TRANSIENT ANALYSIS OF THREE-PHASE INDUCTION MACHINE USING DIFFERENT REFERENCE FRAMES

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ABSTRACT

Three-phase induction machines are generally used as motors for many industrial applications and all this is due to its simple construction and other advantages in contrast to other machines. Popularity of these motors has resulted into a lot of research including the transient behaviour of the machine. Literature survey reveals that most of the researchers adopted only a single reference frame to estimate transient behaviour of the machine. In this work qd axis based modeling is proposed to analyze the transient performance of three-phase squirrel cage induction motor using stationary reference frame, rotor reference frame and synchronously rotating reference frame. Simulated results have been presented to buttress the functionality of these induction motor with the aid of MATLAB/SIMULINK.

Keywords: modeling, induction motor, reference frames, simulation, transient analysis.

INTRODUCTION

During start-up and under severe transient operations induction motor draws large currents, produces voltage dips, oscillatory torques and can even generate harmonics in the power systems. In order to investigate such problems, the *d*, *q axis* model has been found to be well tested and proven to be reliable. The basic concept of transient modeling of the machine and the dynamic behaviour may be analyzed using any one of following reference frames: a) Stationary reference frame, b) Rotor reference frame and c) Synchronous reference frame; specific tests to estimate the machine parameters that proceeds with transient modeling may be necessary [1, 3].

Matlab/Simulink is a very useful tool for modeling electrical machine and it can be used to predict the dynamic behaviour of the machines. In this work Matlab/Simulink based modeling is proposed using all the three reference frames mentioned above. During simulations sufficient time span is included to predict the complete behaviour of the machine [4, 5].

Mathematical Modeling [6]

A three-phase induction motor can be modeled with qd'axis theory. According to qdo axis modeling:

$$\left|F_{qdo}\right| = \left|T_{qdo}\right| * \left[F_{abc}\right]$$

(1)

where

$$\begin{bmatrix} T_{qdo} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos_{\#} & \cos(\# - 2f/3) & \cos(\# + 2f/3) \\ \sin_{\#} & \sin(\# - 2f/3) & \sin(\# + 2f/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$$
(2)

The voltage balance equation for the q, d coils in arbitrary reference frames are as follows [2]:

$$\begin{bmatrix} V^C \end{bmatrix} = \begin{bmatrix} Z^C \end{bmatrix} * \begin{bmatrix} i^C \end{bmatrix}$$
(3)

where

 $\left[V^{\,C}
ight]$ and $\left[i^{\,C}
ight]$ represents '4x1' column matrices of voltage and current and are given as

 $\left[V_{qs}^{C}V_{ds}^{C}V_{qr}^{C}V_{dr}^{C}\right]^{T}$ and $\left[i_{qs}^{C}i_{ds}^{C}i_{qr}^{C}i_{dr}^{C}\right]^{T}$ respectively; and, impedance matrices (4x4), $\left[Z^{C}\right]$ is given as,

$$\begin{bmatrix} Z^{C} \end{bmatrix} = \begin{bmatrix} R_{s} + L_{s}p & \check{\mathsf{S}}_{c}L_{c} & L_{m}p & \check{\mathsf{S}}_{c}L_{m} \\ -\check{\mathsf{S}}_{c}L_{s} & R_{s} + L_{s}p & -\check{\mathsf{S}}_{c}L_{m} & L_{m}p \\ L_{m}p & (\check{\mathsf{S}} - \check{\mathsf{S}}_{r})L_{m} & R_{r} + L_{r}p & (\check{\mathsf{S}}_{c} - \check{\mathsf{S}}_{r})L_{r} \\ -(\check{\mathsf{S}} - \check{\mathsf{S}}_{r})L_{m} & L_{m}p & -(\check{\mathsf{S}}_{c} - \check{\mathsf{S}}_{r})L_{r} & R_{r} + L_{r}p \end{bmatrix}$$

Stationary Reference Frame Model [7, 8]

The speed of the reference frames is that of the stator, which is zero; hence,

$$\tilde{\mathsf{S}}_{c} = 0 \tag{4}$$

Equation (4) is substituted into equation (3). The resulting model is

$$[V] = [Z]^*[i] \tag{5}$$

where

 $\begin{bmatrix} V \end{bmatrix}$ and $\begin{bmatrix} i \end{bmatrix}$ represents '4xl' column matrices of voltage and current and are given as $\begin{bmatrix} V_{qs}, V_{ds}, V_{qr}, V_{dr} \end{bmatrix}^T$ and $\begin{bmatrix} i_{qs}, i_{ds}, i_{qr}, i_{dr} \end{bmatrix}^T$ respectively.

And, impedance matrices (4x4), [Z] is given as

$$[Z] = \begin{bmatrix} R_s + L_s p & 0 & L_m p & 0 \\ 0 & R_s + L_s p & 0 & L_m p \\ L_m p & -\check{S}_r L_m & R_r + L_r p & -\check{S}_r L_r \\ \check{S}_r L_m & L_m p & \check{S}_r L_r & R_r + L_r p \end{bmatrix}$$

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The torque equation is

$$T_{e} = \frac{3}{2} \frac{P}{2} L_{m} (i_{qs} i_{dr} - i_{ds} i_{qr})$$
(6)

In case bus bar voltages are

$$V_{as} = V_m * \cos(\tilde{S}_s t + \})$$

$$V_{bs} = V_m * \cos(\tilde{S}_s t + \} - 2f/3)$$

$$V_{cs} = V_m * \cos(\tilde{S}_s t + \} + 2f/3)$$
(7)

And the motor terminals are connected directly to the bus, then using the transformation of equation (2),

$$V_{qs} = V_m * \cos(\check{S}_s t + \})$$

$$V_{ds} = -V_m * \sin(\check{S}_s t + \})$$
(8)

Rotor Reference Frames Model [9, 10]

The speed of the rotor reference frames is

$$\check{\mathsf{S}}_{c} = \check{\mathsf{S}}_{r} \tag{9}$$

and the angular position is

$$_{n_{c}} = _{n_{r}}$$
 (10)

Substituting in the upper subscript *r* for rotor reference frames and equation (4) in the equation (3), the induction-motor model in rotor reference frames is obtained. The equations are given by

$$\begin{bmatrix} V^r \end{bmatrix} = \begin{bmatrix} Z^r \end{bmatrix} * \begin{bmatrix} i^r \end{bmatrix}$$
(11)

 $\begin{bmatrix} V^r \end{bmatrix} \text{ and } \begin{bmatrix} i^r \end{bmatrix} \text{ represents '4xl' column matrices of voltage and current and are given as} \\ \begin{bmatrix} V_{qs}^r, V_{ds}^r, V_{qr}^r, V_{dr}^r \end{bmatrix}^T \text{ and } \begin{bmatrix} i_{qs}^r, i_{ds}^r, i_{qr}^r, i_{dr}^r \end{bmatrix}^T \text{ respectively; and, impedance matrices (4x4), } \begin{bmatrix} Z^r \end{bmatrix} \text{ is given as} \\ \begin{bmatrix} Z^r \end{bmatrix} = \begin{bmatrix} R_s + L_s p & \tilde{S}_r L_s & L_m p & \tilde{S}_r L_m \\ -\tilde{S}_r L_s & R_s + L_s p & -\tilde{S}_r L_s p & L_m p \\ L_m p & 0 & R_r + L_r p & -0 \\ -0 & L_m p & -0 & R_r + L_r p \end{bmatrix}$

and the electromagnetic torque is

$$T_{e} = \frac{3}{2} \frac{P}{2} L_{m} (i_{qs}^{r} \ i_{dr}^{r} \ -i_{ds}^{r} \ i_{qr}^{r} \) \tag{12}$$

The transformation from *abc* to *qdo* variables is obtained by substituting (10) into $[T_{abc}]$ defined in (2) as

$$\begin{bmatrix} T_{abc}^{r} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos_{w_{r}} & \cos_{w_{r}} - 2f/3 & \cos_{w_{r}} + 2f/3 \\ \sin_{w_{r}} & \sin_{w_{r}} - 2f/3 & \sin_{w_{r}} + 2f/3 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$
(13)

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The terminal voltage equation (7) becomes

$$V_{qs} = V_m * \cos(s \check{S}_s t + \})$$

$$V_{qs} = -V_m * \sin(s \check{S}_s t + \})$$
(14)

The *q*, *d* voltages are therefore of slip frequency and the q- axis rotor current behaves exactly as the phase a rotor current does.

Synchronously Rotating Reference Frame Model [//]

The speed of the reference frame is

$$\check{\mathsf{S}}_{c}=\check{\mathsf{S}}_{s} \tag{15}$$

Stator supply angular frequency (rad /sec) and the instantaneous angular position is

$$J_{c} = J_{s} = \check{S}_{s}t \tag{16}$$

By substituting (15) into (3), the induction motor model in the synchronous reference frames is obtained. By using the superscript e to denote this electrical synchronous reference frame, the model is obtained as

$$\begin{bmatrix} V^e \end{bmatrix} = \begin{bmatrix} Z^e \end{bmatrix}^* \begin{bmatrix} i^e \end{bmatrix}$$
(17)

where

 $\begin{bmatrix} V^{e} \end{bmatrix} \text{ and } \begin{bmatrix} i^{e} \end{bmatrix} \text{ represents '4xl' column matrices of voltage and current and are given as} \\ \begin{bmatrix} V_{qs}^{e}, V_{ds}^{e}, V_{qr}^{e}, V_{dr}^{e} \end{bmatrix}^{T} \text{ and } \begin{bmatrix} i_{qs}^{e}, i_{ds}^{e}, i_{qr}^{e}, i_{dr}^{e} \end{bmatrix}^{T} \text{ respectively, and, impedance matrices (4x4), } \begin{bmatrix} Z^{e} \end{bmatrix} \text{ can be given as,} \\ \begin{bmatrix} Z^{e} \end{bmatrix} = \begin{bmatrix} R_{s} + L_{s}p & \tilde{S}_{s}L_{s} & L_{m}p & \tilde{S}_{w}L_{m} \\ -\tilde{S}_{s}L_{s} & R_{s} + L_{s} & -\tilde{S}_{s}L_{m} & L_{m}p \\ L_{m}p & (\tilde{S}_{s} - \tilde{S}_{r})L_{m} & R_{r} + L_{r}p & (\tilde{S}_{s} - \tilde{S}_{r})L_{r} \\ -(\tilde{S}_{s} - \tilde{S}_{r})L_{m} & L_{m}p & -(\tilde{S}_{s} - \tilde{S}_{r})L_{r} & R_{r} + L_{r}p \end{bmatrix}$

The electromagnetic torque is,

$$T_{e} = \frac{3}{2} \frac{P}{2} L_{m} (i_{qs}^{e} i_{dr}^{e} - i_{ds}^{e} i_{qr}^{e}) \text{ N-m}$$
(18)

The transformation from *abc* to *dqo* variables is found by substituting (16) into equation (2) and is given as

$$\begin{bmatrix} T_{abc}^{e} \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos_{\# s} & \cos(\# s - 2f/3) & \cos(\# s + 2f/3) \\ \sin_{\# s} & \sin(\# s - 2f/3) & \sin(\# s + 2f/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$$
(19)

The terminal voltage in equation (7) become

$$V_{qs} = V_m * \cos \}$$

$$V_{ds} = V_m * \sin \}$$
(20)

This means the stator d, q voltages are dc quantities. The mechanical motion described by

$$p\check{S}_r = (T_e - T_L)/J \tag{21}$$

Simulation

Asynchronous motor is essentially the stator resistance and leakage reactance in series with the rotor resistance and leakage reactance. Consequently with the rated applied voltage, the starting current is large, in some cases in the order of 10 times the rated value. This is observed in Figure 2 stator phase voltage at full-load. Figure 3 is stator phase current at transient and steady state, Figure 4 is torque speed of induction motor; while Figure 5 is electromagnetic produced during transient condition. It is recommended that reduced voltage starting methods such as star/delta, autotransformer and soft start methods be employed to reduce the excess starting current. The rotor accelerates from stall with zero mechanical load torque and since friction and windage losses are not taken into account, the machine accelerates to synchronous speed [12 -14].



Figure 1: Matlab/Simulink model of the three-phase induction motor







Figure 3: Stator phase current at transient and steady state







Figure 5: Electromagnetic produced during transient condition.

CONCLUSION

For the analysis of dynamic behaviour of the asynchronous machine, study of reference frames are essential. This paper has demonstrated the elegance of MATLAB/SIMULINK in the dynamic modeling and simulation of asynchronous motor driving a mechanical load. The results obtained clearly show the elegance of d-q axis transformation theory in machine modeling, the inherent

limitations of asynchronous motors and effect of mechanical loading on various motor output variables.

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