



Optimal Control Strategy for a Typical Drug Abuse Model

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ABSTRACT

This study focused on harnessing certain control strategies for checkmating the menace of substance abuse in a typical demographic society. This research uses a deterministic mathematical model of substance abuse using (the 3 strategies) from the compartmentalized model. Building on the insights from epidemiology, our model entails controlling the spread of abusers' ideologies in society. We introduce a simple compartmental model suitable to describe the abuser group. The population is divided into six compartments: denoting the Susceptible, Light substance users, Heavy substance users, Temporal Quitters' class, Prosecuted compartment and Permanent Quitters populations respectively. The motive is basically to examine whether the substance abuser population will persist or on the other hand cease to exist with the introduction of the three control strategies. The generation Matrix method and algorithm for calculating the basic reproduction number was explicitly enumerated and The Pontryagin Maximum Principle was employed for our optimization of the system hence the plot-visibility of each of the substance-embedded compartments; conclusively showing a significant decline in the abusers' groups

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INTRODUCTION

Living under the influence of stimulants and charging substances have in general been directly associated with societal menace. Drug addiction amongst youths has been identified as one of the global menace that has affected negatively the development and productivity of young people which has come to stay from time pass to the present day and has resulted to various behavioural disorder such as criminality, psychiatric issues and death (Eguda, Ocheme, Sule, Abah and Hamza, 2022) The phenomenon of substance addiction are usually kept in check by the advanced and developed nations, these are usually enforced by strict drug use evaluation and monitoring mechanisms. Unfortunately, a number of developing nations have a very high number of its inhabitants that keeps abusing several substances with impunity.

Substance abuse is simply the use of a drug in amounts or by methods which are harmful

to the individual or others. It is a form of substance related disorder. Different definitions of substance abuse are used in Public Health, Medical and Criminal Justice contexts. In some cases, criminal or antisocial behaviour occurs when the individual is under the influence of a substance, and long-term personality changes in individuals may also occur. In addition to possible physical, social and psychological harm, the use of some drugs may also lead to criminal penalties, although these vary widely depending on the local jurisdiction.

According to (Abderrahim, Abdelfatah, Bouchaib, Omar and Mostafa 2020), it is estimated that 271 million people worldwide, which represents approximately 5.6 percent of the global population aged 15 to 64 years, used drugs at least once during 2017. About 35million of drugs users suffer from drug use disorders. As regards to the number of deaths, the Global Burden of Disease Study 2017 estimated that, globally, in 2017, there were 585,000 deaths and

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42 million years of “healthy” life lost as a result of the use of drugs.

Drugs such as tramadol, legally and legitimately prescribed by doctors for pain relief, are abused by millions in search of a fix or a release from poverty, unemployment and lack of opportunity. According to a survey in 2018 by the National Bureau of Statistics (NBS) and the Centre for Research and Information on Substance Abuse (CRISA) with technical support from the United Nations Office on Drugs and Crime (UNODC 2019), the prevalence of any drug use in Nigeria is estimated at 14.4% , corresponding to 14.3 million people aged 15-64 years who had used a psychoactive substance in the past year for non-medical purposes. The highest levels of drug use were among those aged 25-39 years which we can regard as youths.

Also, up to 67% of adolescents in urban Nigeria have used substances. Factors including experimental curiosity, peer pressure, poor socio-economic conditions at homes and the need for extra energy for daily activities have been identified as contributors to illicit drug use. These are precursors that are being magnified by COVID-19, with an increasing number of the Nigerian youth at risk of turning to substance abuse. In a world drug report by the UNODC in 2019, people worldwide who had used opioids - a class of drugs naturally found in the opium poppy plant - had surged by 56% and this was attributed to improved knowledge of the extent of drug use from new surveys conducted in two highly populated countries, namely India and Nigeria.

Furthermore, there is a strong correlation between drug addiction, rape and crime. Combating drug addiction and trafficking in Nigeria is a good first step to ending the menace of rapes and crimes in the country.

Some other examples of abused substances are Opium, Amphetamine, Cocaine, Alcohol, Ayahuasca, Central Nervous System Depressants, DMT, GHB, Hallucinogens, Heroin, Inhalants, Netamine, Khat, Kratom, LSD, MDMA (Ecstasy/Mocky), Mescaline (Peyote), Methamphetamine, Over - The - Counter Medicines (Dextromethorphan, Loperamide), PCP, Prescription Opioids, Prescription

Stimulants, Psilocybin, Rohypnol (Flunitrazepam), Salvia, Steroids (Anabolic), Synthetic Cathinones (Bat salts), Tobacco; NIDA(2013).

Abderrahim *et al.* (2020) employed four control strategies viz awareness programs through media and education, preventing contact through security campaigns, treatment, and psychological support along with follow-up and used Pontryagin's maximum principle in discrete time to characterize the optimal controls, carried out numerical simulation using MATLAB and consequently deduced the obtained results confirm the effectiveness of the optimization strategy.

Montemayor, Murfree, Nabil and Barry (2023) suggested that PG prior to live collegiate sporting events has clear public health and policy implications; and that future efforts should institute and enforce risk-management policies on college campuses and employ event-specific alcohol and polysubstance use prevention and intervention strategies.

Maarten, Sacha, Michael, Han, Reinout and Peter (2022) reviewed and classified a selection of formal models of addiction focusing on the intra- and inter-individual dynamics, i.e., (neuro) psychological models and social models; Found that these modeling approaches to addiction are too disjoint and argue that in order to unravel the complexities of biopsychosocial processes of addiction, models should integrate intra- and inter-individual factors.

Eguda *et al.* (2022) formulated a 5 sub-class model to analyze the population dynamics of drug addiction among youths and the effect of addiction on their development and productivity towards contributing their quota to the human population. The model was developed using ordinary differential equations approach and the basic reproduction number was obtained using the next generation method. Stability analysis was carried out and it was discovered that drug-free equilibrium exists and is locally and globally asymptotically stable whenever effective basic reproduction number is less than one; and unstable whenever effective basic reproduction number exceeds one. It concluded that drug abuse persisted among youths and are both

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locally and globally asymptotically stable whenever effective basic reproduction number is greater than one and unstable whenever effective basic reproduction number is less than one.

Winiker, Heidari, Pollock, Sodder and Tobin (2023) conducted findings which demonstrate opportunities to improve standards and systems such that primary care providers are better equipped to assess and treat the complex histories of trauma experienced by individuals with OUD.

Fikiri and Dmitry (2021) employed numerical simulation to establish that the dynamical behavior of drug abuse model, their result shows the significant contribution by the rate of adequate contact between the susceptible individual and the drug user, and the rate of recovery of drug user after rehabilitation as the major factors or players that dictate the dynamics of the drug user population in the society. The model also analyzed the dynamical behavior of drug abuse when control is applied, whose result shows the drug users are minimized significantly. Inferences from the work signifies the importance of early control of the drug abuse problem through the establishment of strict laws that will narrow the possibility of the bad practice in societies

Bobashev, Holloway, Solano and Gutkin (2017) have created an abstract nonlinear dynamical model of smoking based on the opponent-process and the allostatic theory. They use a system of five differential equations, each dependent on the previous, with the first representing the hedonic result of drug input. Each 'process' has a delay over the one it is based on. They assign abstract theoretical meaning to these equations, going from short term hedonic drug effect to the opponent-process, habit, and eventually long term hedonic memory of the drug which has a timescale of years. Then using these processes, they define a threshold for self-administration actions and equations for levels of withdrawal and craving.

Abubakar (2021) conducted a systematic search of the literature on PubMed to identify information on drug abuse and drug laws in Nigeria from the inception of the database to March 2020. They identified several drug laws and

policies that were established under government agencies such as the National Drug Law Enforcement Agency (NDLEA), National Agency for Foods and Drugs Administration and Control (NAFDAC), Pharmacists Council of Nigeria (PCN) and a Presidential Advisory Committee; and concluded that the burden of drug abuse is still high despite the existing drug laws, policies, and strategies for prevention. The work inferred that measures to reduce the burden should involve the community, government, religious bodies; and that preventive measures should target the youths, the students, identified sources of the drugs, reasons and risk factors associated with drug abuse in Nigeria.

Klein, Gyorda, Lekkas and Jacobson (2023) demonstrated that reports of dysregulated emotion were linked to a greater likelihood of trying both alcohol and tobacco products; The findings were also present when controlling for individual differences in executive control and socioeconomic status. Their results suggest that well-established links between dysregulated negative emotion and substance use may emerge early as in childhood and also suggest that children who experience excessive episodes of uncontrollable negative emotion may be at greater risk for trying substances early in life.

Keramati, Ahmed and Gutkin (2017) continue on the model of (Zhang, Berridge, Tindell, Smith and Aldridge, 2009) with the homeostatically regulated reinforcement learning model. Instead of a single internal variable, they propose that the reward is calculated depending on the internal state at the time of the action. Here the pathology is not in the rewards system but in the 'needs' machinery. They are able to show that seeking rewards is equivalent to the more fundamental objective that is physiological stability, and that a disruption can lead to over-consumption.

Subasish, Ly-Na and Magdalena (2020) highlighted the landscape of marijuana regulation which is changing now rapidly. Several countries have already legalized marijuana in some form, and marijuana for recreational purposes will soon become legal for a quarter of the U.S. population.

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(2023) discussed the treatment and strategy to reduce the number of drug users. Drug users can recover themselves by undergoing rehabilitation in the form of inpatient or outpatient care. They conducted qualitative analyses including stability analysis of equilibrium points of the model, the basic reproduction number and parameter sensitivity analysis. Mathematical model of drug abuse reduction by concerning type of treatment along with risk level without control has two equilibrium points, namely non-endemic or drug-free equilibrium and endemic equilibrium. Sensitivity analysis was provided to investigate which parameter that most affects the dynamical behaviour of the drug abuse model in terms of stability of the non-endemic and endemic equilibrium point, then imposed an anti-drug campaign on the model as strategy control to reduce the number of drug abusers. Simulation results showed that the anti-drug campaign has a significant effect in reducing both the number of drug abusers who received any treatment and do not get any treatment.

Muketo, Kimathi and Wainaina (2021) developed a SEIGWR model which is a mathematical model of drug abuse as an infectious disease with six compartments; susceptible, exposed, infected, guided and counseled, rehabilitated and recovered. The quantitative analysis of the study found out that the local stability existed when $R_0 < 1$. Numerical analysis of the drug abuse model determined that guidance and counseling affects the dynamics of the model and established that an increase in guidance and counseling leads to a decrease in the number of infections. The work suggested need for all the relevant stakeholders

in the secondary schools to increase the rate at which guidance and counseling is offered in the schools and that guidance and counseling would serve as a control strategy if given at an early age when the students are still in the primary schools.

Research findings by (Guijin, Emily, Jennifer, Marvin, Danielle, Elizabeth and Stelle, 2023) opine that Alcohol can have serious side effects alone and can enhance the side effects of prescription Opioids (like Tramadol) in unpredictable and dangerous ways; they thus suggested that further efforts are needed to minimize the increasing polysubstance-involved overdose mortality among young adults.

This work serves as an extension to the existing work by the inclusion of a prosecution compartment and thereby leading to jailing before the abuser gets to quit; temporally and/or permanently. A different mathematical concept is also employed (see Fig. 3.1).

In the coming sections, the model of substance abuse as a disease will be developed and analyzed with the effect of three control strategies.

METHODOLOGY

This work will divide the societal populace into six compartments depending on their substance use status, determine the common abused substances and their supposed assumption behind them, formulation of the mathematical model of substance abuse with Control Strategies.

The sum total of the entire population, $N(t)$, takes form;

$$N(t) = S(t) + L(t) + H(t) + Q_1(t) + P(t) + Q_2(t) \quad (2.1)$$

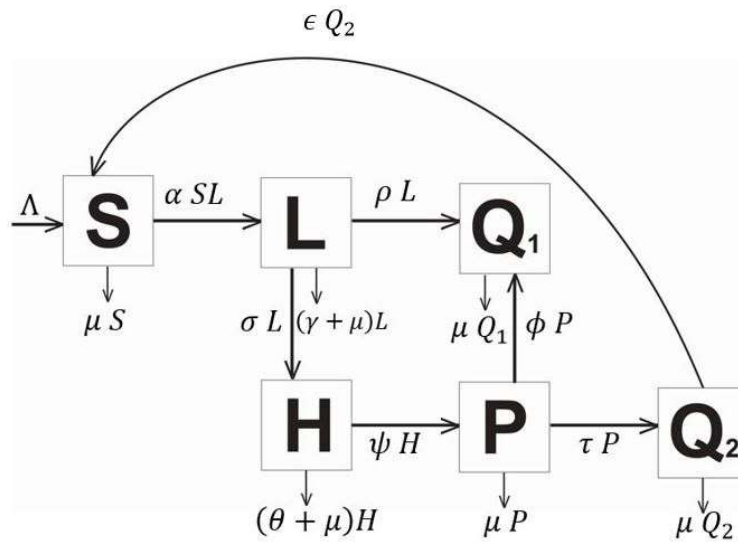


Figure 1: Model Diagram

- $S \Rightarrow$ Susceptible Class
 $L \Rightarrow$ Light Substance Users
 $H \Rightarrow$ Heavy Substance Users
 $Q_1 \Rightarrow$ Temporary quitters
 $P \Rightarrow$ Prosecution Class
 $Q_2 \Rightarrow$ Permanent quitters
- (2.2)

$$\begin{aligned}
 \frac{dS(t)}{dt} &= \Lambda + \epsilon Q_2 - (\mu + \alpha L)S \\
 \frac{dL(t)}{dt} &= \alpha SL - (\sigma + \gamma + \rho + \mu)L \\
 \frac{dH(t)}{dt} &= \sigma L - (\theta + \psi + \mu)H \\
 \frac{dQ_1(t)}{dt} &= \rho L + \phi P - \mu Q_1 \\
 \frac{dP(t)}{dt} &= \psi H - (\phi + \tau + \mu)P \\
 \frac{dQ_2(t)}{dt} &= \tau P - (\epsilon + \mu)Q_2
 \end{aligned}$$

(2.3)

Assumptions

1. The natural mortality rate is uniform for all compartments.
2. It is assumed that mortality can occur as a result of both the light and Heavy abuse of substance
3. Light substance users and heavy can not at any point have same population size.
4. All heavy users population will undergo apprehension.

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5. In the absence of substance, all variable(s) except S term goes to zero
6. There isn't a direct relationship between the Temporary and Permanent Quitters

The Substance Invariant Region

The population size N can be determined by

$$N(t) = S(t) + L(t) + H(t) + Q_1(t) + P(t) + Q_2(t), \quad (2.4)$$

The solution of the system remains positive at any point in time if the initial values of all the variables are positive.

Theorem 1

Consider $\Omega = \{S(t), L(t), H(t), Q_1(t), P(t), Q_2(t) \in \mathbb{R}^6, S(0) > 0, L(0) > 0, H(0) > 0, Q_1(0) > 0, P(0) > 0, Q_2(0) > 0\}$, then, the solution of $\{S(t), L(t), H(t), Q_1(t), P(t), Q_2(t)\}$ are positive for $t \geq 0$.

Proof :

Boundedness refers to the region in which solutions of the model or system is uniformly bounded in the proper subset $\Omega \subset \mathbb{R}_+^6$.

Looking at the total population at any time (t):

$$N(t) = S(t) + L(t) + H(t) + Q_1(t) + P(t) + Q_2(t), \quad (2.5)$$

$$\frac{dN(t)}{dt} = \Lambda - \mu S, \quad (2.6)$$

$$\left. \begin{aligned} \frac{dN}{dt} = & \left. \begin{aligned} & \Lambda + \epsilon Q_2 - (\mu + \alpha L)S + \alpha SL - (\sigma + \gamma + \rho + \mu)L + \sigma L \\ & - (\theta + \psi + \mu)H + \rho L + (\phi P - \mu)Q_1 + \psi H - (\phi + \tau + \mu)P \\ & + \tau P - (\epsilon + \mu)Q_2 \end{aligned} \right\} \\ \frac{dN}{dt} = & \left. \begin{aligned} & \Lambda + \epsilon Q_2 - \epsilon Q_2 - \mu S - \alpha SL + \alpha SL - \sigma L + \sigma L - \gamma L - \rho L \\ & + \rho L - \mu L - \theta H - \psi H + \psi H - \mu H + \phi P - \phi P - \mu Q_1 - \tau P \\ & + \tau P - \mu P - \mu Q_2 \\ & = \Lambda - \mu S - \gamma L - \mu L - \theta H - \mu H - \mu Q_1 - \mu P - \mu Q_2 \\ & \frac{dN(t)}{dt} = \Lambda - \mu S - (\gamma + \mu)L - (\theta + \mu)H - \mu(Q_1 + P + Q_2). \end{aligned} \right\} \end{aligned} \quad (2.7)$$

There exists no Light and Heavy users(L and H), Temporary Quitters Q_1 , Prosecution(P), and Permanent Quitters Q_2 in the absence of drug. Thus $L = 0, H = 0, Q_1 = 0, P = 0, Q_2 = 0$.

The equation thus becomes

$$\frac{dN}{dt} = \Lambda - \mu S. \quad (2.8)$$

$$\frac{dN}{dt} = \Lambda - \mu S \leq \Lambda - \mu N. \quad (2.9)$$

On integrating both sides of the equation

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$$\left. \begin{aligned} \int \frac{dN}{\Lambda - \mu N} &\leq \int dt + C \\ -\frac{\ln(\Lambda - \mu N)}{\mu} &\leq t + C \\ \ln(\Lambda - \mu N) &\leq t + C \\ \ln(\Lambda - \mu N) &\geq -\mu(t + C) \\ (\Lambda - \mu N) &\geq e^{-\mu(t+C)} \\ (\Lambda - \mu N) &\geq e^{-\mu t} \cdot e^{-\mu C} \end{aligned} \right\} \quad (2.10)$$

where $e^{-\mu C} = K$. Then $\Lambda - \mu N \geq K e^{-\mu t}$.

Utilizing the condition at $N(0) = S(0)$, $\Lambda - \mu N(0) = K$.

Therefore, $\Lambda - \mu N = (\Lambda - \mu N(0))e^{-\mu(t)}$. Further simplification yields

$$N \leq \frac{\Lambda}{\mu} - \left(\frac{\Lambda - \mu N(0)}{\mu}\right)e^{-\mu(t)} \quad (2.11)$$

As time tends to infinite, i.e. $t \rightarrow \infty$, then the side of population $N \rightarrow \frac{\Lambda}{\mu}$.

Deductively, $0 \leq N \leq \frac{\Lambda}{\mu}$ and $N(t) \leq \frac{\Lambda}{\mu}$. Hence,

$$\Omega = \left\{ (S, L, H, Q_1, P, Q_2) \in \mathbb{R}_+^6 : S + L + H + Q_1 + P + Q_2 \leq \frac{\Lambda}{\mu} \right\}. \quad (2.12)$$

Basic Reproduction Number of Substance Abuse R_0

This is the average number of secondary cases per infectious case in a population made up of both susceptible and non-susceptible hosts. Here, we term it expected number of new substance abusers that arose by initial abuser in a susceptible population. Thus, R_0 is seen as a Threshold term to determine the condition for abusers outbreak

Here, we will use the concepts of Next Generation Matrix (Shuaibu, Oluwole and David, 2018) to establish the linear stability of the disease-free equilibrium. The basic reproduction

number is the number of secondary infections produced by one infected person in a completely susceptible population.

The reproduction number combines the biology of infections with the social and behavioral factors causing contact rates (Van den Driessche P., 2017), (Victor et. al., 2022).

The basic reproduction number R_0 is defined as the spectral radius of the next generation matrix.

Definition: The spectral radius of a matrix X is defined as the maximum of the absolute values of the matrix

$$i.e. \rho(X) = \sup\{|\lambda| : \lambda \in \rho(X)\} \quad (2.13)$$

with $\rho(X)$ representing the set of eigenvalues of the matrix X .

Substance Abuse Basic Reproduction Number by Next Generation Matrix

Theorem Li and Schneider (2002) stated that suppose F a non zero non negative matrix and V a non-singular M -matrix such that $F - V$ is irreducible.

Then

$$R_0 = \rho(FV^{-1}) \quad (2.14)$$

and R_0 is the reciprocal of the smallest positive root x of the polynomial equation

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Proof:

From the assumptions, FV^{-1} is a non-zero non-negative matrix, and by Li [*et al.* (2002) and Perron-Frobenius theory the principal submatrix corresponding to the non-zero rows of F is irreducible.

Thus $\lambda = \rho(FV^{-1}) > 0$ is the largest positive root of the polynomial equation $\det(\lambda I - FV^{-1}) = 0$. Since V is non-singular and $\lambda > 0$, the polynomial equation is equivalent to $\det(F\lambda^{-1} - V) = 0$

(where $V - F\lambda^{-1}$ is a singular M-matrix) and λ^{-1} is the smallest positive root. The proof suffices from the definition of $R_0 = \rho(FV^{-1})$.

Next Generation Matrix: This is the method used to derive the basic reproduction number, for a compartmental model of the spread of substance abuse.

Deriving R_0 with the next generation matrix (Van den Driessche, 2017), Yekeen, & Ibrahim (2022) is sometimes difficult whenever the dimension of the model is increasing. An option of solution to this problem is employing the graph-theoretic method (de-Camino-Beck, Lewis and Van den Driessche, 2009) to derive the reproduction number R_0 in a simpler form.

Computing R_0 , we first assume $f_i(x)$ the rate of appearance of new abusers in compartment i , and $v_i(x)$ the net reducing rate of substance abusers in compartment i due to substance abusers flow inside system of abusers compartments.

Supposing

$$\begin{aligned} F &= (f_1, \dots, f_{3n}) \quad \text{and} \\ V &= (v_1, \dots, v_{3n}), \end{aligned} \tag{2.15}$$

with

$$\begin{aligned} F &= \begin{pmatrix} \frac{\partial F_1}{\partial L} & \frac{\partial F_1}{\partial H} & \frac{\partial F_1}{\partial J} \\ \frac{\partial F_2}{\partial L} & \frac{\partial F_2}{\partial H} & \frac{\partial F_2}{\partial J} \\ \frac{\partial F_3}{\partial L} & \frac{\partial F_3}{\partial H} & \frac{\partial F_3}{\partial J} \end{pmatrix} (x_0) \\ V &= \begin{pmatrix} \frac{\partial v_1}{\partial L} & \frac{\partial v_1}{\partial H} & \frac{\partial v_1}{\partial J} \\ \frac{\partial v_2}{\partial L} & \frac{\partial v_2}{\partial H} & \frac{\partial v_2}{\partial J} \\ \frac{\partial v_3}{\partial L} & \frac{\partial v_3}{\partial H} & \frac{\partial v_3}{\partial J} \end{pmatrix} (x_0) \end{aligned} \tag{2.16}$$

Suppose $X = (S, L, H, P, R, C,)^T$, thus F and V can be derived from the model equation; also with its Jacobian matrices at substance-free equilibrium.

Thus $R_0 = \Gamma(FV^{-1})$ with Γ , the spectral radius

Using the Next Generation Matrix, we can derive the threshold value for substance abuse.

We recall that set of D.Es for Model one is;

$$\begin{aligned} \frac{dS(t)}{dt} &= \Lambda + \epsilon R - \beta S(1 + \alpha L) - \mu S \\ \frac{dL(t)}{dt} &= \beta S(1 + \alpha L) - (\sigma + \gamma + \mu + \rho)L \\ \frac{dH(t)}{dt} &= \sigma L - (\psi + \theta + \mu)H \\ \frac{dP(t)}{dt} &= \psi H - \mu P + (\phi - \tau)P \\ \frac{dR(t)}{dt} &= \rho L + \phi P - (\epsilon + \mu)R \\ \frac{dC(t)}{dt} &= \tau P - \mu C \end{aligned} \tag{2.17}$$

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For the purpose of R_0 derivation, we consider parts in P to still be infected before movement to Q_2
 Setting the Jacobian matrix of the system of differential equations, we have

$$F(t) = \begin{bmatrix} \alpha L & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.18)$$

$$V(t) = \begin{bmatrix} (\sigma + \gamma + \rho + \mu) & 0 & 0 & 0 \\ -\sigma & (\theta + \psi + \mu) & 0 & 0 \\ -\rho & 0 & \mu & -\phi \\ 0 & -\psi & 0 & \phi + \tau + \mu \end{bmatrix} \quad (2.19)$$

$$F(t) = \begin{bmatrix} \alpha L & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.20)$$

suppose $Z = \frac{1}{V}$ i.e V^{-1} , thus

$$Z = \begin{bmatrix} \frac{(\sigma + \gamma + \rho + \mu)^{-1}}{\sigma} & 0 & 0 & 0 \\ \frac{(\theta + \psi + \mu)(\sigma + \gamma + \rho + \mu)}{(\phi + \tau + \mu)(\theta + \psi + \mu)\rho + (\phi, \psi, \sigma)} & (\theta + \psi + \mu)^{-1} & 0 & 0 \\ \frac{(\phi + \tau + \mu)(\theta + \psi + \mu)\rho + (\phi, \psi, \sigma)}{(\sigma + \gamma + \rho + \mu)(\theta + \psi + \mu)\mu(\phi + \tau + \mu)} & \frac{\phi, \psi}{(\theta + \psi + \mu)(\phi + \tau + \mu)\mu} & (\mu)^{-1} & \frac{\phi}{(\phi + \tau + \mu)\mu} \\ \frac{\phi, \sigma}{(\sigma + \gamma + \rho + \mu)(\theta + \psi + \mu)\mu(\phi + \tau + \mu)} & \frac{\psi}{(\theta + \psi + \mu)(\phi + \tau + \mu)} & 0 & (\phi + \tau + \mu)^{-1} \end{bmatrix} \quad (2.21)$$

$$F \cdot V^{-1} = \begin{bmatrix} \frac{\alpha L}{\sigma + \gamma + \rho + \mu} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.22)$$

$$R_0 = \Gamma(F \cdot V^{-1}) \quad (2.23)$$

$$R_0 = \frac{\alpha L}{\sigma + \gamma + \rho + \mu} \quad (2.24)$$

Thus;

$$R_0 = \Gamma(F \cdot V^{-1}) = \frac{\alpha L}{\sigma + \gamma + \mu + \rho} \quad (2.25)$$

We deduce that; the substance-free equilibrium $E_0 \left(\frac{\Lambda}{\mu}, 0, 0, 0, 0 \right)$ associated with the mode set of D.Es is locally asymptotically stable if $R_0 < 1$.

E_0 becomes unstable if and only if $R_0 > 1$ i.e. the act of abuse of substance is prevalent.

Extension of Model Four to Optimal Control

(2007) optimal control model is a strong mathematical method that can be used in

According to Lenhart and Workman

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analysing a complex situation. The control enters the model equations of ordinary differential equations and touches the dynamics of the system. The goal is to change the control to (maximize or minimize) a given objective functional. Similarly, the problem needs a cost function $J[x(t), u(t)]$, a set of state variables $x(t) \in X$ and a set of control variables $u(t) \in U$ in a time $t_0 \leq t \leq t_f$, where; t_0, t_f are both initial and final time respectively. The main aim is

to find a control $u(t)$ and the related state variables $x(t)$ to minimize a given objective function. The optimal control approaches in this research focuses on;

- (i) Proper counselling,
- (ii) Media sensitization against substance abuse
- (iii) Medical-psychological therapies.

The incorporated optimal control approaches are given by non-linear differential equations in the form;

$$\left. \begin{aligned} \frac{dS}{dt} &= \Lambda + \epsilon Q_2 - (\mu + \alpha(1 - u_2)L)S \\ \frac{dL}{dt} &= \alpha(1 - u_2)LS - (\sigma + \gamma + \rho + \mu + u_1)L \\ \frac{dH}{dt} &= \sigma L - (\theta + \psi + \mu + u_3)H \\ \frac{dQ_1}{dt} &= (\rho + u_1)L + (\phi + u_3)P - \mu Q_1 \\ \frac{dP}{dt} &= (\psi + u_3)H - (\phi + \tau + u_3 + u_3 + \mu)P \\ \frac{dQ_2}{dt} &= (\tau + u_3)P + u_1L - (\mu + \epsilon)Q_2 \end{aligned} \right\} \quad (3.1)$$

Table 1: Parameter Values

Parameter	Value	Unit	Source
Λ	1500	people/year	Assumed
α	0.00008	1/year	Analysis
μ	0.02	1/year	Li & Ma (2018)
σ	0.03	1/year	Li & Ma (2018)
ρ	0.005	1/year	Analysis (Assumed)
γ	0.2	1/year	Li & Ma (2018)
θ	0.4	1/year	Assumed
ψ	0.421	1/year	Li & Ma (2018)
ϕ	0.25	1/year	Li & Ma (2018), i.e $1 - \phi = \tau$
τ	0.75	1/year	Analysis (Assumed)
ϵ	0.2	1/year	Analysis (Assumed)

Table 2: Initial Conditions for each point for variables and source

Variables	Value	Source
$S(t)$	500	UNODC (2019)
$L(t)$	250	''
$H(t)$	150	''
$Q_1(t)$	40	''
$P(t)$	100	''
$Q_2(t)$	100	''

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Three control variables are recommended to get the optimal result necessary to minimize the number of abusers population thereby reducing drug menace in the society. The controls are u_1 , u_2 and u_3 defined as the weight of the individual as afore-highlighted

According to (Abidemi et.al., 2024, Abioye, et.al., 2021; Ayoola, et.al.,2021; Oguntolu, et.al., 2023; Ojo, et.al.,2023; Ojo, et.al.,2022; Peter, et.al.,2020; Peter, et.al.,2021; Peter, et.al.,2023), the objective function is defined as

$$J(u_1, u_2, u_3) = \int_0^T \left(a_1 L + a_2 H + \frac{1}{2} \sum_{i=1}^3 w_i u_i^2 \right) dt \tag{3.2}$$

where (t_0, t) are the initial and final time respectively.

The objective function is to find a set of controls that minimizes substance abuse menace by minimizing and implementing the controls $u_1(t)$, $u_2(t)$ and $u_3(t)$.

Suppose u_1^* , u_2^* , u_3^* are the optimal controls such that

$$J(u_1^*, u_2^*, u_3^*) = \min J(u_1, u_2, u_3) | (u_1, u_2, u_3) \in U \tag{3.3}$$

subject to the system (4.1) where the set of Lebesgue measure on $[t_0, t]$ to $[0,1]$

$$\min J(u_1, u_2, u_3) = \int_{u_0}^w \left[a_1 L + a_2 H + \frac{1}{2} (w_1 u_1^2 + w_2 u_2^2 + w_3 u_3^2) \right] \tag{3.4}$$

$$J(u_1, u_2, u_3) = \min J(u_1, u_2, u_3) | u_1, u_2, u_3 \in U$$

$$\text{Hamiltonian, } H = a_1 L + a_2 H + \frac{1}{2} \sum_{i=1}^3 w_i u_i^2 + \sum_{i=1}^6 \lambda_i f_i \tag{3.5}$$

where $f_1 = \frac{dS}{dt}$, $f_2 = \frac{dL}{dt}$, ..., $f_6 = \frac{dQ_2}{dt}$ and λ_i is the adjoint corresponding to each of the state variable.

The optimal controls u_1^* , u_2^* , u_3^* are then solved from the optimality condition as follow

$$\frac{\partial H}{\partial u_i} = 0 \quad i = 1, 2, 3 \tag{3.6}$$

$$\frac{\partial H}{\partial u_1} = w_1 u_1 - L(\lambda_2 - \lambda_4 - \lambda_6) = 0 \tag{3.7}$$

$$\frac{\partial H}{\partial u_2} = w_2 u_2 - \alpha LS(\lambda_1 - \lambda_2) = 0 \tag{3.8}$$

$$\frac{\partial H}{\partial u_3} = w_3 u_3 + (\lambda_5 - \lambda_3)H + P(\lambda_4 + \lambda_6 - 2\lambda_5) = 0 \tag{3.9}$$

and

$$\begin{aligned} u_1 &= \frac{L(\lambda_2 - \lambda_4 - \lambda_6)}{w_1} \\ u_2 &= \frac{\alpha LS(\lambda_1 - \lambda_2)}{w_2} \\ u_3 &= \frac{H(\lambda_3 - \lambda_5) + P(2\lambda_5 - \lambda_4 - \lambda_6)}{w_3} \end{aligned}$$

Next, consider the bound on the controls to have

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$$\begin{aligned} u_1^* &= \min\{1, \max\{0, u_1\}\} \\ u_2^* &= \min\{1, \max\{0, u_2\}\} \\ u_3^* &= \min\{1, \max\{0, u_3\}\} \end{aligned}$$

Setting $a_1 = 0.1$, $a_2 = 0.5$, $w_1 = 10$, $w_2 = 15$ and $w_3 = 20$.

The incorporated optimal control system is in (3.1) and the adjoint variable system by combining the characterizing control set with initial and transversal condition gives;

$$\left. \begin{aligned} \frac{dS}{dt} &= \Lambda + \epsilon Q_2 - (\mu + \alpha(1 - u_2)L)S \\ \frac{dL}{dt} &= \alpha(1 - u_2)LS - (\sigma + \gamma + \rho + \mu + u_1)L \\ \frac{dH}{dt} &= \sigma L - (\theta + \psi + \mu + u_3)H \\ \frac{dQ_1}{dt} &= (\rho + u_1)L + (\phi + u_3)P - \mu Q_1 \\ \frac{dP}{dt} &= (\psi + u_3)H - (\phi + \tau + u_3 + u_3 + \mu)P \\ \frac{dQ_2}{dt} &= (\tau + u_3)P + u_1L - (\mu + \epsilon)Q_2 \end{aligned} \right\} \quad (3.10)$$

$$\left. \begin{aligned} \frac{d\lambda_1}{dt} &= -\frac{\partial H_*}{\partial S} = \lambda_1(\mu + \alpha(1 - u_2)L) - \lambda_2\alpha(1 - u_2)L \\ \frac{d\lambda_2}{dt} &= -\frac{\partial H_*}{\partial L} = \alpha(1 - u_2)(\lambda_1 - \lambda_2)S + \lambda_2(q_1 + u_1) - \lambda_3\sigma - \lambda_4(u_1 + \rho) - \lambda_6u_1 - a_1 \\ \frac{d\lambda_3}{dt} &= -\frac{\partial H_*}{\partial H} = -a_2 + q_2\lambda_3 + (\lambda_3 - \lambda_5)(\psi + u_3) \\ \frac{d\lambda_4}{dt} &= -\frac{\partial H_*}{\partial Q_1} = \mu\lambda_4 \\ \frac{d\lambda_5}{dt} &= -\frac{\partial H_*}{\partial P} = \lambda_5(q_x + 2u_3 + \mu) - (\lambda_4 + \lambda_6)u_3 - \lambda_4\phi - \lambda_6\tau \\ \frac{d\lambda_6}{dt} &= \lambda_6q_3 - \epsilon\lambda_1 \end{aligned} \right\} \quad (3.11)$$

Subject to the following transversality conditions $\lambda_i(T) = 0$ for all $i = 1, 2, \dots, 6$.

where

$$\begin{aligned} q_1 &= \sigma + \gamma + \rho + \mu \\ q_2 &= \theta + \psi + \mu \\ q_3 &= \mu + \epsilon \\ q_x &= \phi + \tau \end{aligned}$$

Graphical Solution with Effects of Control variables on the Model

Proper counselling ($u_1(t)$)

Media sensitization ($u_2(t)$)

Medical-psychological therapies ($u_3(t)$)

Table 3: List of Optimal Control Strategies

3*Single Control	Strategy I	$u_1 \neq 0, u_2 = 0, u_3 = 0$
	Strategy II	$u_1 = 0, u_2 \neq 0, u_3 = 0$
	Strategy III	$u_1 = 0, u_2 = 0, u_3 \neq 0$
3*Double Control	Strategy IV	$u_1 \neq 0, u_2 \neq 0, u_3 = 0$

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	Strategy V	$u_1 \neq 0, u_2 = 0, u_3 \neq 0$
	Strategy VI	$u_1 = 0, u_2 \neq 0, u_3 \neq 0$
Triple Strategy	Strategy VII	$u_1 \neq 0, u_2 \neq 0, u_3 \neq 0$
Without Control		$u_1 = 0, u_2 = 0, u_3 = 0$

Numerical Simulation

The influence of implementing the single strategies on the population of substance users is depicted in Figure (2). The figure reveals that the population of substance users reduces throughout the simulation period when Strategy I is applied and compared when no control is used. The zoom plot from Figure (2) shows that during the first two years, there are more substance

users when no control is used than when Strategy III is applied. Figure (2) points out that during the first five years, there is no noticeable difference in the population of substance users when no control and Strategy II are used. Lastly, Figure (2) depicts that Strategy I is the best single strategy required to reduce the number of substance users when compared with no control.

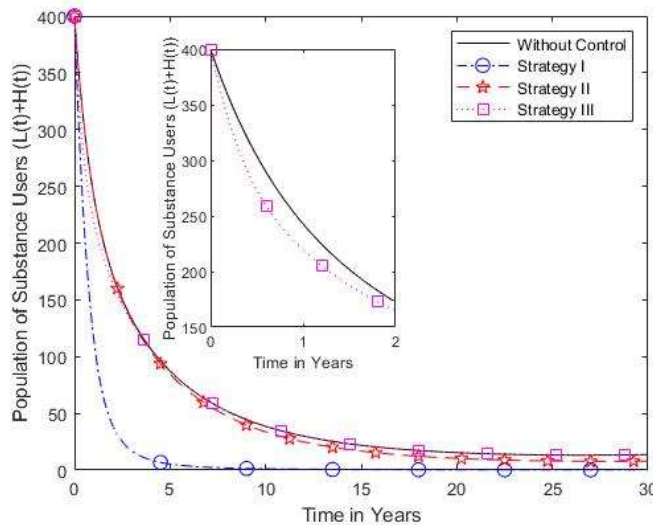


Figure 2: Population of Substance Users with Single Strategies and Without Control

The optimal control profile for each of the single strategies is shown in Figure (3). This figure suggests the best way to implement each of the strategies. Figure (4.9a) reveals that during the first year, the effort to implement only proper counselling (Strategy I) should be intensified before gradually decreasing to zero. The implementation of Strategy II as shown in Figure

(4.9b) points out that the effort required for media sensitization should gradually increase to about 24% during the first ten years before declining to zero while the use of medical-psychological therapies should gradually decrease from the upper bound to zero. Finally, Figure (3) suggests that media sensitization requires a longer time to implement than the other controls.

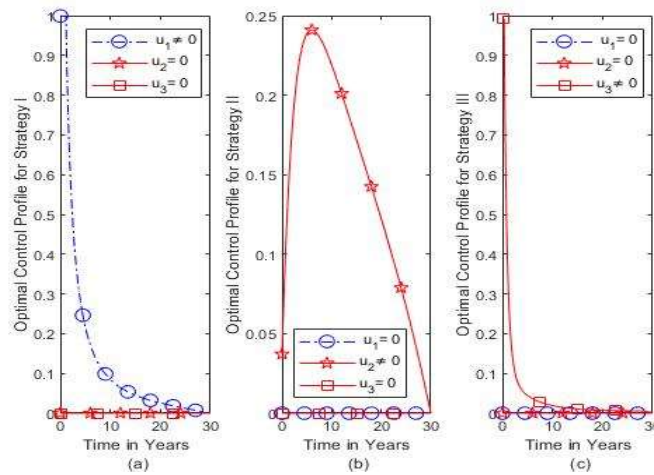


Figure 3: Optimal Control Profile for Single Strategies

Figure (4) shows the impact of the double strategies compared to the without-control strategy. The figure suggests that the application of double strategies is beneficial in reducing the

population of substance users. Strategy V is seen to have the least population of substance users when compared with the other strategies.

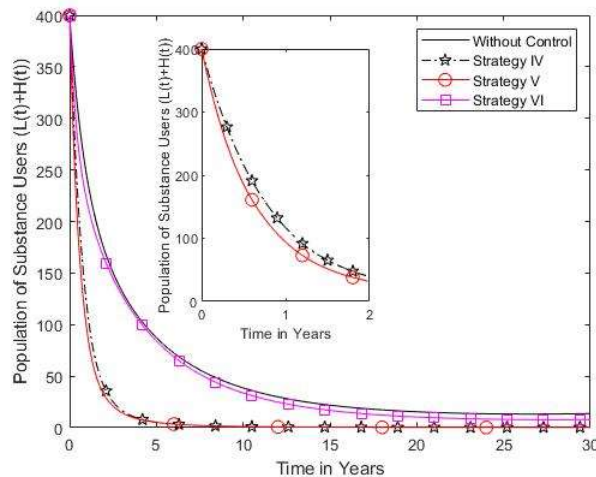


Figure 4: Population of Substance Users with Double Strategies and Without Control

How best to implement the double strategies are displayed in Figure (5). Figure (4.11a) shows that in implementing Strategy IV, proper counselling should be practised from the upper bound for about a year before declining to

zero, while media sensitization should be applied from the lower bound till it gets to 3% before dropping to zero. Figure (4.11b) depicts that Strategy V requires that both medical-psychological therapies and proper counselling

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shall be intensified for about 6 months and a year before rapidly diminishing to zero at about 7 years and 28 years respectively. The implementation of Strategy VI as described by Figure (4.11c) entails that media sensitization is applied from the lower

bound to 25% at about 7 years then declines to zero at about 30 years while effort should be intensified for medical-psychological therapies for about 6 months and decline to zero at about 20 years.

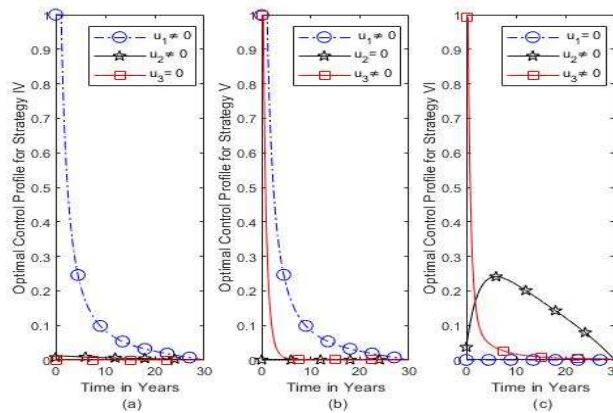


Figure 5: Optimal Control Profile for Double Strategies

In Figure (6), there is a decrease in the population of substance users when the triple controls are used compared to when no control is

used. This means that the application of Strategy VII is beneficial in the reduction of substance users.

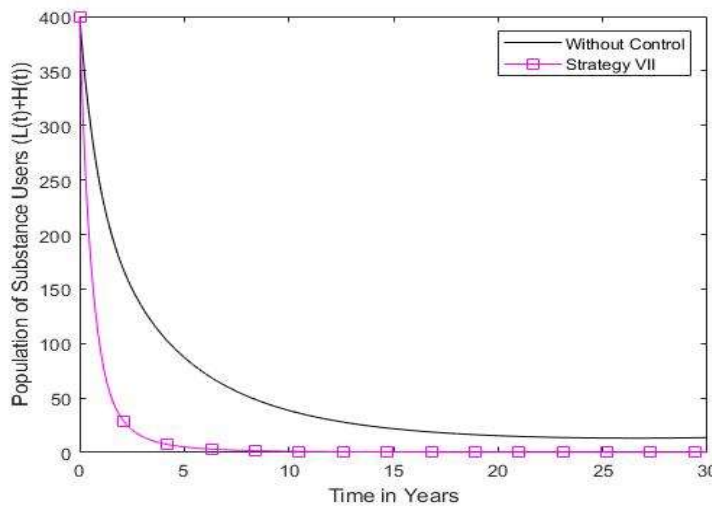


Figure 6: Population of Substance Users with Triple Control and Without Control

Figure (7) shows the optimal control profile for the triple controls (Strategy VII). The

figure suggests that proper counselling should be applied from the upper bound for about a year

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before declining to zero, media sensitization should be applied from 1% for about a month and then decline to zero while medical-psychological therapies from the upper bound for about 4

months before dropping to zero. This means that the implementation of proper counselling requires a longer duration compared to the other controls for Strategy VII to be best utilized.

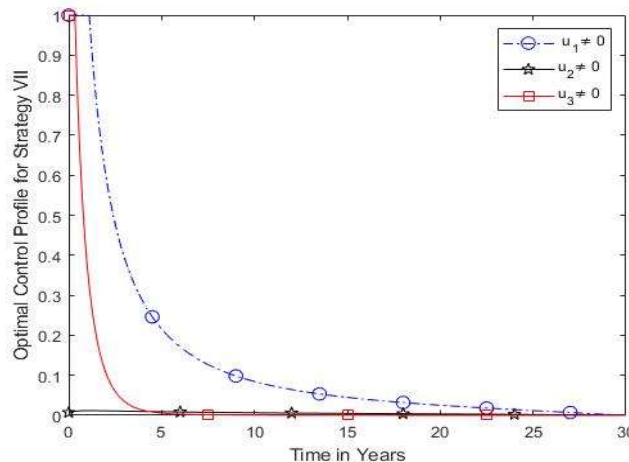


Figure 7: Optimal Control Profile for Triple Controls

RESULTS AND DISCUSSION

This research centers on an Optimal control technique to checkmate the menace of substance abuse in a typical population which with applications of control parameters on the Light and Heavy substance users reveals on analysis that Proper counselling, Media sensitization and also Medical and psychological therapies significantly diminishes the abuse of substance, over intervals of time (see Fig. above) We also considered the approach of deriving the basic reproduction number R_0 , known as the Next Generation Matrix method for modelling substance abuse. This approach is equivalent to Gaussian elimination and digraph reduction method, and since the rules of digraph reduction method does not take the path of direct calculation of matrix inverse or determinants, the latter approach can be practical for larger systems.

CONCLUSION

In this research, the next generation matrix method was employed to compute the threshold number for the use and abuse of substance, R_0 . The use and abuse of substances

would continue to spread in the population if the substance abuse number is greater than one and the abuse of substances would die out of the system if the abuse number is less than one (

The optimal control approach to the spread of substance abusers population was examined using Proper counselling, Media sensitization against substance abuse and also Medical and psychological therapies as strategies for control and they clearly demonstrate significant reductions to the abusers population in the society.

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