $\sigma^2$ , where  $\sigma$  is the standard deviation.

The goal of the multiexponential analysis is to determine the number of exponential components  $M$ , their amplitudes  $A_i$ , and the decay rates  $\gamma$ . Although the implications of these parameters differ according to the phenomenon/experiment, the methods of analyzing the resulting data are the same. Several methods of analyzing these data have been reported (Istratov and Vyvenko, 1999) most of which are in time-domain. This paper is particularly concerned about the frequency-

deep-level transient spectroscopy (DLTS) (Istratov and Vyvenko, 1999), Thermal transient experiment data of the high power light emitting diodes (Lai et al, 2015), fluorescence decay analysis (Lakowicz, 2013), NMR relaxation data (Kroeker and Henkelman, 1986). In these and similar experiments the signal involved is

domain methods in which a convolution model arising from the application of Gardner transform (Gardner et al, 1959) on the signal in (1) is converted to a frequency domain signal by taking the Discrete Fourier transform and analyzed. This approach has been expounded by Jibia and Salami (2012a).

Cramer Rao lower bound (CRLB) has been derived for parameter estimation of transient multiexponential signals (Jibia et al, 2009) and the CRLB

has also been used to determine the length of the useful data after deconvolution for use with three parametric methods, viz. minimum norm, Multiple Signal Classification (MUSIC) and Autoregressive Moving Average (ARMA) methods (Jibia and Salami, 2012). In this paper, these three modeling techniques are tested and

evaluated at very low Signal to Noise Ratios (40dB and below). The methods are analysed in terms of Mean square error (MSE) of the decay rates estimates due to the presence of noise and their failure to estimate the decay rates and number of components due to low SNR.

**THE METHODS**

Application of Gardner transform on the data signal in (1) will result in (Salami and Sidek, 2003)

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau)d\tau + v(t) \tag{2}$$

where

$$y(t) = \exp(\alpha t)S(\exp(\alpha t)) \tag{3a}$$

$$x(t) = \exp((\alpha - 1)t)g(e^{-t}) \tag{3b}$$

$$h(t) = \exp(\alpha t)\exp(-e^t) \tag{3c}$$

$$v(t) = \exp(\alpha t)n(e^t) \tag{3d}$$

This is now a standard deconvolution problem in which  $x(t)$  is the unknown input signal consisting of a series of weighted delta functions,  $h(t)$  is the impulse response and  $y(t)$  is the output observation.

It has been shown (Salami and Sidek, 2003) that the unknown input distribution function is given by

$$x(t) = \sum_{i=1}^M B_i u(t + \ln \tau_i) \tag{4}$$

where  $B_i = A_i(\tau_i)^{-\alpha}$

Eq. (2) can be converted to a discrete-time deconvolution model by sampling  $y(t)$  at the rate of  $f_s = 1/\Delta t$ . This yield

$$y[n] = \sum_{m=n_{\min}}^{n_{\max}} x[m]h[n - m] + v[n] \tag{5}$$

Where  $N = n_{\max} - n_{\min} + 1$  and the sampling interval is  $\Delta t \leq \frac{1}{\alpha}$ .

For frequency-domain analysis, the discrete Fourier transform of equation (5) is taken thus

$$Y(k) = X(k)H(k) + V(k) \tag{6}$$

Of the several methods of deconvolution of (6), the two considered in this paper are Multiparameter

deconvolution (Zhang et al, 2008) and Exponential Compensation Deconvolution (Jibia and Salami, 2012).

$$\hat{X}(k) = \sum_{i=1}^M B_i e^{j \frac{2fk}{N} \ln \gamma_i} + v(k)$$

The noise  $v(k)$  is different from the original noise  $n(t)$  and the  $v(t)$  in (2). This is because the original signal  $S(t)$  has undergone many manipulations including Gardner transformation, discretization and deconvolution, to arrive at (7). Therefore, even if  $n(t)$  is stationary,  $v(k)$  may not be stationary. There is,

$$\hat{X}(k) = \sum_{i=1}^M B_i e^{j \frac{2fk}{N_d} \ln \gamma_i} + e(k) \tag{8}$$

$k = 1, 2, \dots, N_d$ ;  $N_d = 2N_0 + 1$ ,  $N_0$  is the truncation point and  $e(k)$  is the new stationary white noise.

Three parametric models are then applied to the truncated data of Equation (7). They are ARMA model (Salami and Sidek, 2003), MUSIC (Jibia and Salami,

These methods result in a signal of the form

thus, the need to look for a procedure for stationarizing the signal in (7) such that the deterministic signal would be associated with a stationary noise which would be further analysed easier. The method used here is the same as the one used in Jibia and Salami (2012b). The resulting stationary signal is therefore

2012c) and Minimum norm (Jibia, 2012). The reader is referred to these sources for the detailed procedures.

**SIMULATION AND RESULTS**

The signal of equation (9) is used as a test signal; that is

$$S_4(t) = 50e^{-0.025t} + 100e^{-0.1t} + 200e^{-0.2t} + 350e^{-0.7t} + n(t) \tag{9}$$

The CRLB was computed using the steps outlined in chapter five. The noise  $n(t)$  was added to the signal using the MATLAB function *awgn*. Nonlinear change of variables (i.e. the Gardner transform) was performed by multiplying  $S(t)$  by  $e^{rt}$ . The discrete form of equation (5) was obtained by selecting  $r = 0.5$ ,  $\Delta t = 0.25$ ,  $n_{max} = 179$ ,  $n_{min} = -844$ . The DFT of equation (6) was therefore computed using FFT with  $N = 1024$ .

The Mean Square Error (MSE) was computed using 100 Monte Carlo simulations at five different SNRs. These are 0, 10, 20, 30 and 40 dB. This was done for each of the three techniques. Table 1 – Table 5 show the

values of MSEs and CRLBs at the respective SNRs. Figure 1 – Figure 4 show the graphs of CRLB and MSE over a range of SNRs for the three techniques. As expected, the lower the SNR the farther away the MSE is from the bound. Below the SNR of 10 dB for MUSIC and 20 dB for SVD, the pseudo spectral and power distribution plots respectively are too poor for any useful observations to be made, hence the MSE do not extend below those SNRs for the two methods. Most importantly, it can be observed that the closest MSE graph to the CRLB is that of MUSIC, followed by Minimum norm and finally, the SVD-ARMA. This makes MUSIC the most efficient of the three over the observed SNRs.

**Table 1:** MSE (in dB) and CRLB at SNR of 0 dB

Technique	$\} _1$	$\} _2$	$\} _3$	$\} _4$
Minimum norm	-74.208	-44.93	-33.16	-38.02
MUSIC	-	-	-	-
SVD-ARMA	-	-	-	-
CRLB	-105	-86	-82.98	-86.31

**Table 2:** MSE (in dB) and CRLB at SNR of 10 dB

Technique	$\} _1$	$\} _2$	$\} _3$	$\} _4$
Minimum norm	-113.8	-77.1	-65.6	-70.93
MUSIC	-119.96	-70.49	-80.12	-85.06
SVD-ARMA	-	-	-	-
CRLB	-137	-115	-112.13	-115.65

**Table 3:** MSE (in dB) and CRLB at SNR of 20 dB

Technique	$\} _1$	$\} _2$	$\} _3$	$\} _4$
Minimum norm	-144.55	-113.91	101.94	-108.85
MUSIC	-155.30	-126.25	-114.58	-131.03
SVD-ARMA	-121.21	-89.218	-80.27	-90.94
CRLB	-169	-145.50	-143.95	-144.43

**Table 4:** MSE (in dB) and CRLB at SNR of 30 dB

Technique	$\} _1$	$\} _2$	$\} _3$	$\} _4$
Minimum norm	-166.03	-142.07	-137.09	-140
MUSIC	-190.80	-156.49	-149.80	-159
SVD-ARMA	-153.40	-127.72	-115.50	-121
CRLB	-201	-177	-172.11	-175

**Table 5:** MSE (in dB) and CRLB at SNR of 40 dB

Technique	1	$\} _2$	$\} _3$	$\} _4$
Minimum norm	-206.03	-171.06	-171.09	-171.57
MUSIC	-220.40	-185.49	-185.33	-190.46
SVD-ARMA	-190.45	-159.34	-154.44	-150.46
CRLB	-233	-207.5	-202.60	-205.10

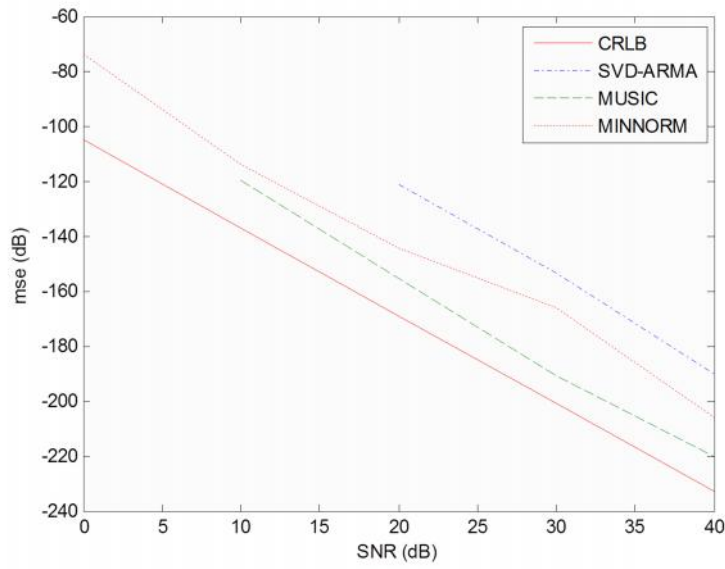


Figure 1. MSE versus SNR for }<sub>1</sub>

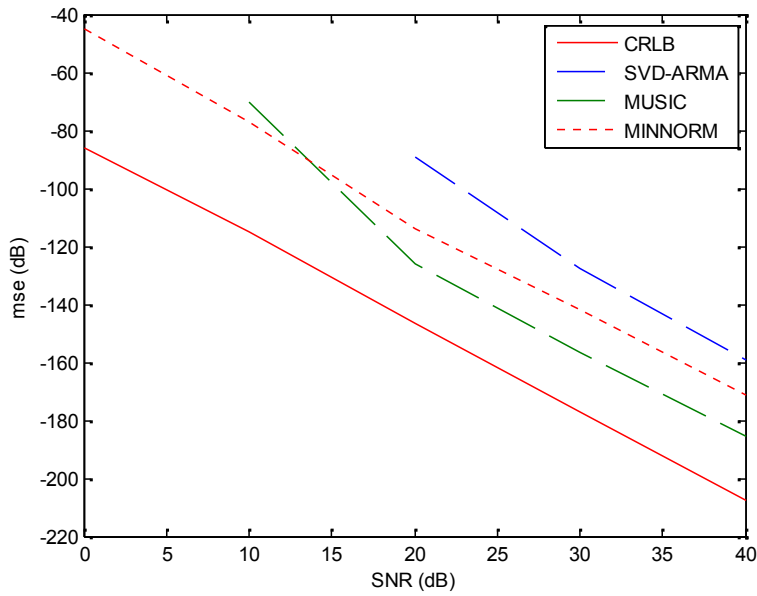


Figure 2. MSE versus SNR for }<sub>2</sub>

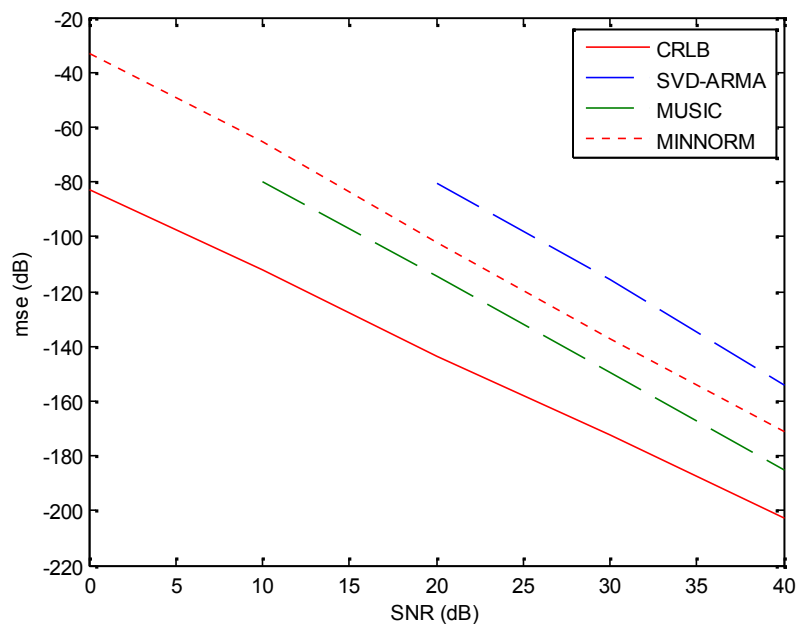


Figure 3. MSE versus SNR for }<sub>3</sub>

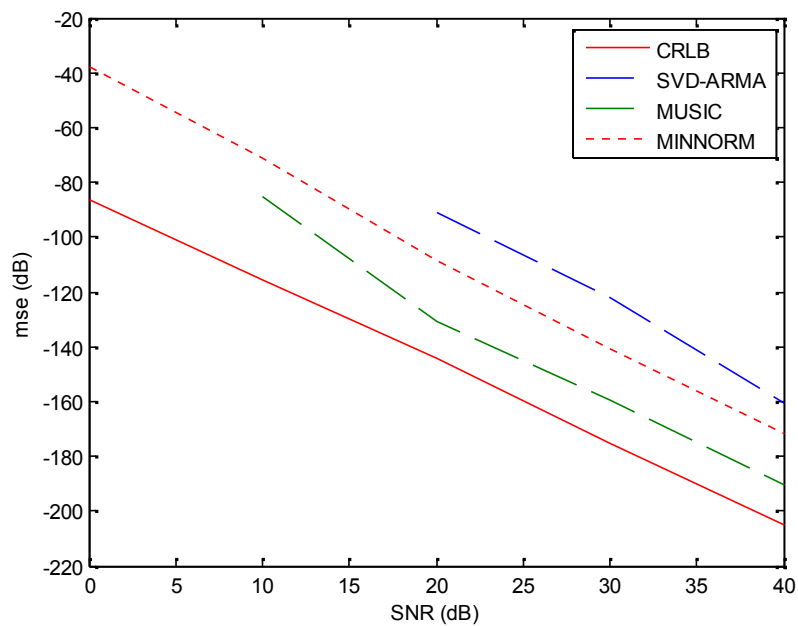


Figure 4. MSE versus SNR for }<sub>4</sub>

A performance summary of the three modelling techniques is given in Table 6.

**Table 6:** Performance Summary of Signal Modeling Techniques

Technique	Performance
MUSIC	Least Noisy. Very sharp peaks indicating low noise and low estimation variance. Performance deteriorates below an SNR of about 15dB for signal of medium resolution. Poor below 40dB for high resolution signals. High spectral dynamic range. Performs better with MPD.
Minimum Norm	High estimation variance. Noisy pseudo spectra. Very low detection threshold; so low it can detect components even below the SNR of 0dB for both low and high resolution signals and signals with large number of components. Low spectral dynamic range. Performs better with MPD.
SVD-ARMA	High estimation variance and noisy especially at low SNRs. Better than minimum norm for higher SNRs but performance is poor below 40dB and is not stable below 20 dB. High spectral dynamic range. Performs better with MPD.

## CONCLUSION

The three modelling techniques may work well with both ECD and MPD at medium and high SNRs but for analysis over low SNRs, MPD outperforms the ECD. Performance comparison using the CRLB confirms that of all the three techniques, minimum norm has the lowest detection threshold while the SVD-ARMA has the highest. MUSIC was shown to have the least estimation variance over low SNRs while SVD-ARMA has the highest. This makes the combination of Minimum norm and MPD the best technique to use at very low SNRs while for higher SNRs MUSIC or SVD-ARMA combined with MPD are better recommended. SVD-ARMA is an excellent method except for its poor performance over very low SNRs.

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