



Application of Differential Transform Method in Solving Measles Model

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ABSTRACT

We present a deterministic model of the dynamics of measles disease transmission in this work. The series solution of the model is attempted using the Differential Transform Method (DTM). Maple 18's implementation of the traditional fourth-order Runge-Kutta method establishes the DTM's validity in solving the model. Comparisons between Runge-Kutta (RK4) and the DTM solution were made. The outcomes show that the DTM is accurate and has the capacity to handle the study of contemporary epidemics.

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INTRODUCTION

Measles is a highly contagious, serious disease caused by a virus. Before the introduction of measles vaccine in 1963 and widespread vaccination, major epidemics occurred approximately every 2–3 years and measles caused an estimated 2.6 million deaths each year (James, 2022). More than 140 000 people died from measles in 2018 – mostly children under the age of 5 years, despite the availability of a safe and effective vaccine. Measles is caused by a virus in the paramyxovirus family and it is normally passed through direct contact and through the air. The virus infects the respiratory tract, then spreads throughout the body. Measles is a human disease and is not known to occur in animals (Garba, 2017; Griffin, 2016). In recent years, there has been an increase in interest in using deterministic compartmental models to analyze the dynamics of the measles and identify strategies for containing and preventing the outbreak (Bauch, 2004). Several mathematical models have been proposed in the literature using different analytical approaches

The aim of this paper is to present the application of Differential Transform Method to the proposed model and to verify the validity of the Differential Transform Method in solving the model using Maple 18's classical fourth-order Runge-

Kutta method as a basis for comparison.

Differential Transform Method was first introduced by (Zhou, 1986) to study the initial value problems in electrical circuit for a computation in linear and non-linear system. Differential transform method (DTM) is proved to be an excellent tool to investigate analytical and numerical solutions of nonlinear ordinary differential equations. The concept of differential transformation is briefly presented and some well-known properties of this DTM are rewritten in a more generalized forms. The Differential Transformation Method is one of the semi-analytical methods commonly used for solving ordinary and partial differential equations in the forms of polynomials as approximations of exact solution (Peter, 2017). Other semi-analytical methods include: Homotopy Analysis Method (HAM), Homotopy Perturbation Method (HPM), Reduced Differential Transform Method (RDTM), Variational Iterational Method (VIM), Parameter Expansion Method (PEM). The methods mentioned above have been used as tools to approximate linear and non-linear problems in Physics and Engineering. The main advantages of DTM over other techniques are, it provides desired accuracy and reliability to the series solution with remarkable convergence rate, it

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requires less size computational work and yield high accuracy (Peter, 2018a, Peter, 2018b) . In this study, we employ the (DTM) to the system of differential equations which describe our model

and approximating the solutions in a sequence of time intervals. In other to illustrate the accuracy of the DTM, the obtained results are compared with fourth-order Runge-Kutta Method.

MODEL FORMULATION

In this section, we solve the model equations proposed by momh et al by DTM. The model equation is given below

$$\left. \begin{aligned} \frac{dS}{dt} &= B - \beta SI - \mu S \\ \frac{dE}{dt} &= \beta SI - (\mu + \alpha + \sigma)E \\ \frac{dI}{dt} &= \alpha E - (\mu + \gamma)I \\ \frac{dR}{dt} &= \gamma I + \sigma E - \mu R \end{aligned} \right\} \quad (1)$$

DIFFERENTIAL TRANSFORM METHOD

The process involved in DTM is as follows: Given an arbitrary function of x , suppose $y(x)$ is a non-linear funtion of x , then $y(x)$ can be expanded in a Taylor series about a point $x = 0$ as

$$y(x) = \sum_{k=0}^{\infty} x^k \frac{1}{k!} \left[\frac{d^k}{dx^k} y(x) \right]_{x=0}$$

Thus, the differential Transform of $y(x)$ is given as:

$$Y(k) = \frac{1}{k!} \left[\frac{d^k}{dx^k} y(x) \right]_{x=0}$$

and the inverse differential Transform is given as

$$y(x) = \sum_{k=0}^{\infty} Y(k)x^k$$

Table 3 gives some operational properties of DTM. In the table, $c(x)$ and $d(x)$ are arbitrary functions with transforms $C(k)$ and $D(k)$ are the transformed functions respectively.

Table 2: Basic operation properties of the DTM

S/No	Original Function	Transformed Function
1	$y(x) = c(x) \pm d(x)$	$Y(k) = C(k) \pm D(k)$
2	$y(x) = \alpha c(x)$	$Y(k) = \alpha C(k)$, α is a constant
3	$y(x) = \frac{dc(x)}{dx}$	$Y(k) = (k+1)C(k+1)$
4	$y(x) = \frac{d^2c(x)}{dx^2}$	$Y(k) = (k+1)(k+2)C(k+2)$
5	$y(x) = \frac{d^nc(x)}{dx^n}$	$Y(k) = (k+1)(k+2)\dots(k+n)C(k+n)$

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6	$y(x) = 1$	$Y(k) = \delta(k)$
7	$y(x) = x$	$Y(k) = \delta(k - 1)$, δ is the Kronecker delta
8	$y(x) = e^{\lambda x}$	$Y(k) = \frac{\lambda^k}{k!}$
9	$y(x) = c(x)d(x)$	$Y(k) = \sum_{n=0}^k D(n)C(k-n)$
10	$y(x) = (1+x)^n$	$Y(k) = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!}$

IMPLEMENTATION OF THE METHOD

Using the operational properties (1), (2), (3) and (6) in Table 1 and applying them to the system of differential equations in (1) we obtain the following system of transformed equations below,

$$S(k+1) = \frac{1}{k+1} [Bb(k,0) - \sum_{l=0}^k S(l)(\beta I(k-l)) - \mu S(k)]$$

$$E(k+1) = \frac{1}{k+1} [\sum_{l=0}^k S(l)(\beta I_c(k-l)) - K_1 E(k)]$$

$$I(k+1) = \frac{1}{k+1} [\alpha E(k) + K_2 I(k)]$$

$$R(k+1) = \frac{1}{k+1} [\gamma I(k) + \sigma E(k) - \mu R(k)]$$

where

$$K_1 = \mu + \alpha + \sigma, \quad K_2 = (\mu + \gamma)$$

By using the following initial condition and the parameter values in the (Momoh et. al., 2013), the initial conditions $S(0) = 60$, $E(0) = 40$, $I(0) = 20$, $R(0) = 10$ and with the help of Maple 18, we obtain the following expansion up to 5th order.

$$S(t) = \sum_{n=0}^k S(k)t^k = 60 + 2.6693t + \dots$$

$$E(t) = \sum_{n=0}^k I(k)t^k = 40 + 6.50t + 3.7498940t^2 + \dots$$

$$I(t) = \sum_{n=0}^k R(k)t^k = 20 + 10.40t + 1.749856100 \times 10^5 t^2 + \dots$$

$$R(t) = \sum_{n=0}^5 R(k)t^k = 10 + 71.580t + 1.499t^2 + \dots$$

Numerical Simulation and Graphical Illustration of Model

In this section, we present the numerical simulation which demonstrate the analytical results for the measles model (1). This is achieved by using the set of parameter values given in the Table which are derived from literature and well as assumptions. We considered the following initial conditions for the different compartments. $S(0) = 60$, $I_c(0) = 40$, $I(0) = 20$, $R(0) = 10$. The

DTM is demonstrated against Maple 18's fourth order Runge-Kutta procedure for the solution of measles Model (1). Fig (1) to (4) shows the combined plots of the solutions of $S(t)$, $I_c(t)$, $I(t)$ and $R(t)$ by DTM and RK4

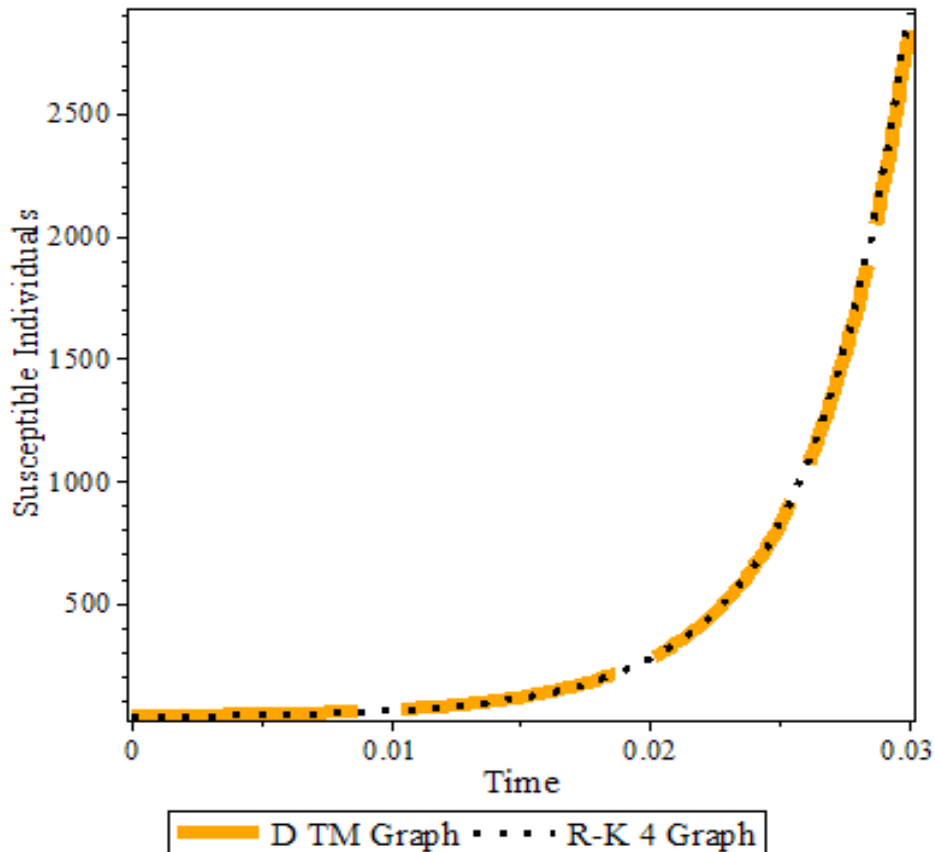


Fig1: Solution of Susceptible Population by DTM and RK4

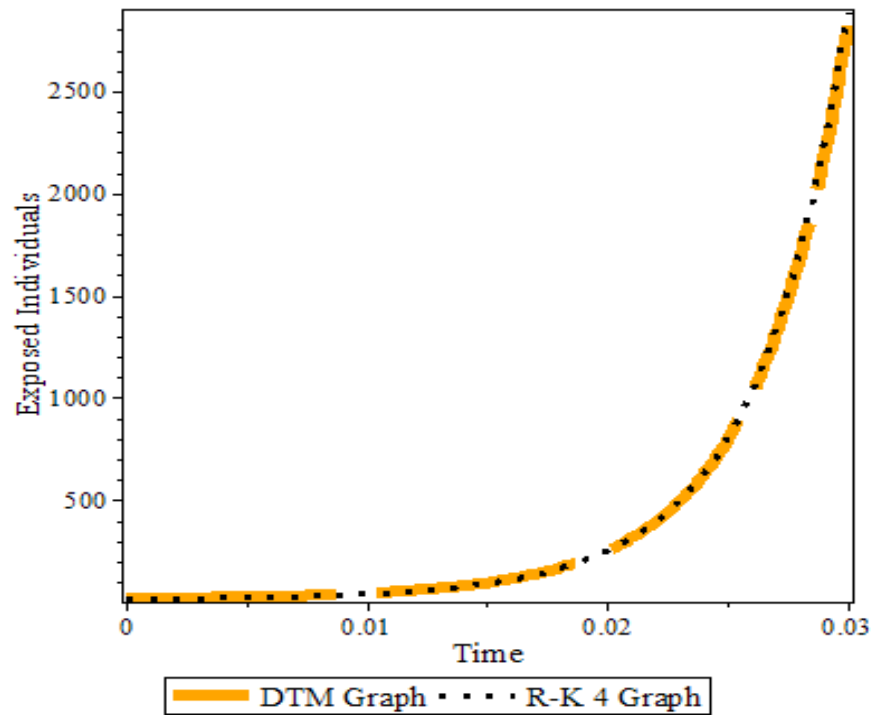


Fig 2: Solution of Carrier Population by DTM and RK4

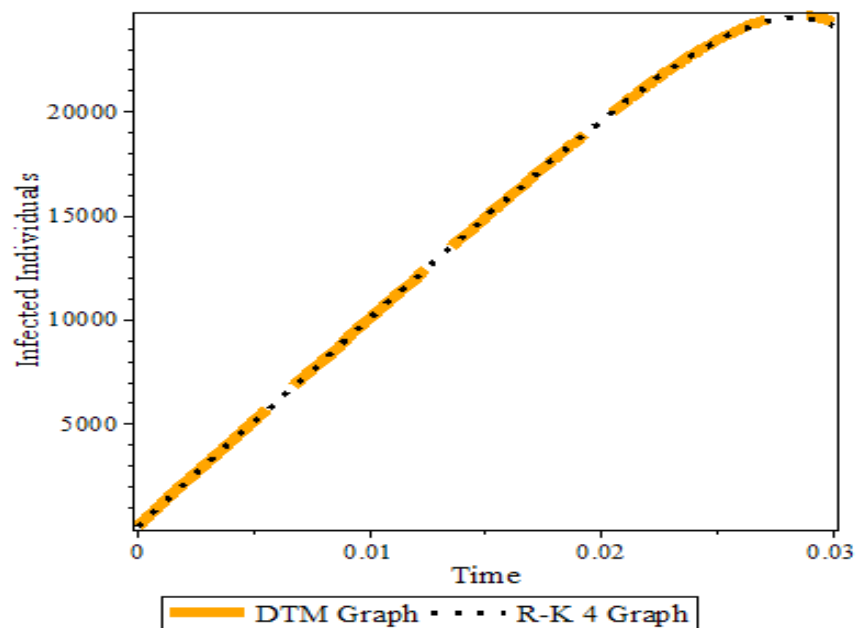


Fig 3: Solution of Infected Population by DTM and RK4

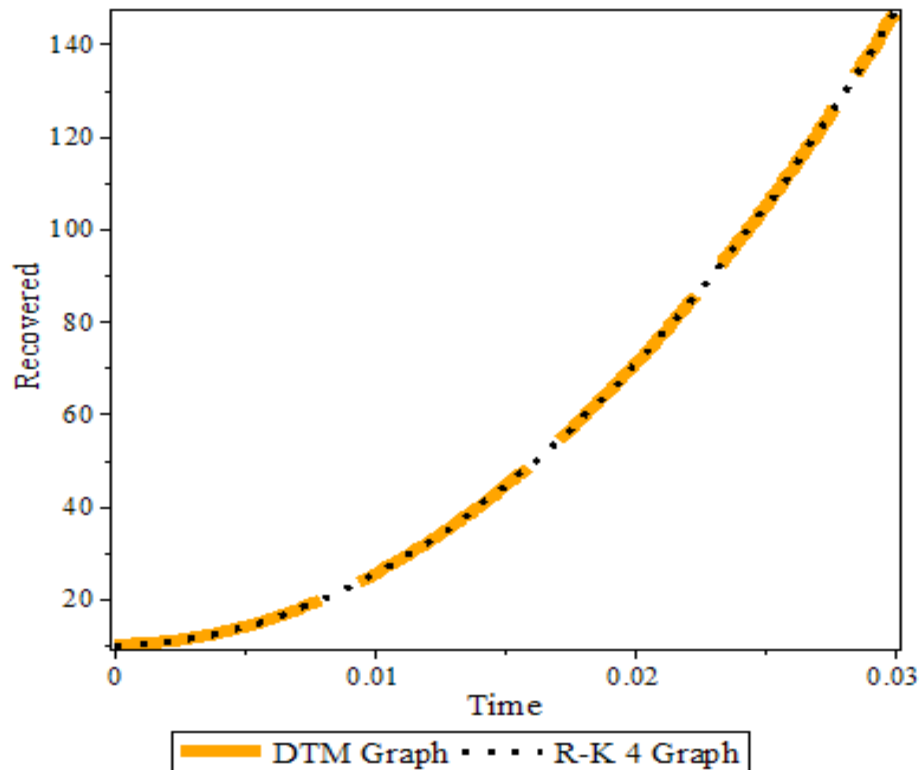


Fig 4: Solution of Recovered Population by DTM and RK4

DISCUSSION OF RESULTS

The solutions obtained by using Differential Transform Method with given initial conditions compared favourably with the solution obtained by using classical fourth-order Runge-Kutta method. The solutions of the two methods follows the same pattern and behaviour. This shows that Differential Transform Method is suitable and efficient to conduct the analysis of measles models.

CONCLUSION

We present a mathematical model of measles transmission dynamics. Numerical simulations were carried out to compare the results obtained by Differential Transform Method with the result of classical fourth-order Runge-Kutta method. The results of the simulations were displayed graphically. The results obtained confirm the accuracy and potential of the DTM to cope with the analysis of modern epidemics.

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