
STRUCTURAL DEFORMATION MONITORING OF KIRI DAM, ADAMAWA STATE, NIGERIA

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Abstract

Deformation of structures is a major hazard worldwide and has in recent years become a challenge to surveyors, civil engineers, geologist, and other professionals of geosciences and environmental engineers. In this paper, a study was conducted on Kiri Dam where monitoring data for 12 epochs were used for both vertical and horizontal displacements. These data were collected using heights leveling and offset distance measurements for the vertical oscillation and horizontal displacements respectively. Observation equation method of least squares adjustment was used for adjusting the heights difference and the offset distances were adjusted by arithmetic mean method. The study revealed the minimum and maximum vertical movement of -238mm and 131mm which occurred in 1998 and 1996 at chainages 800m and 1300m respectively. For the horizontal displacement, there was a minimum and maximum of -161mm and 186mm which occurred in 1996 and 1989 at chainages 1400m and 1500m respectively. The results of the adjustments were tested using the null hypothesis, H_0 at 5% significance level in which ANOVA method was used for both vertical and horizontal displacements. The results of the statistical test also revealed that the structure is experiencing small random movement in both vertical and horizontal sections which is not imposing significant force on the structure. It is recommended that the dam structures be monitored periodically to avert any catastrophic consequences.

Introduction

Engineering structures (such as dams, viaducts, bridges oil farm tanks, e.t.c.) are subject to deformation when adversely affected by physical and chemical processes. This deformation could be caused by deflection in both horizontal and vertical axes due to factors such as changes of ground water level, tidal phenomena, loading and pressure, tectonic phenomena etc, which may occur gradually over a long period of time.

Monitoring and analysis of these structures constitute a special branch of geo-science. Several techniques for measuring and monitoring deformation are grouped into two, geodetic and non-geodetic techniques (Ono, et al 2005).

Generally, dams are constructed for various purposes such as hydroelectricity, agriculture, water supply system etc. Consequently, dams need to be monitored periodically to mitigate any danger that may result from deformation. Steward and Sakiri (2002) stated some factors that could precipitate the failure of dams as ground water pressure and stresses within the ground or structure itself, surface displacement structure or the surrounding bedrock. Monitoring and measurement of these effects are the fields of civil engineers. The concern of surveyors is to determine whether there are displacements in horizontal or vertical, in relation to a fixed reference system in space (Kulkarni, 1999). These displacements refer to the downstream face of the dam, its crest could be foundation bedrock. Measurements made are aimed at determining the variation in positions with time. Any detected

displacement will provide an avenue for analysis and prediction.

The primary applications of distances and directions measurements as suggested by Chrzanowski (1993) and Wan Aziz et al (2000) are to provide a continuing surveillance of the structure and warning of any dangerous developments.

Statement of Problem

Kiri dam was constructed between 1976 and 1980 by Upper Benue River Basin Development Authority of Nigeria. One of the reasons for constructing the dam was for irrigation farming at Savannah Sugar Company Limited. Over the years, the top of the dam become an access route to Shelleng and other neighbouring towns in Shelleng Local Government Area of Adamawa State. The external forces (loads) exerted on the dam structure as a result of daily vehicular traffic becomes a matter of concern. Recently, Adamawa State Government in its efforts to provide the State with alternative source of electricity proposed to develop a hydro-electric plant at Kiri dam (Abubakar, 2011). The proposal intends to modify the existing dam structure in order to accommodate the installation of intake facilities in the reservoir (Dilli, 2011). Based on these observations, it becomes imperative that an immediate study be conducted on the dam to ascertain its stability by checking the reliability of the dam.

The information obtained could be used to indicate whether:

1- the structure is stable and consequently safe.

- 2- the structure is experiencing small random movements which are not imposing significant forces on the structure.
- 3- the structure is experiencing small, localized systematic deformation i.e. as caused by seasonal effect like the volume of water which may not be significant, or.
- 4- the structure is experiencing deformation which increases as a function of time. They may indicate that remedial measures have to be taken in some cases in order to avoid catastrophic consequences.

Aim of the Study

This paper aims at vertical and horizontal displacement observations and analysis of Kiri Dam Structures in Adamawa State, Nigeria.



Fig. 1a: Side view of the dam wall



Fig. 1b: Extension of the dam wall

Data Collection Procedure

For the purpose of this study, the authors' intention was to use the data collected from Upper Benue River Basin Development Authority of Nigeria for a period of 20 years (1989-2009). But due to unavailability of some data, only 12 years data was used, i.e. 1989, 1991, 1996-2004 and 2009. And at the beginning of each set of observations, in-situ test

The Study Area

Kiri Dam is located in Shelleng Local Government Area of Adamawa State, Nigeria. The dam wall is a linear figure located between longitudes $12^{\circ} 09^1E$ and $12^{\circ} 10^1E$, and latitudes $09^{\circ} 42^1N$ and $09^{\circ} 48^1N$. The dam is bounded by Shelleng District in the North, Borong District in the South, Longuda District in the West, and Yungur District in the Eastern part. The wall of Kiri dam covers a distance of 1km across river Gongola, and it has an embankment of 20m above river bed. Below are photographs of some elevations of the dam.

was carried out on the two control points (H_{DB2} and H_{DB3}) located on the opposite sides of the dam, which are off the structure, and are connected to the A-Chain of the Nigerian Primary Triangulation Network. For each test conducted, it was found that the two controls were in-situ. The heights of the controls H_{DB2} and H_{DB3} provided by Upper Benue

River Basin Development Authority of Nigeria are 174.228m and 174.441m respectively. While the computed heights from the observations carried out by a precise automatic level of 1mm accuracy are 174.22804127m for H_{DB2} and 174.44093015m for H_{DB3} .

Observations for each campaign (year) were taken on monthly bases by Kiri dam survey unit. The mean of the observed offsets and levels for 12 campaigns for both vertical and horizontal movements were obtained and used to monitor the following:

1. Deflection in horizontal axes: This involved the use of two control points at the abutment of the dam which define the axis, along with monitoring stations established at 100m interval on the crest of the dam. During the period of measurements, a theodolite was used in defining a fixed reference axis with stations H_{DB2} and H_{DB3} as controls. This was followed by taking offsets from each monument to the centre line.
2. Vertical displacement: The relative settlement of the control points on the crest of the dam were measured with precise automatic level instrument relative to the bench marks. The relative elevation change for each bench mark at each survey period was measured. Vertical displacement / settlement were then computed.

Data Quality

For horizontal movement, a Kern theodolite with an accuracy of 0.5sec was used. And for vertical movement, a Kern automatic level instrument with an accuracy of 1mm was also used. The two instruments are of second order, but it is recommended that second order instruments can be used for first order work provided redundant observations are carried out. Hence, 30 different sets of redundant observations were made for both horizontal and vertical movements respectively to improve the accuracy of the work.

Computations

The mathematical equation associated with observation equation can be written in matrix form as:

$$AX = L + V \quad 1.0$$

Where:

A = Design matrix.

X = Vector of unknown.

L = Vector of the observations.

V = Vector of the discrepancies which is also the unknown.

The system of normal equation is represented in matrix form as follows:

$$(A^T P A) X = A^T P L \quad 1.1$$

$$\text{i.e. } X = (A^T P A)^{-1} A^T P L \quad 1.3$$

The results of the mean height differences, offset differences, adjusted offsets, mean offset displacements, and the least squares estimates (X) for 12 epochs are presented in appendices 1, 2, 3, and 5 respectively.

Precision: - Based on Ayeni, (2001) it involved computing

i. The residual (v) = $Ax - L$ (Appendix 6)

ii. A posteriori variance (σ^2) = $\frac{V^T P V}{M - N}$ (Appendix 7)

iii. Variance –co variance (σ) = $(A^T P A)^{-1}$ (Appendix 8)

iv. M and N are number of observations and unknown parameters respectively.

Reliability

$$\text{Reliability} = r_1 / r_2$$

Where r_1 = a posteriori standard error of observation, and r_2 = a priori standard error of the observation. Appendix 9 shows the reliability result.

The greater the number of observations, the lower the ratio should be. And low ratio indicates high reliability, but a ratio of one indicates complete unreliability.

Numerical Analysis

The results of both vertical and horizontal movements were tested using null hypothesis, H_0 which stated that, $\mu_1 = \mu_2 = \mu_3 = \dots = \mu_n$ (i.e there is no difference in measurements for all the epochs).

The statistic of accepting or rejecting the stated hypothesis was based on the analysis of the variance (ANOVA). That is, if $F_{cal} > F$ the null hypothesis, H_0 will be rejected at 5% level of significant. That is, there is significant difference between the observations.

Analysis of the Vertical Displacement

Shepher, (1981) explained that test failure can be expected since it is based on the tested population and not on the entire population. Based on the hypothesis earlier stated, the result was tested using Analysis of Variance (ANOVA).

Thus: The Hypothesis;

H_0 : $\mu_1 = \mu_2 = \mu_3 = \dots = \mu_{13}$ (there is no difference in offset distances measured for all epoch).

Subscript

H_i : not all μ_s are equal.

The ANOVA table is shown below;

**Table 1 ANOVA Single Factor
SUMMARY**

<i>Groups</i>	<i>Count</i>	<i>Sum</i>	<i>Average</i>	<i>Variance</i>
0.000333	10	-0.01767	-0.00177	0.001414
0.101311	10	-0.17389	-0.01739	0.000354
0.101311	10	-0.43389	-0.04339	0.006901
-0.05125	10	-0.04917	-0.00492	0.003746
-0.15481	10	-1.65233	-0.16523	0.000891
-0.19035	10	-1.86177	-0.18618	0.00078
-0.08223	10	-0.7588	-0.07588	0.002972
-0.09473	10	-1.05817	-0.10582	0.000996
-0.08844	10	-0.911	-0.0911	0.000812
0.015057	10	0.305417	0.030542	0.000439
-0.03683	10	-0.83367	-0.08337	0.001067
-0.09817	10	-0.90953	-0.09095	0.001043
0.0066	10	0.103	0.0103	3.2E-05

<i>Source of Variation</i>	<i>SS</i>	<i>ANOVA</i>	<i>MS</i>	<i>F</i>	<i>P-value</i>	<i>F crit</i>
Between Groups	0.533173	12	0.044431	26.93075	3.5E-28	1.835813
Within Groups	0.19303	117	0.00165			
Total	0.726203	129				

From the ANOVA table above;

$$F_{\text{cal}} = 26.93075$$

$$F = 1.835813$$

Since $F_{\text{cal}} > F$ the null hypothesis, H_0 was rejected at 5% level of significant. That is, there is significant difference between the observations.

From the adjusted movement, the graph of vertical movement is plotted against the chainage distance for

every epoch. This portrayed the minimum and maximum movement occurred at the respective chainages for each epoch. The summary is shown in table 2 below:

Table 2: Minimum & Maximum Vertical Movements

EPOCH	Minimum Movement(mm)	Chainage (m)	Maximum Movement(mm)	Chainage (m)
1989	-50	1100	101	500
1991	-274	1500	16	1300
1996	-64	700	131	1300
1997	-200	1100	-126	1300
1998	-238	800	-140	1500
1999	-117	900	-33	1500
2000	-150	1100	47	1500
2001	-122	900	44	1500
2002	0	1100	75	700
2003	-126	1100	32	1500
2004	-130	900	39	1500
2009	07	500	20	1100

The maximum vertical movement of the dam is ranged from -274 to 131mm which occurred in 1991

and 1996 respectively. Thus this table 2 depicted that there is random significant oscillation.

GRAPHICAL REPRESENTATION OF VERTICAL DISPLACEMENT

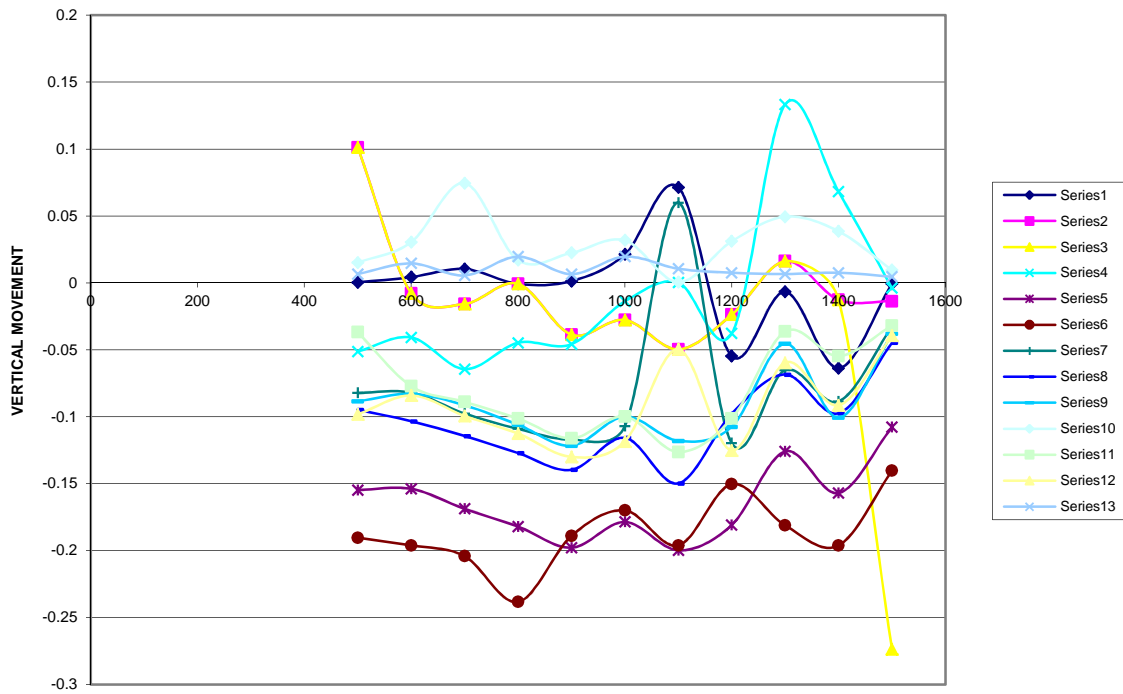


Fig. 3: The graph of vertical movement.

Analysis of the Horizontal Displacement

The analysis of the downstream offset movement was tested using the method of the one way Analysis of the variance (ANOVA). It involved splitting of the total variance of the data into meaningful components that measured the different source of variation. The ANOVA was computed using

Microsoft word excel which is shown in table 3 below.

The statistical test was carried out as follows:

Hypothesis:

$H_0: \mu_1 = \mu_2 = \mu_3 = \dots = \mu_{12}$ (there is no difference in offset distance measured for all epoch).

Subscript

H: not all μ_s are equal.

Table 3: ANOVA of the horizontal displacement:

Anova: Single Factor						
Groups	Count	Sum	Average	Variance		
-40.9545	10	739.4818	73.94818	2479.741		
37.04545	10	967.4818	96.74818	3184.257		
37.04545	10	-458.518	-45.8518	5051.909		
-20.9545	10	-9.51818	-0.95182	2355.832		
-70.9545	10	-608.518	-60.8518	1964.754		
-40.9545	10	-271.518	-27.1518	2359.29		
-35.9545	10	-783.518	-78.3518	888.7114		
-50.9545	10	-740.518	-74.0518	487.1186		
-16.4545	10	-571.818	-57.1818	1250.801		
-50.9545	10	-370.518	-37.0518	2350.993		
4.045455	10	-234.518	-23.4518	1405.309		
-12.9545	10	-380.518	-38.0518	1289.432		
Source of Variation	SS	df	MS	F	P-value	F crit
Between Groups	334584.6	11	30416.78	14.56036	7.27E-17	1.878388
Within Groups	225613.3	108	2089.012			
Total	560197.9	119				

From the ANOVA table 3, the analysis was done, thus:

$$F_{\text{cal}} = 14.56036$$

$$F = 1.878388$$

Since $F_{\text{cal}} > F$, the null hypothesis, H_0 was rejected at 5% level of significant. That is, there is significant difference between the observations.

Table 4 Minimum & Maximum Horizontal Movements

Epoch	Minimum Movement(mm)	Chainage (m)	Maximum Movement(mm)	Chainage (m)
1989	-48	500	186	1500
1991	-30	1300	155	1100
1996	-161	1400	70	500
1997	-25	500	105	1300
1998	-70	500	-169	1400
1999	-40	500	60	1400
2000	-40	500	-119	1100
2001	-96	500	-27	1500
2002	-118	1400	-16	500
2003	-97	800	63	1500
2004	-74	900	53	1500
2009	-92	1400	20	1500

From fig. 4, for the first epoch, the maximum horizontal displacement of 186mm existed at chainage 1500. In the second epoch (1991) the maximum horizontal displacement of 155mm exists at chainage 1100 that is in the same direction to the first epoch. But in the third epoch (1996), the maximum displacement of -161mm exists at chainage 1400 that is in opposite direction to the first two epochs. The 4th epoch reveals the displacement of 105mm at chainage 1300 in the same direction to the first two epochs. At 5th epoch (1998) and 6th epoch (1999), the movement is ranged from -169mm to -40mm at 1400 & 500 chainages respectively. Also, at 7th epoch, the movement is ranged from -

119mm to -23mm at chainage 1100 & 1500 respectively. At 8th epoch, the movement is range from -96mm to -27mm at 1100 & 1500 respectively. At 9th epoch (2002) the movement is range from -118mm to -16mm at 1400 & 500 respectively. At 10th epoch (2003) the movement is ranged from -97mm to 63mm at 800 and 1500 respectively. At 11th epoch (2004) the movement is range from -74mm to 53mm at 900 and 1500 respectively. The displacement ranged from -92mm-21mm at chainages 1400 and 1500 respectively at 12th epoch. Fig. 2 shows that the displacement of the other epochs were not significant as their graphs are about to superimpose.

In nutshell the results indicated that the horizontal movement is significant but in a random manner.

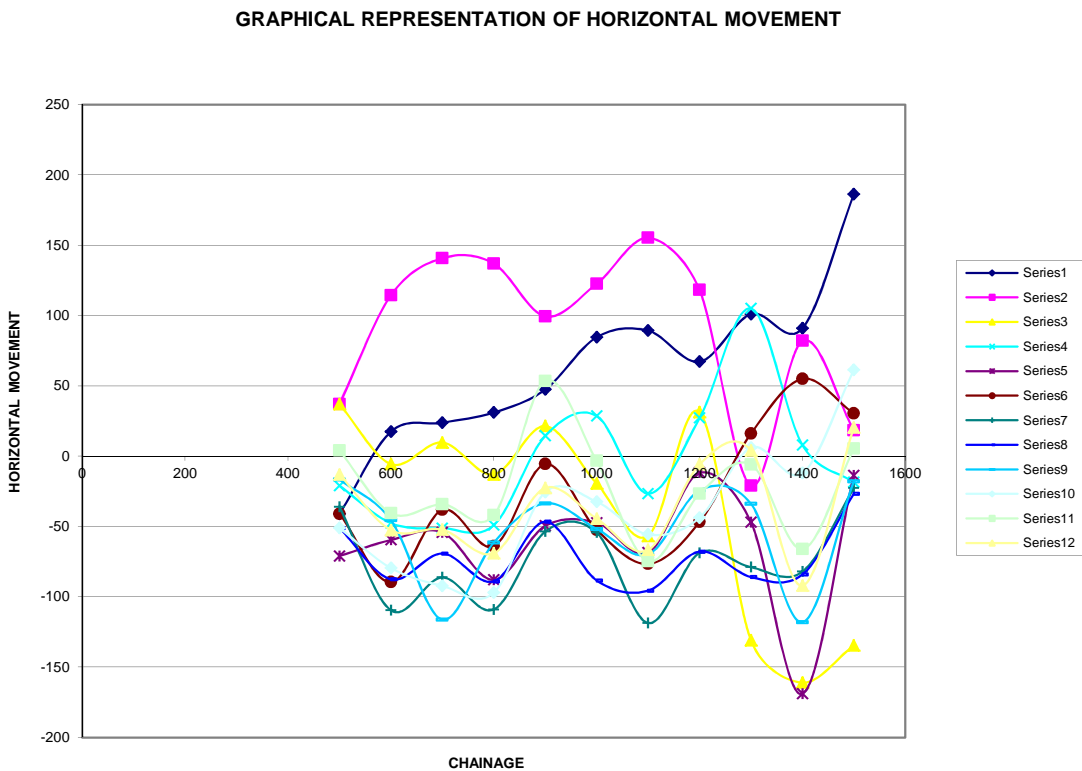


Fig. 4: The graph of horizontal movement.

Research Finding

The major findings of this research are;

1. The dam is not stable
2. The dam is experiencing small random movements which are not imposing significant force on the dam.
3. Statistically, there is deformation (both horizontally and vertically) at 5% significant level during the period under study.
4. Based on the graphical representation, the chainage 600,800, 900, 1000 are more vertically stable.
5. The vertical movement indicated at chainages 1500 and 1300 was as a result of the load on the structure.

Conclusion

From the observed data and analysis of the results carried out, it was revealed that the structure is experiencing small random movement which, as ascertained by Chm-Chyang(2000) and Brandon (2002), is not imposing significant force on the dam. This is indicated by the graphical representation of the results especially at chainages 1500 and 1300 for the vertical movement and chainages 1500, 1400, 1100, 800 and 700 for horizontal movement.

The statistical test based on the null hypothesis portrayed that, there is significant difference between observations made on each 12 epochs for vertical and horizontal formation respectively at 5% level of significance. The graphical representations also showed that the wall is vertically moving down and horizontally moving in the left hand(downstream) side of the wall. Therefore the sudden movements especially at the chainages mentioned above require regular and rigorous monitoring.

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Appendix 1 Averages of Height Differences

Line	CPGN 1989	CPGN 1991	CPGN 1996	CPGN 1997	CPGN 1998	CPGN 1999	CPGN 2000	CPGN 2001	CPGN 2002	CPGN 2003	CPGN 2004	CPGN 2009
DB2-500	174.297	174.310	174.310	174.309	174.306	174.304	174.294	174.303	174.200	174.303	174.295	174.295
500-600	0.115	-0.043	-0.048	-0.047	-0.050	-0.050	-0.050	-0.051	0.006	-0.005	-0.059	0.002
600-700	0.006	-0.029	-0.037	-0.046	-0.056	-0.040	-0.059	-0.045	0.021	-0.045	-0.045	0.010
700-800	-0.002	-0.042	-0.061	-0.061	-0.064	-0.065	-0.060	-0.054	0.065	-0.057	-0.060	0.001
800-900	0.013	-0.044	-0.041	-0.074	-0.098	-0.076	-0.082	-0.068	0.007	-0.060	-0.073	0.015
900-1000	-0.025	-0.057	-0.042	-0.090	-0.049	-0.084	-0.095	-0.084	0.013	-0.084	-0.091	0.002
1000-1100	-0.014	-0.033	-0.010	-0.071	-0.030	-0.074	-0.071	-0.062	0.022	-0.068	-0.079	0.015
1100-1200	-0.036	-0.053	0.004	-0.092	-0.056	-0.093	-0.105	-0.080	0.009	-0.095	-0.010	0.006
1200-1300	-0.010	-0.028	-0.034	-0.073	-0.010	-0.087	-0.053	-0.060	0.022	-0.060	-0.086	0.003
1300-1400	0.030	0.014	0.137	-0.018	-0.041	-0.033	-0.024	-0.007	0.030	-0.005	-0.020	0.002
1400-1500	0.001	-0.014	0.072	-0.049	-0.056	-0.056	-0.053	-0.063	0.029	-0.023	-0.052	0.003
1500-DB3	174.580	174.320	174.310	174.324	174.324	174.310	174.300	174.290	174.280	174.260	174.311	174.285

Appendix 2 Offset Differences

Chainage	Original Off-set	Epoch 1989	Epoch 1991	Epoch 1996	Epoch 1997	Epoch 1998	Epoch 1999	Epoch 2000	Epoch 2001	Epoch 2002	Epoch 2003	Epoch 2004	Epoch 2009
500	1.310	0.030	-0.010	-0.048	0.010	0.060	0.030	0.025	0.040	0.006	0.044	-0.015	0.002
600	1.320	0.060	0.000	-0.037	0.005	0.017	0.047	0.067	0.045	0.003	0.037	-0.002	0.010
700	1.339	0.056	-0.019	-0.061	0.000	0.003	-0.013	0.035	0.018	0.065	0.041	-0.017	0.001
800	1.350	0.065	-0.020	-0.041	-0.005	0.034	0.010	0.055	0.035	0.007	0.043	-0.012	0.015
900	1.348	0.010	-0.058	-0.040	-0.035	0.029	-0.015	0.033	0.026	0.013	0.005	-0.074	0.002
1000	1.347	0.028	-0.086	-0.010	-0.058	0.018	0.023	0.025	0.059	0.022	0.003	-0.026	0.015
1100	1.328	0.062	-0.088	-0.004	-0.034	0.008	0.016	0.058	0.035	0.009	-0.005	0.014	0.006
1200	1.323	0.017	-0.053	-0.034	-0.030	0.009	0.044	0.066	0.065	0.022	0.041	0.024	0.003
1300	1.310	0.015	-0.080	0.137	-0.099	0.053	-0.010	0.085	0.092	0.040	0.000	0.012	0.002
1400	1.387	0.063	-0.117	0.072	-0.097	0.080	-0.144	0.007	-0.005	0.029	-0.077	-0.023	0.003
1500	1.307	-0.007	-0.167	0.161	-0.045	0.040	-0.004	0.049	0.053	0.044	-0.035	0.021	0.006

Appendix 3: Adjusted Offset

Chainage	Camp. 1989	Camp. 1991	Camp. 1996	Camp. 1997	Camp. 1998	Camp. 1999	Camp. 2000	Camp. 2001	Camp. 2002	Camp. 2003	Camp. 2004	Camp. 2009
500	1.340	1.370	1.262	1.320	1.370	1.340	1.335	1.350	1.316	1.365	1.295	1.312
600	1.380	1.320	1.283	1.325	1.337	1.367	1.387	1.365	1.323	1.357	1.318	1.330
700	1.395	1.320	1.278	1.339	1342	1.326	1.374	1.357	1.404	1.380	1.322	1.340
800	1.415	1.330	1.309	1.345	1.384	1.360	1.405	1.385	1.358	1.393	1.338	1.365
900	1.358	1.290	1.306	1.313	1.377	1.333	1.381	1.374	1.367	1.353	1.374	1.350
1000	1.375	1.261	1.337	1.289	1.365	1.370	1.372	1.406	1.361	1.350	1.321	1.362
1100	1.390	1.240	1.324	1.294	1.336	1.344	1.386	1.363	1.338	1.323	1.342	1.334
1200	1.340	1.270	1.289	1.293	1.332	1.367	1.389	1.388	1.345	1.364	1.347	1.326
1300	1.325	1.230	1.447	1.211	1.363	1.300	1.395	1.402	1.341	1.310	1.322	1.312
1400	1.450	1.270	1.459	1.290	1.467	1.243	1.380	1.382	1.416	1.310	1.364	1.390
1500	1.300	1.140	1.468	1.352	1.347	1.303	1.356	1.360	1.352	1.272	1.328	1.313

Appendix 4: Mean Offset Movement and Displacement

Mean X	X-X ₁	X-X ₂	X-X ₃	X-X ₄	X-X ₅	X-X ₆	X-X ₇	X-X ₈	X-X ₉	X-X ₁₀	X-X ₁₁	X-X ₁₂
-0.01095	-0.04095	0.03705	-0.07250	-0.02095	-0.07095	-0.04095	-0.03595	-0.05095	-0.01645	-0.05095	0.00405	-0.01295
-0.04255	0.01745	0.11444	-0.43110	-0.04755	-0.05955	-0.08955	-0.10955	-0.08755	-0.04545	-0.07955	-0.04055	-0.05255
-0.05118	0.02382	0.14081	-0.50200	-0.05118	-0.05418	-0.03818	-0.08618	-0.06918	-0.11618	-0.09218	-0.03418	-0.05218
-0.05396	0.03104	0.13706	-0.55260	-0.04896	-0.08796	-0.06396	-0.10896	-0.08896	-0.06136	-0.09696	-0.04196	-0.06896
-0.02055	0.04745	0.09945	-0.18410	0.01445	-0.04955	-0.00555	-0.05355	-0.04655	-0.03345	-0.02555	0.05345	-0.02255
-0.02934	0.08464	0.12264	-0.31270	0.02864	-0.04734	-0.05234	-0.05434	-0.08834	-0.05164	-0.03234	-0.00333	-0.04434
-0.06063	0.08937	0.15537	-0.66290	-0.02663	-0.06863	-0.07663	-0.11863	-0.09563	-0.06973	-0.05563	-0.07462	-0.06663
-0.00277	0.06727	0.11287	0.00350	0.02727	-0.01177	-0.04677	-0.06877	-0.06777	-0.02427	-0.04377	-0.02677	-0.00577
0.00608	0.10108	0.10508	-0.07017	0.10508	-0.04692	0.01608	-0.07892	-0.08592	-0.03382	-0.00608	-0.00591	0.00408
-0.08900	0.09100	0.00800	-1.05100	0.00800	-0.16900	0.05500	-0.08200	-0.08400	-0.11800	-0.01200	-0.06600	-0.09200
0.26394	0.18639	-0.01861	0.12930	-0.01861	-0.01361	0.03039	-0.02261	-0.02661	-0.01791	0.06139	-0.00539	0.02039

Appendix: 5 Least Squares Estimate: X for 12 Epochs

Chainage	1989 X_1	1991 X_2	1996 X_3	1997 X_4	1998 X_5	1999 X_6	2000 X_7	2001 X_8	2002 X_9	2003 X_{10}	2004 X_{11}	2009 X_{12}
500	174.351	174.400	174.325	174.740	174.446	174.435	174.473	174.455	174.170	174.430	174.453	174.277
600	174.480	174.380	174.281	174.801	174.431	174.418	174.468	174.442	174.166	174.457	174.433	174.274
700	174.500	174.374	174.249	174.862	174.401	174.402	174.454	174.435	174.177	174.444	174.428	174.280
800	174.511	174.354	174.191	174.909	174.382	174.370	174.429	174.420	174.233	174.419	174.407	174.276
900	174.538	174.333	174.154	174.943	174.319	174.325	174.391	174.490	174.230	174.381	174.373	174.286
1000	174.527	174.298	174.116	174.961	174.305	174.275	174.341	174.344	174.234	174.325	174.322	174.284
1100	174.527	174.288	174.109	174.998	174.311	174.233	174.315	174.320	174.247	174.292	174.282	174.294
1200	174.504	174.258	174.117	174.013	174.290	174.358	174.255	174.278	174.228	174.230	174.312	174.295
1300	174.508	174.252	174.087	174.048	174.315	174.304	174.246	174.246	174.240	174.192	174.285	174.294
1400	174.552	174.289	174.228	174.138	174.310	174.304	174.257	174.277	174.270	174.219	174.284	174.291
1500	174.566	174.297	174.303	174.197	174.289	174.281	174.249	174.252	174.290	174.228	174.272	174.290

Appendix 6: Residual, V for 12 Epochs

Chainage	1989 V_1	1991 V_2	1996 V_3	1997 V_4	1998 V_5	1999 V_6	2000 V_7	2001 V_8	2002 V_9	2003 V_{10}	2004 V_{11}	2009 V_{12}
500	-0.05475	-0.09040	-0.01499	-0.43125	-0.14035	-0.13093	-0.17891	-0.15176	0.03823	-0.12731	-0.15749	0.01840
600	-0.01369	-0.02260	-0.00375	-0.10781	-0.14035	-0.03273	-0.04473	-0.03794	0.00956	-0.03183	-0.03937	0.00460
700	-0.01369	-0.02260	-0.00375	-0.10781	-0.14035	-0.03273	-0.04473	-0.03794	0.00956	-0.03183	-0.03937	0.00460
800	-0.01369	-0.02260	-0.00375	-0.10781	-0.14035	-0.03273	-0.04473	-0.03794	0.00956	-0.03183	-0.03937	0.00460
900	-0.01369	-0.02260	-0.00375	-0.10781	-0.14035	-0.03273	-0.04473	-0.03794	0.00956	-0.03183	-0.03937	0.00460
1000	-0.01369	-0.02260	-0.00375	-0.10781	-0.14035	-0.03273	-0.04473	-0.03794	0.00956	-0.03183	-0.03937	0.00460
1100	-0.01369	-0.02260	-0.00375	-0.10781	-0.14035	-0.03273	-0.04473	-0.03794	0.00956	-0.03183	-0.03937	0.00460
1200	-0.01369	-0.02260	-0.00375	-0.10781	-0.14035	-0.03273	-0.04473	-0.03794	0.00956	-0.03183	-0.03937	0.00460
1300	-0.01369	-0.02260	-0.00375	-0.10781	-0.14035	-0.03273	-0.04473	-0.03794	0.00956	-0.03183	-0.03937	0.00460
1400	-0.01369	-0.02260	-0.00375	-0.10781	-0.14035	-0.03273	-0.04473	-0.03794	0.00956	-0.03183	-0.03937	0.00460
1500	-0.01369	-0.02260	-0.00375	-0.10781	-0.14035	-0.03273	-0.04473	-0.03794	0.00956	-0.03183	-0.03937	0.00460

Appendix 7: A Posteriori Variance

1989 ₂ 1	1991 ₂ 2	1996 ₂ 3	1997 ₂ 4	1998 ₂ 5	1999 ₂ 6	2000 ₂ 7	2001 ₂ 8	2002 ₂ 9	2003 ₂ 10	2004 ₂ 11	2009 ₂ 12
0.00028	0.00076	0.000000	0.01743	0.022159	0.001607	0.0030007	0.002159	0.000137	0.001519	0.002324	0.000000

Appendix 9: Variance- Covariance

Chainage	1989	1991	1996	1997	1998	1999	2000	2001	2002	2003	2004	2009
	1	2	3	4	5	6	7	8	9	10	11	12
500	0.00825	0.02247	0.00062	0.51143	0.65000	0.04714	0.08802	0.06334	0.00402	0.04457	0.06821	0.00093
600	0.00936	0.02553	0.00700	0.58119	0.73864	0.05356	0.10002	0.07197	0.00457	0.05065	0.07751	0.00106
700	0.01012	0.02758	0.00076	0.62768	0.79773	0.05786	0.10802	0.07773	0.00493	0.05467	0.08371	0.00114
800	0.01045	0.02860	0.00079	0.65093	0.82728	0.05997	0.11202	0.08060	0.00511	0.05672	0.08682	0.00185
900	0.01045	0.02860	0.00079	0.65093	0.82728	0.05997	0.11202	0.08060	0.00493	0.05672	0.08682	0.00185
1000	0.01012	0.02758	0.00076	0.62761	0.79774	0.05786	0.10803	0.07773	0.00456	0.05467	0.08371	0.00114
1100	0.00937	0.02553	0.00072	0.58119	0.73865	0.05357	0.10002	0.07197	0.00419	0.05065	0.07751	0.00105
1200	0.00826	0.02273	0.00062	0.51143	0.65000	0.04714	0.08802	0.06334	0.00328	0.04457	0.06821	0.00931
1300	0.00675	0.01838	0.00056	0.41845	0.53182	0.03866	0.07201	0.05182	0.00237	0.03645	0.05580	0.00762
1400	0.00487	0.01328	0.00037	0.30222	0.02786	0.02785	0.05201	0.03743	0.00128	0.02636	0.04030	0.00055
1500	0.00262	0.00715	0.00012	0.16273	0.20682	0.01499	0.28017	0.02152	0.00457	0.01418	0.02170	0.00030

Appendix 9: Reliability

Chainages	1989	1991	1996	1997	1998	1999	2000	2001	2002	2003	2004	2009
	R ₁	R ₂	R ₃	R ₄	R ₅	R ₆	R ₇	R ₈	R ₉	R ₁₀	R ₁₁	R ₁₂
500	0.03000	0.03409	0.03409	0.03409	0.03409	0.03409	0.03409	0.03409	0.03409	0.03409	0.03409	0.03409
600	0.03000	0.03000	0.03000	0.03000	0.03000	0.03000	0.03000	0.03000	0.03000	0.03000	0.03000	0.03000
700	0.02778	0.02778	0.02778	0.02778	0.02778	0.02778	0.02778	0.02778	0.02778	0.02778	0.02778	0.02778
800	0.02579	0.02579	0.02579	0.02579	0.02579	0.02579	0.02579	0.02579	0.02579	0.02579	0.02579	0.02579
900	0.02579	0.02579	0.02579	0.02579	0.02579	0.02579	0.02579	0.02579	0.02579	0.02579	0.02579	0.02579
1000	0.02778	0.02778	0.02778	0.02778	0.02778	0.02778	0.02778	0.02778	0.02778	0.02778	0.02778	0.02778
1100	0.03000	0.03000	0.03000	0.03000	0.03000	0.03000	0.03000	0.03000	0.03000	0.03000	0.03000	0.03000
1200	0.03409	0.03409	0.03409	0.03409	0.03409	0.03409	0.03409	0.03409	0.03409	0.03409	0.03409	0.03409
1300	0.04167	0.04167	0.04167	0.04167	0.04167	0.04167	0.04167	0.04167	0.04167	0.04167	0.04167	0.04167
1400	0.05769	0.05769	0.05769	0.05769	0.05769	0.05769	0.05769	0.05769	0.05769	0.05769	0.05769	0.05769
1500	0.10714	0.10714	0.10714	0.10714	0.10714	0.10714	0.10714	0.10714	0.10714	0.10714	0.10714	0.10714