



# Modeling the Endemic Stability of Covid-19 in Nigeria

Okorie Charity Ebelechukwu, 2Haruna Yusuf, 2Bala Ma'aji Abdulhamid, 3Hina Aliyu Danladi Department of Mathematics & Statistics, Federal University Wukari Department of Mathematics, Abubakar Tafawa Balewa University, Bauchi Department of Mathematical Sciences, Gombe State University, Gombe

## **ABSTRACT**

COVID-19 came as the most fearful infectious disease. It started with massive elimination of lives. Based on the rate of spread, people all over the world thought that the world has come to an end. Researchers were working round the clock in order to look for a way to control the spread of the disease. Many researchers have used many methods to study the transmission dynamics of infectious diseases. In this research paper, a compartmental mathematical model was formulated which will provide the needed solution of control of the disease. To check the spread and control of the disease, apply stability analysis of COVID-19 in Nigeria. The researchers also checked the basic reproduction number of the model which is a determining factor for the stability analysis. The result shows that COVID-19 is globally asymptotically stable, meaning that the spread is dying down.

## **INTRODUCTION**

Corona virus is a disease that made the whole country to be stationary. This is because, started with rampant destruction of lives, stampeding economic and social activities. This made COVID-19 to be the most dreaded disease that has ever surfaced in the face of the world. The emergency of COVID-19 motivated the researchers into carrying out research work so as to see if there is a way to control its spread thereby preventing the incessant death associated with COVID-19. From the literatures, we observed that a lot of mathematical models have been formulated. The essence of all these is to seek for a lasting solution to stop the spread of the disease. COVID-19 is a new form of a typical pneumonia. It was first reported in the city of Wuhan in December 2019, and has spread rapidly to other areas of China and multiple countries (Chen, 2019). Corona virus disease is likely to emerge as a watershed moment in the history of the planet. COVID-19, the abbreviation of corona virus disease (2019), is caused by a severe acute respiratory syndrome corona virus 2 (SARS-CoV2) (WHO, 2019), which hit the globe with a bang. In December 2019, the first outbreak was

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noticed in Hubei province, Wuhan, China (Hui et al, 2020).

The COVID-19 outbreak has generated significant pandemic worry. Pandemic worry refers to the constant, uncontrollable, and unwanted negative thoughts about the future outcomes of disease (Scherr et al, 2017).

Despite the fact that there were multiple crises in recent years, such as natural disasters, economic crises, and even epidemics, the corona virus pandemic is the first in 100 years to severely affect the entire world. The economic effects of the COVID-19 Pandemic concern an impending global recession caused by the lockdown of nonessential industries and the disruption of production and supply chains (Wong and Tang (2005).

To slow the spread of COVID-19, the Centre for Disease Control and Prevention (CDC) has recommended that individuals adopt certain preventive health behaviours. These recommendations include hand washing with soap and water or hand sanitizer, keeping a sixfeet social distance from other people, and covering the mouth and nose with a cloth face mask when around others (CDC, 2019). Some

Corresponding author: Okorie Charity Ebelechukwu

Department of Mathematics and Statistics, Federal University, Wukari, Taraba State, Nigeria.





studies showed that seeking social support is one of the most common strategies used to deal with the corona virus pandemic (Ogueji and Okoloba, 2021;Chew et al (2020).

Person-to-person household transmission of SARS-CoV-2 has been described in China (Ji et al (2020), Quan et al (2020).

 Okorie et al (2024). In their work titled "COMPARATIVE STUDY AND SIMULATION OF COVID-19 MODELS") gleaned two existing Covid-19 models and carryout some modifications on them. The authors employed a simulation case study methodology to conduct an in-depth analysis of the models in a bid to determine the dynamics of the disease, how fast it can be controlled, which model among them performs better, and what key policy lessons can be drawn from the behavioral dynamics of all the models. Their result shows that the models were good but the modified model performed better than the existing models.

# MATERIALS AMD METHODS

The model equations

$$
\frac{dS}{dt} = \pi - (1 - P_1)(\beta_1 I_A + S\beta_3 I_S)S - (1 - P_2)S\beta_1 V + ZV_c + nQ + R\varpi_1 - (\Gamma + \mu_1)S
$$
\n(1)

$$
\frac{dE}{dt} = (1 - p_{1})(\beta_{DA}I_{A} + \beta_{DS}I_{S})S + (1 - p_{2})S\beta_{1}V - (\delta + \varpi_{2} + \mu_{1})E
$$
\n(2)

$$
\frac{dI_A}{dt} = (1-r)\delta E - (\rho_A + \gamma_A + \omega_1 + \mu_1)I_A
$$
\n(3)

$$
\frac{dI_s}{dt} = r\delta E - (\rho_s + \mu_s + \gamma_s + \omega_z + \mu_l)I_s
$$
\n(4)

$$
\frac{dI_H}{dt} = \alpha_A \gamma_A I_A + \alpha_S \gamma_S I_S + \nu I_W - (\rho_H + \mu_H + \mu_I) I_H
$$
\n(5)

$$
\frac{dI_w}{dt} = (1 - \alpha_A) \gamma_A I_A + (1 - \alpha_S) \gamma_S I_S + mQ - (\nu + \rho_w + \mu_w + \mu) I_w
$$
\n(6)

$$
\frac{dR}{dt} = \rho_A I_A + \rho_s I_s + \rho_H I_H + \rho_w I_w - (\varpi_1 + \mu_1)R
$$
\n(7)

Corresponding author: Okorie Charity Ebelechukwu

Department of Mathematics and Statistics, Federal University, Wukari, Taraba State, Nigeria.

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Figure 1: Flow diagram of the modified model

## THE BASIC REPRODUCTION NUMBER

The basic reproduction number is defined as the number of cases directly caused by an infected individual throughout his infected period (Dharmaratne et al, 2020). Basic reproduction number is used to determine the ability of a disease to spread with a given population. It is also used to study the way of disease spread (Esteban, 2016).



cannot invade the population, but if  $R_0 > 1$ , then the DFE is unstable and invasion is possible. if  $R_0 < 1$ , then on average, an infected individual produces less than one new infected individual over the cause of the infectious period, and once the infection cannot grow (Driessche and Watmough,2008).

Calculating the Reproduction Number using the Next Generation Matrix Approach (Van den Driessche and Watmough, 2002). Since there are

Corresponding author: Okorie Charity Ebelechukwu

okoriec636@gmail.com

Department of Mathematics and Statistics, Federal University, Wukari, Taraba State, Nigeria.

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only 6 infected compartments  $E, \int_A, \int_S, \int_H, \int_W, V$  it follows that we are going to have  $6 \times 6$  matrices. Recall DFE is  $E^* = I_A^* = I_S^* = I_H^* = I_W^* = V = 0$  and  $S^* = V_C^* \neq 0$ 

Following the notations of Vanden Driesseche and Watmough (2002), we have the following

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\ninfected compartments  
\n
$$
\overline{I}_H, \overline{I}_W, \overline{I'}_W, \overline{V} \text{ it follows that we are}
$$
\n
$$
\overline{I}^* = \overline{I}_A^* = \overline{I}_S^* = \overline{I}_H^* = \overline{I}_W^* = \overline{V} = 0 \text{ and } \overline{S}^* = \overline{V}_C^* \neq 0
$$
\n
$$
\text{vations of Vanden Driesseche and Watmough (2002), we have the following}
$$
\n
$$
\begin{pmatrix}\n(1 - p_1)(\beta_{DA}I_A + \beta_{DS}I_A) + (1 - p_2)S\beta_1I' \\
0 \\
0 \\
0 \\
0 \\
0\n\end{pmatrix}
$$
\n
$$
f = \begin{pmatrix}\n(1 - p_1)(\beta_{DA}I_A + \beta_{DS}I_A) + (1 - p_2)S\beta_1I' \\
0 \\
0 \\
0 \\
0 \\
0 \\
0\n\end{pmatrix}
$$

(11)

This is the inflow of disease into the infected compartment, that is the force of infection.



This is obtained by partially differentiating the inflow with respect to each of the infected compartments.

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\n
$$
V = \begin{pmatrix}\n(\delta + \mu_1 + \varpi_2) E \\
-(1-r)\delta E + (\rho_A + \gamma_A + \omega_1 + \mu_1) I_A \\
-r\delta E + (\rho_S + \mu_S + \gamma_A + \omega_1 + \mu_1) I_S \\
-\alpha_A \gamma_A I_A - \alpha_S \gamma_S I_S - \nu I_W + (\rho_H + \mu_H + \mu_1) I_H \\
-(1-\alpha_A)\gamma_A I_A - (1-\alpha_S)\gamma_S I_S - mQ + (\nu + \rho_W + \mu_W + \mu_1) I_W \\
-\omega_1 I_A - \omega_2 I_S - (\varphi + \mu_1) V\n\end{pmatrix}
$$
\n(14)

Similarly, the partial derivatives of v with respect to the infected compartments at disease- free equilibrium  $\varepsilon_0$ gives;

$$
V = \begin{pmatrix} k_2 & 0 & 0 & 0 & 0 & 0 \\ -b_1 & k_3 & 0 & 0 & 0 & 0 \\ -b_2 & 0 & k_4 & 0 & 0 & 0 \\ 0 & -c_1 & -c_2 & k_5 & -\nu & 0 \\ 0 & -d_1 & -d_2 & 0 & k_6 & 0 \\ 0 & -\omega_1 & -\omega_2 & 0 & 0 & k_8 \end{pmatrix}
$$
(15)

Where;

$$
k_1 = \Gamma + \mu_1, k_2 = \delta + \mu_1 + \sigma_2, k_3 = \rho_A + \gamma_A + \omega_1 + \mu_1
$$
  
\n
$$
k_4 = \rho_S + \gamma_S + \mu_S + \omega_2 + \mu_1, k_5 = \rho_H + \mu_H + \mu_1
$$
  
\n
$$
k_6 = \nu + I_W + \mu_W + \mu_1, k_7 = \sigma_1 + \mu_1
$$
  
\n
$$
k_8 = \varphi + \mu_1, k_9 = Z + \mu_1, k_{10} = n + m + \mu_\partial
$$
  
\n
$$
\alpha_1 = \alpha_A \gamma_A, \alpha_2 = \alpha_S \gamma_S, b_1 = (1 - r)\delta, b_2 = r\delta,
$$
  
\n
$$
c_1 = 1 - p_1, c_2 = 1 - p_2, d_1 = (1 - \alpha_A)\gamma_A, d_2 = (1 - \alpha_S)\gamma_S
$$

Corresponding author: Okorie Charity Ebelechukwu

Department of Mathematics and Statistics, Federal University, Wukari, Taraba State, Nigeria.



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$$
FV^{-1} = \begin{bmatrix} A & B & C & 0 & 0 & D \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{16}
$$

Where;

$$
\begin{bmatrix}\n0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
A & = \frac{(1-p_1)\beta_2 S_0 b_1}{k_2 k_3} + \frac{(1-p_1)\beta_3 S_0 b_2}{k_2 k_4} + \frac{(1-p_2)\beta_1 S_0 (b_2 \omega_2 k_3 + k_4 b_1 \omega_1)}{k_2 k_3 k_4 k_8}\n\end{bmatrix}
$$
\n
$$
B = \frac{(1-p_1)\beta_2 S_0}{k_3} + \frac{(1-p_2)\beta_1 S_0 \omega_1}{k_3 k_4 k_8}
$$
\n
$$
C = \frac{(1-p_1)\beta_3 S_0}{k_4} + \frac{(1-p_2)\beta_1 S_0}{k_4 k_4 k_8}
$$
\n
$$
D = \frac{(1-p_2)\beta_1 S_0}{k_8}
$$
\nTherefore, it follows that  $R_0$  is the spectral radius of  $F V^{-1}$  and it is given as;  
\n
$$
R_0 = A = R_{0A} + R_{0S} + R_{0V}
$$
\n
$$
R_0 = \rho (F V^{-1}) = \frac{(1-p_1)\beta_2 S_0 b_1}{k_2 k_3} + \frac{(1-p_1)\beta_3 S_0 b_2}{k_2 k_4} + \frac{(1-p_2)\beta_1 S_0 (b_2 \omega_2 k_3 + k_4 b_1 \omega_1)}{k_2 k_3 k_4 k_8}
$$

Therefore, it follows that  $\,R_0\,$  is the spectral radius of  $\,F\,{V}^{-1}\,$  and it is given as;

$$
R_0 = A = R_{oA} + R_{0S} + R_{0V}
$$
  
\n
$$
R_0 = \rho (FV^{-1}) = \frac{(1 - p_1)\beta_2 S_0 b_1}{k_2 k_3} + \frac{(1 - p_1)\beta_3 S_0 b_2}{k_2 k_4} + \frac{(1 - p_2)\beta_1 S_0 (b_2 \omega_2 k_3 + k_4 b_1 \omega_1)}{k_2 k_3 k_4 k_8}
$$

(17)

### The Global Asymptotic Stability of the Endemic Equilibrium State

To determine the global Stability of the Endemic State, we use the method of Lyapunov function. The Lyapunov method is used to assess the stability of various differential equations and systems

Theorem 1:

If  $\ R$   $_0$   $>$   $1$  , the endemic equilibrium  $(E_1)$  of the system (1)-(10) is globally asymptotically stable. Proof Consider the following function

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$$
V(t) = \sum_{k=1}^{n} d_k (x_k - x_k^* l \ln x_k)
$$

At endemic equilibrium,

$$
E_1 = (S^*, E^*, I_A^*, I_S^*, I_H^*, I_W^*, R^*, V^*, V_C^*, Q^*)
$$

we have;

$$
V = \sum_{k=1}^{n} d_k (x_k - x_k^* \ln x_k) = d_1 (x_1 - x_1^* \ln x_1) + d_2 (x_2 - x_2^* \ln x_2)
$$
  
+  $d_3 (x_3 - x_3^* \ln x_3) + d_4 (x_4 - x_4^* \ln x_4) + d_5 (x_5 - x_5^* \ln x_5)$   
+  $d_6 (x_6 - x_6^* \ln x_6) + d_7 (x_7 - x_7^* \ln x_7) + d_8 (x_8 - x_8^* \ln x_8)$   
+  $d_9 (x_9 - x_9^* \ln x_9) + d_{10} (x_{10} - x_{10}^* \ln x_{10})$ 

Let

$$
+d_3(x_3 - x_3^* \ln x_3) + d_4(x_4 - x_4^* \ln x_4) + d_5(x_5 - x_5^* \ln x_5)
$$
  
\n
$$
+d_6(x_6 - x_6^* \ln x_6) + d_7(x_7 - x_7^* \ln x_7) + d_8(x_8 - x_8^* \ln x_8)
$$
  
\n
$$
+d_9(x_9 - x_9^* \ln x_9) + d_{10}(x_{10} - x_{10}^* \ln x_{10})
$$
  
\nLet  
\n
$$
x_1 = S, x_2 = E, x_3 = I_A, x_4 = I_S, x_5 = I_H, x_6 = I_W, x_7 = R, x_8 = V, x_9 = V_C, x_{10} = Q
$$
  
\nTherefore;  
\n
$$
V(t) = d_1(S - S^* \ln S) + d_2(E - E^* \ln E) + d_3(I_A - I_A^* \ln I_A) + d_4(I_S - I_S^* \ln I_S)
$$
  
\n
$$
+d_5(I_H - I_H^* \ln I_H) + d_6(I_W - I_W^* \ln I_W) + d_7(R - R^* \ln R)
$$
  
\n
$$
+d_8(V - V^* \ln V) + d_9(V_C = V_C^* \ln V_C) + d_{10}(Q - Q^* \ln Q)
$$
  
\nThe time derivative of  $V(t)$  along the solution of the model is given by  
\n
$$
V' = d_1(S - S^* \ln S) + d_2(E - E^* \ln E) + d_3(I_A - I_A^* \ln I_A)
$$
  
\n
$$
+d_4(I_S - I_S^* \ln I_S) + d_5(I_H - I_H^* \ln I_H) + d_6(I_W - I_W^* \ln I_H) + d_7(R - R^* \ln R)
$$
  
\n
$$
d_8(V - V^* \ln V) + d_9(V_C - V_C^* \ln V_C) + d_{10}(Q - Q^* \ln Q)
$$
  
\n
$$
L(t) = d_1(S - S^* \ln S)
$$
  
\n
$$
L'(t) = d_1\left(\frac{dS}{dt} - S^* \cdot \frac{1}{S} \frac{dS}{dt}\right)
$$
  
\nNote that L is a function of t and ln S is a function of S, that is a

Note that L is a function of t and  $\ln S$  is a function of S, that is a function of a function. Thus  $L(t) = \ln S$ 

$$
L'(t) = \frac{dL}{dS} \cdot \frac{dS}{dt} = \frac{1}{S} \frac{dS}{dt}
$$
  

$$
L'(t) = d_1 \left( \frac{dS}{dt} - \frac{S^*}{S} \cdot \frac{dS}{dt} \right) = d_1 \left( 1 - \frac{S^*}{S} \right) \frac{dS}{dt}
$$

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At endemic equilibrium State  $E^{}_1 = (S^*,E^*,I^{\;\;*}_{\scriptscriptstyle{A}},I^{\;\;*}_{\scriptscriptstyle{S}},I^{\;\;*}_{\scriptscriptstyle{H}},I^{\;\;*}_{\scriptscriptstyle{W}},R^*,V^*,V^*_C^*,Q^*) \neq 0$ The model satisfies the following relation.  $S' = 0 = \pi - (1 - p_1)S\beta_{DA}I_A - (1 - p_1)S\beta_{DS}I_S - (1 - p_2)S\beta_1V - \Gamma S + \varpi_1R + ZV_C + nQ - \mu_1S$ (18)  $\pi = (1 - p_1)S\beta_{D_1}L_1 + (1 - p_1)S\beta_{D_2}L_2 + (1 - p_2)S\beta_1V + \Gamma S - \overline{\omega}_1R - ZV_C - nQ + \mu_1S$ (19) Substituting (19) into (18), we have; \*  $Q = I^* + (1 - n)S^* Q = I^* + (1 - n)S^* Q V^* = D^*$  $S' = (1 - p_1)S^* \beta_{DA} I^*_{A} + (1 - p_1)S^* \beta_{DS} I^*_{S} + (1 - p_2)S^* \beta_{1} V^* - \varpi_{1} R^*$ \*  $nO^*$  +  $(\mu + \Gamma)S^*$  $-ZV_c^* - nQ^* + (\mu_1 + \Gamma)S^* - (1 - p_1)S\beta_{DA}I_A - (1 - p_1)S\beta_{DS}I_S$  $-(1-p_2)S\beta_1V - (\mu_1 + \Gamma)S + \varpi_1R + ZV_c + nQ$  $E' = 0 = (1 - p_1)S\beta_{0d}L_4 + (1 - p_1)S\beta_{0s}L_5 + (1 - p_2)S\beta_1V - (\delta + \mu_1 + \varpi_2)E$  (20) \*  $R = I^*$   $(1 - n)S^*R = I^*$   $(1 - n)S^*RV^*$  $(\delta + \mu_1 + \varpi_2) = \frac{(1 - p_1)S^* \beta_{DA} \overline{f}_A^*}{\overline{F}_A^*} + \frac{(1 - p_1)S^* \beta_{DS} \overline{f}_S^*}{\overline{F}_A^*} + \frac{(1 - p_2)S^* \beta_1 V^*}{\overline{F}_A^*}$  $E^*$  +  $E^*$  +  $E^*$  +  $E^*$  $\delta + \mu_1 + \varpi_2 = \frac{(1 - p_1)S^* \beta_{DA} \tilde{f}_A}{\Gamma^*} + \frac{(1 - p_1)S^* \beta_{DS} \tilde{f}_S}{\Gamma^*} + \frac{(1 - p_2)S^* \beta_1 V^*}{\Gamma^*}$  (21) Similarly,  $I_1' = 0 = (1 - r)\delta E - (\rho_1 + \gamma_2 + \omega_1 + \mu_1)I_1$  (22) \*  $(\rho_A + \gamma_A + \omega_1 + \mu_1) = \frac{(1 - r)\delta E^*}{r^*}$ A I  $\rho_A + \gamma_A + \omega_1 + \mu_1 = \frac{(1-r)\delta E^*}{4}$  (23) Substituting (23) into (22) gives. \*  $\mathcal{L}_A' = (1 - r) \delta E - \left( \frac{(1 - r) \delta E^*}{r^*} \right) I_A$ A  $I_A' = (1-r)\delta E - \left(\frac{(1-r)\delta E^*}{r^*}\right)I_A$ I  $\delta E - \left(\frac{(1-r)\delta E^*}{r}\right)_{L}$  $I_A' = (1-r)\delta E - \left(\frac{(1-r)\delta E}{I_A^*}\right)I_A$  $I'_{S} = 0 = r\delta E - (\rho_{S} + \mu_{S} + \gamma_{S} + \omega_{2} + \mu_{1})I_{S}$  (24) \*

$$
(\rho_S + \mu_S + \gamma_S + \omega_2 + \mu_1) = \frac{r \delta E^*}{I^*_{S}}
$$
 (25)

Substituting (25) into (24) gives,

$$
I'_{S} = r\delta E - \left(\frac{r\delta E^*}{I^*_{S}}\right)I_{S}
$$
  
\n
$$
I'_{H} = 0 = \alpha_{A}\gamma_{A}I_{A} + \alpha_{S}\gamma_{S}I_{S} + vI_{W} - (\rho_{H} + \mu_{H} + \mu_{1})I_{H}
$$
\n(26)

$$
(\rho_H + \mu_H + \mu_I) = \left(\frac{\alpha_A \gamma_A I^*_{A}}{I^*_{H}} + \frac{\alpha_S \gamma_S I^*_{S}}{I^*_{H}} + \frac{\nu I^*_{W}}{I^*_{H}}\right) I_H
$$
\n(27)

Substituting (27) into (26) gives,

$$
I'_{H} = \alpha_{A} \gamma_{A} I_{A} + \alpha_{S} \gamma_{S} I_{S} + \nu I_{W} - \left( \frac{\alpha_{A} \gamma_{A} I^{*}_{A}}{I^{*}_{H}} + \frac{\alpha_{S} \gamma_{S} I^{*}_{S}}{I^{*}_{H}} + \frac{\nu I^{*}_{W}}{I^{*}_{H}} \right) I_{H}
$$

Corresponding author: Okorie Charity Ebelechukwu

Department of Mathematics and Statistics, Federal University, Wukari, Taraba State, Nigeria.

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$$
I'_{W} = 0 = (1 - \alpha_{A})\gamma_{A}I_{A} + (1 - \alpha_{S})\gamma_{S}I_{S} + mQ - (\nu + \rho_{W} + \mu_{W} + \mu_{1})I_{W}
$$
(28)

$$
(\nu + \rho_W + \mu_W + \mu_1)I_W = \left(\frac{(1 - \alpha_A)\gamma_A I^*}{I^*_{W}} + \frac{(1 - \alpha_S)\gamma_S I^*_{S}}{I^*_{W}} + \frac{mQ^*}{I^*_{W}}\right)I_W
$$
(29)

Substituting (29) into (28) gives,

$$
I'_{W} = (1 - \alpha_{A})\gamma_{A}I_{A} + (1 - \alpha_{S})\gamma_{S}I_{S} + mQ -
$$
  
\n
$$
\left(\frac{(1 - \alpha_{A})\gamma_{A}I^{*}_{A}}{I^{*}_{W}} + \frac{(1 - \alpha_{S})\gamma_{S}I^{*}_{S}}{I^{*}_{W}} + \frac{mQ^{*}}{I^{*}_{W}}\right)I_{W}
$$
  
\n
$$
R' = 0 = \rho_{A}I_{A} + \rho_{S}I_{S} + \rho_{H}I_{H}\rho_{W}I_{W} - (\varpi_{1} + \mu_{1})R
$$
\n(30)

$$
R' = (\varpi_1 + \mu_1) = \frac{\rho_A I^*}{R^*} + \frac{\rho_S I^*}{R^*} \frac{\rho_H I^*}{R^*} + \frac{\rho_W I^*}{R^*}
$$
(31)

Substituting (31) into (30) gives,

$$
R' = \rho_A I_A + \rho_S I_S + \rho_H I_H + \rho_W I_W - \left(\frac{\rho_A I^*_{A}}{R^*} + \frac{\rho_S I^*_{S}}{R^*} \frac{\rho_H I^*_{H}}{R^*} + \frac{\rho_W I^*_{W}}{R^*}\right) R
$$
  

$$
V' = 0 = \rho I_A + \rho_S I_A + \rho_W I_W - \left(\frac{\rho_A I^*_{A}}{R^*} + \frac{\rho_S I^*_{S}}{R^*} \frac{\rho_H I^*_{H}}{R^*}\right) R
$$
 (22)

$$
V' = 0 = \omega_1 I_A + \omega_2 I_S - \mu_1 V
$$
\n(32)

$$
\mu_1 = \frac{\omega_1 I^*_{A}}{V^*} + \frac{\omega_2 I^*_{S}}{V^*}
$$
\n(33)

Substituting (33) into (32) gives,

$$
\omega_1 I_A + \omega_2 I_S - \left(\frac{\omega_1 I^*}{V^*} + \frac{\omega_2 I^*}{V^*}\right) V \nQ' = 0 = \omega_2 E - (n + m + \mu_1) Q
$$
\n(34)

$$
(n+m+\mu_1) = \left(\frac{\varpi_2 E}{Q^*}\right) Q \tag{35}
$$

Substituting (35) into (34) gives,

$$
R' = \rho_A I_A + \rho_S I_S + \rho_H I_H + \rho_W I_W - \left(\frac{P_A + A}{R^*} + \frac{P_S + S}{R^*} + \frac{P_H + H}{R^*} + \frac{P_W + W}{R^*}\right)R
$$
  
\n
$$
V' = 0 = \omega_1 I_A + \omega_2 I_S - \mu_1 V
$$
  
\n
$$
\mu_1 = \frac{\omega_1 I_A^*}{V^*} + \frac{\omega_2 I_S^*}{V^*}
$$
  
\nSubstituting (33) into (32) gives,  
\n
$$
\omega_1 I_A + \omega_2 I_S - \left(\frac{\omega_1 I_A^*}{V^*} + \frac{\omega_2 I_S^*}{V^*}\right) V
$$
  
\n
$$
Q' = 0 = \omega_2 E - (n + m + \mu_1)Q
$$
  
\n
$$
(n + m + \mu_1) = \left(\frac{\omega_2 E}{Q^*}\right)Q
$$
  
\nSubstituting (35) into (34) gives,  
\n
$$
\omega_2 E - \left(\frac{\omega_2 E}{Q^*}\right)Q
$$
  
\n
$$
V'_C = 0 = \Gamma S - (Z + \mu_1) V_C
$$
  
\n
$$
(Z + \mu_1) = \frac{\Gamma S^*}{V^* C}
$$
  
\nSubstituting (37) into (36) gives,  
\n
$$
\Gamma S - \left(\frac{\Gamma S^*}{V^*}\right) V_C
$$
  
\n(36)  
\n(27)

$$
\left(Z + \mu_1\right) = \frac{\Gamma S^*}{V^*_{C}}\tag{37}
$$

Substituting (37) into (36) gives,

$$
\Gamma S - \left(\frac{\Gamma S^*}{V^*_{C}}\right) V_C
$$
\n(38)

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$$
V'(t) = d_{i}\left(1-\frac{S^{*}}{S}\right)\left[\frac{(1-P_{i})S^{*}\beta_{0A}I^{*} + (1-P_{i})S^{*}\beta_{0B}I^{*} + (1-P_{i})S^{*}\beta_{B}I^{*}}{(1-P_{i})S\beta_{0A}I_{A} - (1-P_{i})S\beta_{0A}I_{A} - (1-P_{i})S\beta_{0B}I_{B} - (1-P_{i})S\beta_{0B}I_{B} - (1-P_{i})S\beta_{0B}I_{B} - (1-P_{i})S\beta_{0B}I_{B} - (1-P_{i})S\beta_{0B}I_{A} + (1-P_{i})S\beta_{0B}I_{B} + (1-P_{i})S^{*}\beta_{0B}I^{*} + (1
$$

Corresponding author: Okorie Charity Ebelechukwu<br>
⊠ okoriec636@gmail.com<br>
Department of Mathematics and Statistics, Federal University, Wukari, Taraba State, Nigeria.<br>
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$$
L'_{1} = d_{1}(\mu_{1} + \Gamma)\left(S - \frac{S^{*}}{S}\right)^{2} + d_{1}\varpi_{1}\left(1 - \frac{S^{*}}{S}\right)(R - R^{*})
$$
  
+ $d_{1}Z\left(1 - \frac{S^{*}}{S}\right)(V_{C} - V^{*}_{C}) + d_{1}\eta\left(1 - \frac{S^{*}}{S}\right)(Q - Q^{*}) + d_{1}\beta_{DA}$   
 $(1-P_{1})S^{*}\Gamma_{A}\left(1 - \frac{S^{*}}{S}\right)\left(1 - \frac{SI_{A}}{SI^{*}_{A}}\right)$   
+ $d_{1}\beta_{DS}\left(1-P_{1}\right)S^{*}\Gamma_{S}\left(1 - \frac{S^{*}}{S}\right)\left(1 - \frac{SI_{S}}{SI^{*}_{S}}\right) + d_{1}\beta_{1}\left(1-P_{2}\right)S^{*}V^{*}\left(1 - \frac{S^{*}}{S}\right)\left(1 - \frac{SV}{SV^{*}}\right)$   

$$
L_{2} = d_{2}\left(1 - \frac{E^{*}}{E}\right)\left[\frac{(1-P_{1})\beta_{DA}S^{*}\Gamma^{*}\left(\frac{SI_{A}}{SI^{*}_{A}} - \frac{E}{E^{*}}\right) + (1-P_{1})\beta_{DS}S^{*}\Gamma_{S}\left(\frac{SI_{S}}{SI^{*}_{S}} - \frac{E}{E^{*}}\right)}{1 + (1-P_{2})\beta_{1}S^{*}V^{*}\left(\frac{SV}{SV^{*}} - \frac{E}{E^{*}}\right)}
$$
  

$$
L_{2} = d_{2}\left(1-P_{1}\right)\beta_{M}S^{*}\Gamma_{A}\left(1 - \frac{E^{*}}{E}\right)\left(\frac{SI_{A}}{SI^{*}_{A}} - \frac{E}{E^{*}}\right) + d_{2}\left(1-P_{1}\right)\beta_{DS}S^{*}\Gamma_{S}\left(1 - \frac{E^{*}}{E}\right)\left(\frac{SI_{S}}{SI^{*}_{S}} - \frac{E}{E^{*}}\right)
$$
  
+
$$
+d_{2}\left(1-P_{2}\right)\beta_{N}S^{*}V\left(1 - \frac{E^{*}}{E}\right)\left(\frac{SV}{S^{*}V} - \frac{E}{E^{*}}\right)
$$
  
+
$$
L_{3} = d_{3}\left(1 - \frac
$$

$$
L_{5} = a_{5} \left[ 1 - \frac{1}{I_{H}} \right] \left[ a_{A} Y_{A} I_{A} \left( \frac{1}{I_{A}} - \frac{1}{I_{H}} \right) + a_{5} Y_{S} I_{S} \left( \frac{1}{I_{S}} - \frac{1}{I_{H}} \right) + V I_{W} \left( \frac{1}{I_{W}} - \frac{1}{I_{H}} \right) \right]
$$
  
\n
$$
L_{6}' = d_{6} \left[ 1 - \frac{I_{W}}{I_{W}} \right] \left[ (1 - \alpha_{A}) \gamma_{A} I_{A} \left( \frac{I_{A}}{I_{A}} - \frac{I_{W}}{I_{W}} \right) + (1 - \alpha_{S}) \gamma_{S} I_{S} \left( \frac{I_{S}}{I_{S}} - \frac{I_{W}}{I_{W}} \right) + m Q \left( \frac{Q}{Q} - \frac{I_{W}}{I_{W}} \right) \right] \right] \tag{46}
$$
  
\n
$$
L_{7}' = d_{7} \left( 1 - \frac{R^{*}}{R} \right) \left[ \rho_{A} I_{A} \left( \frac{I_{A}}{I_{A}} - \frac{R}{R^{*}} \right) + \rho_{S} I_{S} \left( \frac{I_{S}}{I_{S}} - \frac{R}{R^{*}} \right) + \rho_{H} I_{H} \left( \frac{I_{H}}{I_{H}} - \frac{R}{R^{*}} \right) + \rho_{W} I_{W} \left( \frac{I_{W}}{I_{W}} - \frac{R}{R^{*}} \right) \right] \tag{48}
$$

Corresponding author: Okorie Charity Ebelechukwu<br>
⊠ okoriec636@gmail.com<br>
Department of Mathematics and Statistics, Federal University, Wukari, Taraba State, Nigeria.<br>
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$$
L'_{8} = d_{8} \left( 1 - \frac{V^*}{V} \right) \left[ \omega_1 \boldsymbol{\mathit{I}}_{A}^* \left( \frac{\boldsymbol{\mathit{I}}_{A}}{\boldsymbol{\mathit{I}}_{A}^*} - \frac{V}{V^*} \right) + \omega_2 \boldsymbol{\mathit{I}}_{S}^* \left( \frac{\boldsymbol{\mathit{I}}_{S}}{\boldsymbol{\mathit{I}}_{S}^*} - \frac{V}{V^*} \right) \right]
$$
  

$$
L'_{9} = d_{9} \left( 1 - \frac{Q^*}{Q} \right) \left[ \omega_2 E^* \left( \frac{E}{E^*} - \frac{Q}{Q^*} \right) \right]
$$

 $(50)$ 

 $(51)$ 

 $(49)$ 

$$
L'_{10} = d_{10} \left( 1 - \frac{V_c^*}{V_c} \left[ \Gamma S^* \left( \frac{S}{S^*} - \frac{V_c^*}{V_c} \right) \right] \right)
$$

$$
L'_{1} = -d_{1}(\mu_{1} + \Gamma) \frac{(s - s^{*})^{2}}{s} + d_{1}(1 - P_{1}) \beta_{24} s^{*} I^{*} \left(1 - \frac{S I_{A}}{s^{*} I_{A}} - \frac{S^{*}}{s} + \frac{S^{*}}{s} \cdot \frac{S I_{A}}{s^{*} I_{A}}\right)
$$
  
+
$$
d_{1}(1 - P_{1}) \beta_{25} s^{*} I^{*} \left(1 - \frac{S I_{S}}{s^{*} I_{S}} - \frac{S^{*}}{s} + \frac{S^{*}}{s} \cdot \frac{S I_{S}}{s^{*} I_{S}}\right) + d_{1}(1 - P_{2}) \beta V \left(1 - \frac{S V}{s^{*} V^{*}} - \frac{S^{*}}{s} + \frac{S^{*}}{s} \cdot \frac{S V}{s^{*} V}\right)
$$
  
+
$$
d_{1} \alpha_{1} R^{*} \left(\frac{R}{R^{*}} - 1 - \frac{R}{R^{*}} \cdot \frac{S^{*}}{s} + \frac{S^{*}}{s}\right) + d_{1} Z V \left(\frac{V_{C}}{V_{C}^{*}} - 1 - \frac{V_{C}}{V_{C}^{*}} \cdot \frac{S^{*}}{s} + \frac{S^{*}}{s}\right) + d_{1} \alpha_{2} \beta \left(\frac{Q}{Q^{*}} - 1 - \frac{Q}{Q} \cdot \frac{S^{*}}{s} + \frac{S^{*}}{s}\right)
$$
(52)

$$
L'_{2} = d_{2} (1 - P_{1}) \beta_{DA} S^{*} \boldsymbol{I}_{A}^{*} \left( \frac{S I_{A}}{S^{*} I^{*}_{A}} - \frac{E}{E^{*}} - \frac{E^{*}}{E} \cdot \frac{S I_{A}}{S^{*} I^{*}_{A}} + 1 \right)
$$
  
+
$$
d_{2} (1 - P_{1}) \beta_{DS} S^{*} \boldsymbol{I}_{S}^{*} \left( \frac{S I_{S}}{S^{*} I^{*}_{S}} - \frac{E}{E^{*}} - \frac{E^{*}}{E} \cdot \frac{S I_{S}}{S^{*} I^{*}_{S}} + 1 \right)
$$
  
+
$$
d_{2} (1 - P_{2}) \beta_{1} S^{*} V^{*} \left( \frac{S V}{S^{*} V^{*}} - \frac{E}{E^{*}} - \frac{E^{*}}{E} \cdot \frac{S V}{S^{*} V^{*}} + 1 \right)
$$
(53)

$$
L'_{3} = d_{3} (1-r) \delta E^{*} \left( \frac{E}{E^{*}} - \frac{I_{A}}{I_{A}^{*}} - \frac{E}{E^{*}} \cdot \frac{I_{A}}{I_{A}^{*}} + 1 \right)
$$
  
\n
$$
L'_{4} = d_{4} r \delta E^{*} \left( \frac{E}{E^{*}} - \frac{I_{S}}{I_{S}^{*}} - \frac{E}{E^{*}} \cdot \frac{I_{S}}{I_{S}^{*}} + 1 \right)
$$
\n(54)

Corresponding author: Okorie Charity Ebelechukwu<br>
⊠ okoriec636@gmail.com<br>
Department of Mathematics and Statistics, Federal University, Wukari, Taraba State, Nigeria.<br>
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$$
L'_{5} = d_{5}\alpha_{A}\gamma_{A} \int_{A}^{*} \left( \frac{I_{A}}{I_{A}^{*}} - \frac{I_{H}}{I_{H}} - \frac{I_{H}^{*}}{I_{H}} \cdot \frac{I_{A}}{I_{A}^{*}} + 1 \right) + d_{5}\alpha_{5}\gamma_{5} \int_{S}^{*} \left( \frac{I_{S}}{I_{S}^{*}} - \frac{I_{H}}{I_{H}} - \frac{I_{S}}{I_{S}} \cdot \frac{I_{H}^{*}}{I_{H}} + 1 \right) + d_{5}\gamma_{5} \int_{S}^{*} \left( \frac{I_{W}}{I_{W}} - \frac{I_{W}}{I_{H}} - \frac{I_{W}}{I_{W}} \cdot \frac{I_{H}}{I_{H}} + 1 \right) + d_{5}\gamma_{6}\gamma_{7} \int_{A}^{*} \left( \frac{I_{A}}{I_{B}^{*}} - \frac{I_{W}}{I_{H}} \cdot \frac{I_{H}}{I_{H}} + 1 \right) \right]
$$
\n
$$
L'_{6} = d_{6} \left( 1 - \alpha_{A} \right) \gamma_{A} \int_{A}^{*} \left( \frac{I_{A}}{I_{A}^{*}} - \frac{I_{W}}{I_{W}} - \frac{I_{W}}{I_{W}} \cdot \frac{I_{A}}{I_{A}^{*}} + 1 \right) + d_{7}\rho_{5} \int_{S}^{*} \left( \frac{I_{S}}{I_{S}^{*}} - \frac{R}{R} - \frac{R^{*}}{I_{S}} \cdot \frac{I_{S}}{I_{S}} + 1 \right) + d_{7}\rho_{H} \int_{A}^{*} \left( \frac{I_{W}}{I_{W}} - \frac{R}{R^{*}} - \frac{R^{*}}{I_{W}} \cdot \frac{I_{W}}{I_{W}} + 1 \right) + d_{7}\rho_{H} \int_{A}^{*} \left( \frac{I_{W}}{I_{W}} - \frac{R}{R^{*}} - \frac{R^{*}}{I_{W}} \cdot \frac{I_{W}}{I_{W}} + 1 \right) + d_{7}\rho_{H} \int_{A}^{*} \left( \frac{I_{W}}{I_{W}} - \frac{R}{R^{*}} - \frac{R^{*}}{I_{W}} \cdot \frac{I_{W}}{I_{W}} + 1 \right) \tag{58}
$$
\

$$
L'_{9} = d_{9}\varpi_{2}E^{*}\left(\frac{E}{E^{*}} - \frac{\mathcal{Q}}{Q^{*}} - \frac{\mathcal{Q}}{Q} \cdot \frac{E}{E^{*}} + 1\right)
$$
  
\n
$$
L' = d_{1}\Gamma S^{*}\left(\frac{S}{E^{*}} - \frac{V_{C}}{Q^{*}} - \frac{V^{*}}{Q^{*}}\frac{S}{E^{*}} + 1\right)
$$
\n(60)

 $L'_{10} = d_{10} \Gamma S^* \left( \frac{c}{S^*} - \frac{c}{V_c^*} - \frac{c}{V_c} \cdot \frac{c}{S^*} + 1 \right)$  $(61)$ 

$$
x = \frac{S}{S^*}, y = \frac{E}{E^*}, z = \frac{I_A}{I_A^*}, w = \frac{I_S}{I_B^*}, p = \frac{I_H}{I_H^*}, d = \frac{I_W}{I_W^*}, e = \frac{R}{R^*}, f = \frac{V}{V^*}, g = \frac{Q}{Q^*}, h = \frac{V_C}{V^*},
$$
  
\n
$$
L_1' = -d_1(\mu_1 + \Gamma) \frac{(S - S^*)^2}{S} + d_1(1 = p_1) \beta_{DA} S^* I_A^* \left(1 - xz - \frac{1}{x} + \frac{1}{x} \cdot xz\right)
$$
  
\n
$$
+d_1(1 = p_1) \beta_{DS} S^* I_A^* \left(1 - xw - \frac{1}{x} + \frac{1}{x} \cdot xw\right) + d_1(1 = p_2) \beta_1 S^* V^* \left(1 - xf - \frac{1}{x} + \frac{1}{x} \cdot xz\right)
$$
  
\n
$$
+d_1 \overline{\omega}_1 R^* \left(e - 1 - e \cdot \frac{1}{x} + \frac{1}{x}\right) + d_1 Z V^* \left(k - 1 - k \cdot \frac{1}{x} + \frac{1}{x}\right) + d_1 n Q^* \left(h - 1 - h \cdot \frac{1}{x} + \frac{1}{x}\right)
$$
  
\n(62)

Corresponding author: Okorie Charity Ebelechukwu<br>
⊠ okoriec636@gmail.com<br>
Department of Mathematics and Statistics, Federal University, Wukari, Taraba State, Nigeria.<br>
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\n
$$
L'_{2} = d_{2}(1-p_{1})\beta_{DA}S^{*}I^{*}{}_{A}\left(xz - y - \frac{xz}{y} + 1\right) + d_{2}(1-p_{1})\beta_{DS}S^{*}I^{*}{}_{S}\left(xw - y - \frac{xw}{y} + 1\right)
$$
\n+ $d_{2}(1-p_{2})\beta_{i}S^{*}V^{*}\left(xf - y - \frac{xf}{y} + 1\right)$   
\n
$$
L'_{3} = d_{3}(1-r)\delta E^{*}\left(y - z - \frac{y}{z} + 1\right)
$$
\n(63)  
\n
$$
L'_{4} = d_{4}r\delta E^{*}\left(y - w - \frac{y}{w} + 1\right)
$$
\n
$$
L'_{5} = d_{5}\alpha_{A}\gamma_{A}I^{*}{}_{A}\left(z - p - \frac{z}{p} + 1\right) + d_{5}\alpha_{5}\gamma_{5}I^{*}{}_{S}\left(w - p - \frac{w}{p} + 1\right)
$$
\n(65)  
\n
$$
L'_{5} = d_{5}\alpha_{A}\gamma_{A}I^{*}{}_{A}\left(z - p - \frac{z}{p} + 1\right) + d_{6}\alpha_{5}\gamma_{5}I^{*}{}_{S}\left(w - p - \frac{w}{p} + 1\right)
$$
\n(66)  
\n
$$
L'_{6} = d_{6}(1-\alpha_{A})\gamma_{A}I^{*}{}_{A}\left(z - d - \frac{z}{d} + 1\right) + d_{6}(1-\alpha_{5})\gamma_{5}I^{*}{}_{S}\left(w - d - \frac{w}{d} + 1\right)
$$
\n(67)  
\n+ $d_{6}mQ^{*}\left(h - d - \frac{h}{d} + 1\right)$ 

$$
L'_{3} = d_{3} (1 - r) \delta E^{*} \left( y - z - \frac{y}{z} + 1 \right)
$$
\n(64)

$$
L'_{4} = d_{4}r\delta E^{*} \left( y - w - \frac{y}{w} + 1 \right)
$$
  
\n
$$
L'_{5} = d_{5}\alpha_{A}\gamma_{A}I^{*} \left( z - p - \frac{z}{w} + 1 \right) + d_{5}\alpha_{S}\gamma_{S}I^{*} \left( w - p - \frac{w}{w} + 1 \right)
$$
\n(65)

$$
L'_{5} = d_{5} \alpha_{A} \gamma_{A} I^{*}{}_{A} \left( z - p - \frac{z}{p} + 1 \right) + d_{5} \alpha_{S} \gamma_{S} I^{*}{}_{S} \left( w - p - \frac{w}{p} + 1 \right)
$$
  
+  $d_{5} \nu I^{*}{}_{W} \left( d - p - \frac{d}{p} + 1 \right)$  (66)

$$
L'_{6} = d_{6} (1 - \alpha_{A}) \gamma_{A} I^{*} \left( z - d - \frac{z}{d} + 1 \right) + d_{6} (1 - \alpha_{S}) \gamma_{S} I^{*} \left( w - d - \frac{w}{d} + 1 \right)
$$
  
+  $d_{6} m Q^{*} \left( h - d - \frac{h}{d} + 1 \right)$  (67)

$$
L'_{6} = d_{6} (1 - \alpha_{A}) \gamma_{A} I^{*}{}_{A} \left[ z - d - \frac{z}{d} + 1 \right] + d_{6} (1 - \alpha_{S}) \gamma_{S} I^{*}{}_{S} \left[ w - d - \frac{n}{d} + 1 \right]
$$
  
\n
$$
+ d_{6} m Q^{*} \left( h - d - \frac{h}{d} + 1 \right)
$$
  
\n
$$
L'_{7} = d_{7} \rho_{A} I^{*}{}_{A} \left( z - e - \frac{z}{e} + 1 \right) + d_{7} \rho_{S} I^{*}{}_{S} \left( w - e - \frac{w}{e} + 1 \right)
$$
  
\n
$$
+ d_{7} \rho_{H} I^{*}{}_{H} \left( p - e - \frac{p}{e} + 1 \right) + d_{7} \rho_{W} I^{*}{}_{W} \left( d - e - \frac{d}{e} + 1 \right)
$$
  
\n
$$
L'_{8} = d_{8} \omega_{I} I^{*}{}_{A} \left( z - f - \frac{z}{f} + 1 \right) + d_{8} \omega_{2} I^{*}{}_{S} \left( w - f - \frac{w}{f} + 1 \right)
$$
  
\n
$$
L'_{9} = \sigma_{2} E^{*} \left( g - h - \frac{g}{h} + 1 \right)
$$
  
\n
$$
L'_{10} = d_{10} \Gamma S^{*} \left( x - k - \frac{x}{k} + 1 \right)
$$
  
\nSetting the Coefficients  
\n
$$
z : d_{1} \left[ (1 - p_{1}) \beta_{DA} S^{*} I^{*}{}_{A} \right] + d_{5} \alpha_{A} \gamma_{A} I^{*}{}_{A} + d_{6} (1 - \alpha_{A}) \gamma_{A} I^{*}{}_{A} + d_{7} \rho_{A} I^{*}{}_{A}
$$
  
\n
$$
+ d_{8} \omega_{I} I^{*}{}_{A} - d_{3} (1 - r) \delta E^{*}
$$
  
\n
$$
\frac{\Gamma_{00} \text{responding author: Oborie Charity Eblelechukwu}}{\Gamma_{01} \text{Conreconflag author: Oborie Charkyu}}
$$
  
\n
$$
\frac{\Gamma_{10} \Gamma_{11} \Gamma_{12}
$$

$$
+a_{7}\rho_{H}I_{H}\left(P-e-\frac{z}{e}+1\right)+a_{7}\rho_{W}I_{W}\left(a-e-\frac{z}{e}+1\right)
$$
\n
$$
L_{8}' = d_{8}\omega_{1}I_{A}^{*}\left(z-f-\frac{z}{f}+1\right)+d_{8}\omega_{2}I_{S}^{*}\left(w-f-\frac{w}{f}+1\right)
$$
\n(68)

$$
L_8 - a_8 \omega_1 I_{A} \left( 2 - J - \frac{1}{f} + 1 \right) + a_8 \omega_2 I_{S} \left( W - J - \frac{1}{f} + 1 \right)
$$
\n(69)

$$
L'_{9} = \varpi_{2} E^{*} \left( g - h - \frac{g}{h} + 1 \right)
$$
\n(70)

$$
L'_{10} = d_{10} \Gamma S^* \left( x - k - \frac{x}{k} + 1 \right)
$$
\n(71)

Setting the Coefficients

$$
z : d_1 \left[ \left( 1 - p_1 \right) \beta_{DA} S^* I^*_{A} \right] + d_5 \alpha_A \gamma_A I^*_{A} + d_6 \left( 1 - \alpha_A \right) \gamma_A I^*_{A} + d_7 \rho_A I^*_{A} + d_8 \omega_1 I^*_{A} - d_3 \left( 1 - r \right) \delta E^* \tag{72}
$$

okoriec636@gmail.com

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Corresponding author: Okorie Charity Ebelechukwu

Department of Mathematics and Statistics, Federal University, Wukari, Taraba State, Nigeria.





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\n
$$
w : d_1 (1 - p_1) \beta_{\text{ns}} S^* I^*_{s} + d_s \alpha_s \gamma_s I^*_{s} + d_6 (1 - \alpha_s) \gamma_s I^*_{s} + d_7 \rho_s I^*_{s}
$$
\n
$$
f : d_1 (1 - p_2) \beta_1 S^* V^* - d_s \omega_1 I^*_{s} - d_s \omega_2 I^*_{s}
$$
\n
$$
e : d_1 \varpi_1 R^* - d_1 (\rho_A I^*_{s} + \rho_s I^*_{s} + \rho_H I^*_{H} + \rho_W I^*_{W})
$$
\n
$$
k : d_1 Z V^*_{c} - d_{10} \Gamma S^*
$$
\n
$$
h : d_1 n Q^* + d_6 m Q^* - d_5 \varpi E^*
$$
\n
$$
y : -d_2 ((1 - p_1) \beta_{\text{D4}} S^* I^*_{s} + (1 - p_1) \beta_{\text{D5}} S^* I^*_{s} + (1 - p_2) \beta_1 S^* V^*)
$$
\n
$$
+d_3 (1 = r) \delta E^* + d_4 r \delta E^*
$$
\n
$$
p : -d_s (\alpha_A \gamma_A I^*_{s} + \alpha_s \gamma_s I^*_{s} + v I^*_{W}) + d_7 \rho_H I^*_{H} + d_7 \rho_W I^*_{W}
$$
\n(83)\n
$$
d : d_s V I^*_{W} - d_6 [(1 - \alpha_A) \gamma_A I^*_{s} + (1 - \alpha_s) \gamma_s I^*_{s} + m Q^*]
$$
\n
$$
g : d_5 \varpi_2 E^*
$$
\n
$$
x : d_{10} \Gamma S^*
$$
\n
$$
ln (235), d_{10} = 0
$$
\n(86)

$$
f : d_1 (1 - p_2) \beta_1 S^* V^* - d_8 \omega_1 I^*_{A} - d_8 \omega_2 I^*_{S}
$$
\n(78)

$$
e: d_1\varpi_1 R^* - d_7\left(\rho_A I^*_{A} + \rho_S I^*_{S} + \rho_H I^*_{H} + \rho_W I^*_{W}\right)
$$
\n(79)

$$
k: d_1 Z V^*_{C} - d_{10} \Gamma S^* \tag{80}
$$

$$
h: d_1 n Q^* + d_6 m Q^* - d_9 \varpi E^*
$$
\n(81)

k: 
$$
d_1ZV^*_{c} - d_{10}\Gamma S^*
$$
  
\n $h: d_1nQ^* + d_6mQ^* - d_9\varpi E^*$   
\ny:  $-d_2((1-p_1)\beta_{DA}S^*I^*_{A} + (1-p_1)\beta_{DS}S^*I^*_{S} + (1-p_2)\beta_1S^*V^*)$   
\n $+d_3(1=r)\delta E^* + d_4r\delta E^*$   
\np:  $-d_5(\alpha_A\gamma_AI^*_{A} + \alpha_S\gamma_SI^*_{S} + \nu I^*_{W}) + d_7\rho_HI^*_{H} + d_7\rho_WI^*_{W}$   
\n $d: d_5VI^*_{W} - d_6[(1-\alpha_A)\gamma_AI^*_{A} + (1-\alpha_S)\gamma_SI^*_{S} + mQ^*]$   
\ng:  $d_3\varpi_2E^*$   
\n $x: d_{10}\Gamma S^*$   
\n $x: d_{10}\Gamma S^*$   
\n(a86)  
\nln (235),  $d_{10} = 0$   
\nln (228), since  $d_1 = 0$ .  $d_1 = 0$   
\nln (228), since  $d_1 = 0$ .  $d_3 = 0$   
\n $\varpi = 0$   
\n $\var$ 

$$
+d_3(1=r)\delta E^* + d_4r\delta E^*
$$

$$
p: -d_{5} \left( \alpha_{A} \gamma_{A} I^{*}{}_{A} + \alpha_{S} \gamma_{S} I^{*}{}_{S} + \nu I^{*}{}_{W} \right) + d_{7} \rho_{H} I^{*}{}_{H} + d_{7} \rho_{W} I^{*}{}_{W}
$$
\n(83)

$$
d: d_{5}vI^{*}_{W} - d_{6}\left[\left(1-\alpha_{A}\right)\gamma_{A}I^{*}_{A} + \left(1-\alpha_{S}\right)\gamma_{S}I^{*}_{S} + mQ^{*}\right]
$$
\n(84)

$$
g: d_{9}\varpi_{2}E^{*} \tag{85}
$$

$$
x: d_{10} \Gamma S^* \tag{86}
$$

$$
\ln(235),\ d_{10}=0
$$

- In (229), since  $d_{10} = 0 : d_1 = 0$
- In (228), since  $d_1 = 0$  : .  $d_7 = 0$

In (3), since 
$$
d_1 = 0
$$
 :  $d_8 = 0$ 

Since  $d_1, d_7, d_8, d_{10} = 0$  , we substitute them in all equations (72-86). Hence,

$$
z: d_{5}\alpha_{A}\gamma_{A}I^{*}{}_{A} + d_{6}(1-\alpha_{A})\gamma_{A}I^{*}{}_{A} - d_{3}(1-r)\delta E^{*}
$$
\n(87)

$$
w: d_s \alpha_s \gamma_s I^* + d_6 (1 - \alpha_s) \gamma_s I^* - d_4 r \delta E^*
$$
\n(88)

$$
h: d_6 m Q^* - d_9 \varpi_2 E^* \tag{89}
$$

 \* \* \* \* \* \* \* \* \* 1 1 3 4 2 \* \* 2 1 1 1 : 1 1 DA A DS S p S I p S I y d r E d r E d p S V (90) \* \* \* \* \* \* \*

$$
p: -d_s \left( \alpha_A \gamma_A I^* + \alpha_S \gamma_S I^* + \nu I^* \right) \tag{91}
$$

$$
d: d_{5}vI^{*}_{W} - d_{6}\left[\left(1-\alpha_{A}\right)\gamma_{A}I^{*}_{A} + \left(1-\alpha_{S}\right)\gamma_{S}I^{*}_{S} + mQ^{*}\right]
$$
\n(92)

$$
g: d_{9}\varpi_{2}E^{*}
$$
 (93)

From (242),  $d_9 = 0$ 

Substituting (93) into (89), it shows that  $d_6 = 0$ 

From (92), substituting the value of  $d_6^+$  into (92), then  $d_5^+ = 0$  , it also shows that (91) gives  $|0\>$ 

Corresponding author: Okorie Charity Ebelechukwu

okoriec636@gmail.com

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Substituting the values of  $d_5$ ,  $d_6$  into (87) gives  $d_3 = 0$ Substituting the values of  $d_5$ ,  $d_6$  into (88) gives  $d_4 = 0$ Substituting the values of  $d_3$ ,  $d_4$  into (89) gives  $d_2 = 0$ 

#### RESULTS AND DISCUSSION

Basic reproduction number of the model was calculated using the next generation matrix. From the result, it was observed that  $R_0 < 1$ ,

meaning that the disease is dying down. From the calculation on the endemic stability, we

observe that  $\frac{dL}{dt} = 0$ dt  $= 0$  , and this follows the

following conditions

(i)  
\n
$$
\frac{dL}{dt} = 0 \text{ iff } S = S^*, E = E^*, I_A = I^*, I_S = I^*_{S}, I_H = I^*_{H},
$$
\n
$$
I_W = I^*_{W}, R = R^*, V = V^*, Q = Q^*, V_C = V^*_{C}
$$
\n
$$
\frac{dL}{dt} \le 0 \text{ iff } S \le S^*, E \le E^*, I_A \le I^*, I_S \le I^*_{S}, I_H \le I^*_{H}
$$
\n
$$
I \le I^*_{S} = R^*_{S} V \le V^*_{S} = R^*_{S} V \le V^*_{S} V \le V^*_{S}
$$

 $, I_w \leq I^*_{W}, R \leq R^*, V \leq V^*, Q \leq Q^*, V_c \leq V^*_{C}$ 

This means that the endemic equilibrium of system of equations (1)-(10) is the largest compact

invariant set in 
$$
\left\{S, E, I_A, I_S, I_H, I_W, R, V, Q, V_C\right\} \in \Omega \frac{dL}{dt} \le 0 \right\}
$$

Therefore,  $E_{\rm 1}^{\phantom{\dag}}$  is globally asymptotically stable in

$$
\Omega \text{ if } S \le S^*, E \le E^*, I_A \le I_{A^*}^* I_S \le I_{S^*}^* I_H \le I_{H^*}^*
$$
  

$$
I_W \le I_{W^*}^* R \le R^*, V \le V^*, Q \le Q^*, V_C \le V_{C^*}^*
$$

#### **CONCLUSION**

 In this paper, we formulated a new mathematical model which incorporated some important factors that plays significant role in the transmission dynamics and control of COVID-19. The result we got from the basic reproduction number shows that the disease is COVID-19 is dying down in Nigeria. We obtained condition for global stability of the endemic equilibrium, using Lyapunov function approach respectively. The result we got from the global stability of the endemic equilibrium shows that the disease is globally asymptotically stable. We recommend that people should not relax in taking precautionary measures in order to avoid contracting the disease.

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Corresponding author: Okorie Charity Ebelechukwu

okoriec636@gmail.com

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