

## ESTIMATION OF THE SHAPE PARAMETER OF GENERALIZED RAYLEIGH DISTRIBUTION UNDER CONJUGATE PRIOR

By

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E-mail: [yakubualiyu@abu.edu.ng](mailto:yakubualiyu@abu.edu.ng)**ABSTRACT**

*Modeling of real life scenario help us to better understand and explain unforeseen eventualities when they take place, thereby enabling us to reproduce such a scenario on a large and/ or on a simplified scale aimed at describing critical parts of the phenomenon. Generalized Rayleigh Distribution can be used quiet effectively in modeling real life scenario. Every distribution model has a set of parameters that needs to be estimated. In this work, a Bayesian estimate of the shape parameter of Generalized Rayleigh Distribution was considered under the assumption of conjugate (gamma) prior. The Bayes estimates were obtained under both symmetric and asymmetric loss functions. The performances of these estimates were compared using Monte Carlo simulation. It is observed that the estimates under the asymmetric loss function slightly perform better than those under the symmetric loss function.*

**Keywords:** Bayes estimators, Conjugate prior, Squared error loss function, Entropy loss function, Precautionary loss function

**INTRODUCTION**

Several reseachers such as Burr (1942), Ana, *et.al* (2013), Hamdy and Mahmoud (2014) and so on have studied and developed different distribution functions that can be used in modeling lifetime data. Twelve different families of this kind were suggested by Burr (1942). Among these suggested families of distribution developed, Burr Type X and Burr Type XII distributions were the most popular distributions and hence, they receives the most attention.

$$F(x; \varphi, \gamma) = (1 - e^{-(\gamma x)^2})^\varphi \quad \text{for } x, \varphi, \gamma > 0 \quad (1)$$

Its probability density function (pdf) is given by:

$$f(x; \varphi, \gamma) = 2\varphi\gamma^2 x (1 - e^{-(\gamma x)^2})^{\varphi-1} e^{-(\gamma x)^2} \quad (2)$$

where  $\varphi$  and  $\gamma$  are shape and scale parameters respectively.

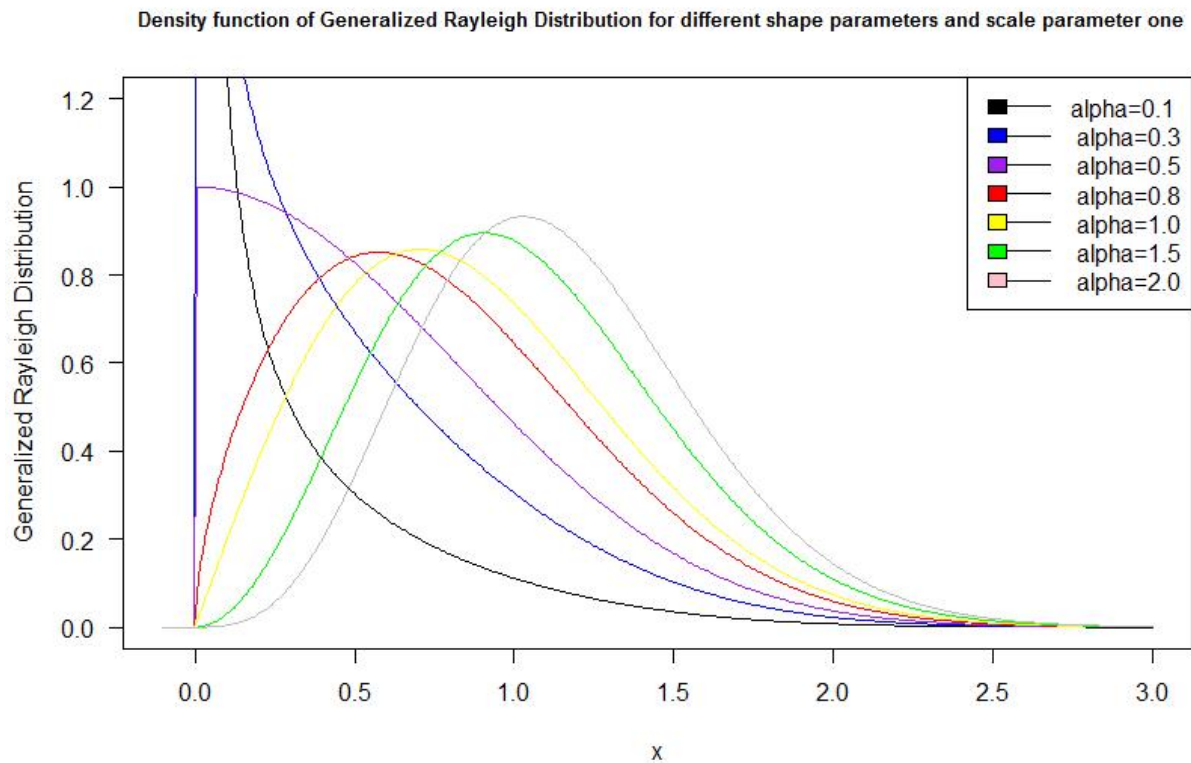
$$\text{Survival function } S(x; \varphi, \gamma) = 1 - F(x; \varphi, \gamma) = 1 - (1 - e^{-(\gamma x)^2})^\varphi \quad (3)$$

In 2001, Surles and Padgett extended Burr Type X distribution to a two parameter distribution. This distribution was named Generalized Rayleigh Distribution by Kundu and Raqab (2005), Lio, *et.al* (2011) and Abdel-Hady (2013), this name will be adopted in this work. For  $\varphi > 0$  and  $\gamma > 0$ , the two-parameter GRD will be denoted by  $GR(\varphi, \gamma)$  and its cumulative distribution function is given by:

$$\text{Hazard function } h(x; \varphi, \gamma) = \frac{f(x; \varphi, \gamma)}{S(x; \varphi, \gamma)} = \frac{2\varphi\gamma^2 x(1-e^{-(\gamma x)^2})^{\varphi-1} e^{-(\gamma x)^2}}{1-(1-e^{-(\gamma x)^2})^\varphi} \tag{4}$$

GRD is widely used in modeling events that occur in different fields such as medicine, social and natural sciences. In Physics for instance, the Rayleigh distribution is used in the study of various types of radiation such as light and sound measurements. It is used as a model for wind speed and is often applied to

wind driven electrical generation. For details, see Samaila and Genac (2006). It is also used in modeling strength and lifetime data (Surles and Padgett (2001), Lio, *et.al.*, (2011), Kundu and Raqab, (2005)). Hence, the GRD has a survival function and hazard function as shown in (3) and (4) respectively.



**Figure 1:** The Graph of Generalized Rayleigh Distribution for different values of shape parameters when the scale parameter takes the value one

The graph of the distribution is shown in figure 1 for different shape parameter values. It is clear from figure 1 that the probability density function of a GRD is a decreasing function if  $\alpha \leq \frac{1}{2}$  and it is right skewed unimodal when  $\alpha > \frac{1}{2}$  see also Kundu and Raqab, (2005). They used six Non-bayesian methods of estimation in estimating the parameters of GRD. These methods are; maximum likelihood method, modified moment estimator,

modified L-moment estimator, least squares estimator, weighted least squares estimator and percentile-based estimator. In the study, Monte-Carlo simulation was used to compare the performances of the different estimators mainly with respect to their biases and mean square errors for different sample sizes and different parameter values. It was found that, the performances of most of the methods are quite bad when the sample

size is small (say  $n \leq 10$ ). In particular, the estimation of the shape parameter becomes very difficult for small sample sizes. It was observed that, the biases of all the methods are quite severe for small and moderate sample sizes ( $n \leq 20$ ). For moderate sample size, it was shown that the least squares estimate performs quite well and for large sample size, the weighted least squares estimate outperforms the least squares estimate marginally. If mean square estimate is considered, the weighted least squares perform better than the rest in most cases considered.

Raqab and Kundu (2005) studied different methods of estimating  $R = P(Y < X)$  when  $Y$  and  $X$  both follow GRD with different shape parameters but the same scale parameter. It is observed that the MLEs of the three unknown parameters can be obtained by solving one non-linear equation for an unknown scale parameter value. To solve for the unknown MLEs of the parameters and MLE of  $R$ , the Researchers developed an iterative procedure. Also, to compute the asymptotic confidence intervals, the asymptotic distribution of  $R$  was derived. The asymptotic confidence intervals were found to work quite well even when the sample size is small. Two bootstrap confidence intervals were also proposed and their performances were also found to be satisfactory. On the other hand, they compare MLE and Uniformly Minimum Variance Unbiased Estimators with different Bayes estimators when the scale parameter is known. It is observed that the MLEs behave quite similarly with the Bayes estimators with non-informative priors.

Mahdi, (2006) estimate the parameters of a Rayleigh distribution using five classical point estimation techniques. These methods are: maximum likelihood technique, moment method, probability weighted moment method, least square method and least absolute deviation method. He proposed the modified maximum likelihood estimation method for the parameters and compares it with the five classical techniques mentioned

above. In his comparison of the different methods considered, all the methods performed reasonably well except the method of moments. On the other hand, the modified maximum likelihood method provides better estimates for the parameters when the sample sizes are moderate or large, while in the case of small samples, the probability weighted moment method outperforms the maximum likelihood method for the estimation of the threshold parameter and performs almost as good as the maximum likelihood method for the estimation of the scale parameter. Hence, Mahdi recommends using the modified maximum likelihood method for the parameter estimation of the Rayleigh distribution if the sample size is large.

A conventional method for estimating the two parameters of generalized Rayleigh distribution for small sample size, medium sample size and large sample size was suggested by (Al- Naqeeb and Hamed (2009)). The estimators were compared by using mean square error (MSE) as a performance measure based on the simulated data used in the study.

Pak, *et.al.*, (2013) study the problem of estimating the scale parameter of Rayleigh distribution under doubly type-II censoring scheme when the lifetime observations are fuzzy and are assumed to be related to underlying crisp realization of a random sample. A new method to determine the maximum likelihood estimate of the scale parameter was proposed and its asymptotic variance was derived using the missing information principle. Abdel-Hady (2013) extended the Marshall and Olkin's bivariate exponential model to the Generalized Bivariate Rayleigh (GBR) Distribution. The CDF, pdf and the conditional distribution of the GBR distribution were computed. The maximum likelihood estimation procedure for the estimation of the GBR parameters when all parameters are unknown and the observed Fisher Information Matrix were also derived.

Al - Kanani and Abbas, (2014) studied the Bayesian and non-Bayesian (classical) methods in

estimating the parameters of the GRD. The non-Bayesian methods considered are the maximum likelihood estimator and the moment estimator while the single stage shrinkage estimator was determined and derived for the Bayesian method. Reliability function and hazard function were also obtained. Pathak and Chaturvedi, (2014) explored and compared the performance of uniformly minimum variance unbiased as well as maximum likelihood estimators of the reliability functions  $R(t) = P(X > t)$  and  $P =$

$P(X > Y)$  for the two-parameter exponentiated Rayleigh distribution. It was observed that, the uniformly minimum variance unbiased estimators of  $R(t)$  and  $P$  are superior to their maximum likelihood counterparts. The rest of the paper is organized as follows: in section two, we briefly describe the MLE. In section 3 we describe the Bayesian method; Numerical comparisons are presented in section 4 and we finally conclude in section 5.

### BAYESIAN ESTIMATION OF THE PARAMETER

Bayesian estimation technique will be used to estimate the shape parameter of the generalized Rayleigh distribution assuming conjugate (gamma) and

non-conjugate (uniform) priors under the squared error loss function, Entropy loss function and Precautionary loss function will be considered.

#### Squared Error Loss Function

The squared error loss function is defined as

$$L(\vartheta, \hat{\vartheta}) = (\hat{\vartheta} - \vartheta)^2 \tag{8}$$

the Bayes estimator of  $\vartheta$  denoted by  $\hat{\vartheta}_{SL}$  relative to the squared error loss function is given by

$$\hat{\vartheta}_{SL} = E(\vartheta/x) \tag{9}$$

The squared error loss function is classified as symmetric loss function, because it assigns equal importance to the losses due to overestimation and underestimation. This is impractical in real life situations since overestimation of a parameter can lead to more severe or less severe consequences than under estimation or vice versa as in the estimation of reliability or failure rate function. Hence, the loss function which associate greater importance to

overestimation or underestimation and serves as alternative to symmetric loss function is the asymmetric loss function. Several asymmetric loss functions has been proposed and studied by many researchers such as Entropy loss function, Precautionary loss function, LINEX loss function, weighted loss function, etc. This paper will consider only Entropy loss function and Precautionary loss function

#### Entropy loss function

The Entropy loss which was discussed by Dey & Liu (1992) and Dey *et.al*(1987) is defined as

$$L_{EL}(\hat{\vartheta}, \vartheta) \propto \left(\frac{\hat{\vartheta}}{\vartheta}\right)^k - k \log\left(\frac{\hat{\vartheta}}{\vartheta}\right) - 1 \quad k \neq 0 \tag{10}$$

where  $\hat{\vartheta}$  is an estimate of  $\vartheta$ .

The Bayes estimator of  $\vartheta$  denoted by  $\hat{\vartheta}_{EL}$  relative to the entropy loss function is given by

$$\hat{\vartheta}_{EL} = [E_{\vartheta}(\vartheta^{-k})]^{-\frac{1}{k}} \tag{11}$$

provided  $E_{\vartheta}(\vartheta^{-k})$  exist. If  $k=1$ , the bayes estimator (II) coincides with the bayes estimator under the squared error loss function and for  $k=1$ , it coincides with the weighted squared error loss function.

**Precautionary Loss Function**

Norstrom, (1996) introduced an asymmetric loss function called Precautionary loss function which is defined as

$$\mathcal{L}(\hat{\vartheta}, \vartheta) = \frac{(\hat{\vartheta} - \vartheta)^2}{\hat{\vartheta}} \tag{12}$$

the Bayes estimator of  $\vartheta$  denoted by  $\hat{\vartheta}_{PL}$  relative to the precautionary error loss function is given by

$$\hat{\vartheta} = (E(\vartheta^2/x))^{1/2} \tag{13}$$

**Bayes estimates under the Gamma prior**

Let  $\varphi$  have the following prior distribution,

$$\pi(\varphi) = \frac{b^a}{\Gamma(a)} \varphi^{a-1} e^{-b\varphi} \text{ for } a, b, \varphi > 0 \tag{29}$$

and let  $x_1, x_2, \dots, x_n$  be a random sample from  $GR(\varphi, 1)$ , then the likelihood of the observed data is as obtained in (15) and the posterior density function of  $\varphi$  is given by equation (30)

$$f(\varphi / x_1, x_2, \dots, x_n) = \frac{\varphi^{n+a-1} H^{n+a} e^{-N\varphi}}{\Gamma(n+a)} \tag{30}$$

where  $H = b + \sum_{i=1}^n \ln(1 - e^{-x_i})^{-1} = b + M$

from the above distribution, it is clear that the posterior distribution under the gamma prior is gamma distributed with parameters  $(n + a)$  and  $H$ . Hence, the gamma prior is said to be a *conjugate prior*, since the posterior distribution is in the same class of distribution with the prior distribution.

The Bayesian estimators of the shape parameter  $\varphi$  under the Squared error, Entropy and Precautionary loss functions using gamma prior are given by equations (31), (32) and (33) respectively

$$\hat{\varphi}_{GSL} = \frac{n+a}{H} \tag{31}$$

$$\hat{\varphi}_{GEL} = \frac{1}{H} \left( \frac{\sqrt{n+a}}{\sqrt{n+a-k}} \right)^{\frac{1}{k}} \tag{32}$$

$$\hat{\varphi}_{GPL} = (E(\varphi^2))^{1/2} = \sqrt{\frac{(n+a+1)(n+a)}{H^2}} = \frac{1}{H} \sqrt{(n+a+1)(n+a)} \tag{33}$$

If  $H = b + M$ , and  $M \sim \text{Gamma}(n, \varphi)$  then, the distribution of  $H$  given by

$$f(H) = \frac{\varphi^n (H-b)^{n-1} e^{-\varphi(H-b)}}{\Gamma(n)} \tag{34}$$

$$E\left(\frac{1}{H}\right) = \left\{ \frac{1}{n-1} - \frac{\xi b}{(n-1)(n-2)} + \frac{(\xi b)^2}{(n-1)(n-2)(n-3)} - \frac{(\xi b)^3}{(n-1)(n-2)(n-3)(n-4)} + \dots \right\} \quad (35)$$

$$E\left(\frac{1}{H^2}\right) = \xi^2 \left[ \frac{1}{(n-1)(n-2)} - \frac{2\xi b}{(n-1)(n-2)(n-3)} + \frac{3(\xi b)^2}{(n-1)(n-2)(n-3)(n-4)} - \frac{4(\xi b)^3}{(n-1)(n-2)(n-3)(n-4)(n-5)} + \dots \right] \quad (36)$$

Taking two terms in  $E(H^{-1})$  and three terms in  $E(H^{-2})$ , then,  $Var\left(\frac{1}{H}\right)$  is given by

$$Var\left(\frac{1}{H}\right) = \frac{\xi^2}{(n-1)^2(n-2)} - \frac{4\xi^3 b}{(n-1)^2(n-2)(n-3)} - \frac{\xi^4 b^2}{(n-1)^2(n-2)^2} \quad (37)$$

3.5.1 Variance of the shape parameter under the Squared error, Entropy and Precautionary loss functions are given by

$$var(\hat{\xi}_{GSL}) = (n+a)^2 \left( \frac{\xi^2}{(n-1)^2(n-2)} - \frac{4\xi^3 b}{(n-1)^2(n-2)(n-3)} + \frac{\xi^4 b^2(2n^2-2n-6)}{(n-1)^2(n-2)^2(n-3)(n-4)} \right) \quad (38)$$

$$var(\hat{\xi}_{GEL}) = \left( \frac{\sqrt{n+a}}{\sqrt{n-k+a}} \right)^{\frac{2}{k}} \left( \frac{\xi^2}{(n-1)^2(n-2)} - \frac{4\xi^3 b}{(n-1)^2(n-2)(n-3)} + \frac{\xi^4 b^2(2n^2-2n-6)}{(n-1)^2(n-2)^2(n-3)(n-4)} \right) \quad (39)$$

$$Var(\hat{\xi}_{GPL}) = (n+a+1)(n+a) \left( \frac{\xi^2}{(n-1)^2(n-2)} - \frac{4\xi^3 b}{(n-1)^2(n-2)(n-3)} + \frac{\xi^4 b^2(2n^2-2n-6)}{(n-1)^2(n-2)^2(n-3)(n-4)} \right) \quad (40)$$

3.5.2 Risk of the estimators under the gamma prior

$$R(\hat{\xi}_{GSL}) = E(\xi^2) - (E(\xi))^2 = \frac{n+a}{H^2} \quad (41)$$

$$R(\hat{\xi}_{GEL}) = w \left[ \frac{\sqrt{n+a} \sqrt{n-k}}{\sqrt{n+a-k} \sqrt{n}} - \ln \left( \frac{\sqrt{n+a}}{\sqrt{n+a-k}} \right) + k \ln(n-1) - 1 \right] \quad (42)$$

$$R(\hat{\xi}_{GPL}) = 2 \left( \frac{\sqrt{(n+a+1)(n+a)}}{H} - \frac{n+a}{H} \right) \quad (43)$$

### NUMERICAL COMPARISONS

In this section, results of some numerical experiments were obtained in order to compare the performance of the estimators proposed in the previous section. Extensive Monte Carlo simulations were carried out to compare the performance of the different

estimators with respect to biases and mean-squared errors (MSEs) for different sample sizes ( $n=15, 20, 30, 50$  and  $100$ ) and different shape-parameter values ( $0.25, 0.5, 1, 1.5, 2,$  and  $2.5$ ). All results are replicated 10000 times and averaged over.

**Table 1:** Average estimates and corresponding MSEs (within parenthesis) for  $\varphi$

n	method	$\xi = 0.25$	$\xi = 0.5$	$\xi = 1.0$	$\xi = 1.5$	$\xi = 2$	$\xi = 2.5$
15	GSL a=b=.5	0.2742	0.5408	1.0625	1.5659	2.0624	2.5196
		(0.0135)	(0.0498)	(0.1850)	(0.3902)	(0.6984)	(1.0563)
	GSL a=b=1	0.2802	0.5489	1.0649	1.5356	1.9808	2.4022
		(0.0146)	(0.0520)	(0.1940)	(0.4439)	(0.8383)	(1.4826)
	K=1	0.2677	0.5318	1.0423	1.5289	2.0098	2.4670
		(0.0124)	(0.0464)	(0.1725)	(0.3582)	(0.6267)	(0.9861)
	GEL a=b=.5	0.2748	0.5356	1.0332	1.4968	1.9336	2.3395
		(0.0132)	(0.0477)	(0.1755)	(0.3957)	(0.7688)	(1.3227)
	K=-1	0.2666	0.5288	1.0400	1.5342	2.0121	2.4723
		(0.0121)	(0.0465)	(0.1674)	(0.3623)	(0.6447)	(0.9968)
	GEL a=b=1	0.2733	0.5360	1.0417	1.5028	1.9364	2.3421
		(0.0131)	(0.0474)	(0.1780)	(0.4057)	(0.7757)	(1.3174)
	GPL a=b=.5	0.2831	0.5610	1.0971	1.6182	2.1172	2.6025
		(0.0157)	(0.0584)	(0.2100)	(0.4480)	(0.7628)	(1.2036)
	GPL a=b=1	0.2892	0.5662	1.0980	1.5816	2.0364	2.4639
		(0.0165)	(0.0605)	(0.2242)	(0.5057)	(0.9619)	(1.6558)

As expected, from tables 1, 2 and 3, it is observed that the performances of all the Bayesian estimates become better when sample size increases. Also, the Bayesian estimates become closer as sample size increases. Hence, the Bayesian estimators become more stable as sample size increases. Careful examination of the estimates reveals that when the

hyper-parameter values takes 0.5 (that is a=b=0.5), the estimates are slightly better than the estimates when the hyper-parameters takes the value 1(that is a=b=1). It is also observed under the gamma prior that the Bayes estimates under the precautionary loss function is more stable than the estimates under

**Table 2:** Average estimates and corresponding MSEs (within parenthesis) for  $\varphi$

n	method	$\xi = 0.25$	$\xi = 0.5$	$\xi = 1.0$	$\xi = 1.5$	$\xi = 2$	$\xi = 2.5$	
20	GSL a=b=.5	0.2676 (0.0091)	0.5310 (0.0344)	1.0516 (0.1283)	1.5554 (0.2763)	2.0409 (0.4661)	2.5230 (0.7369)	
	GSL a=b=1	0.2727 (0.0096)	0.5357 (0.0352)	1.0444 (0.1280)	1.5310 (0.2840)	1.9898 (0.5223)	2.4380 (0.8864)	
	K=1	0.2616 (0.0084)	0.5209 (0.0316)	1.0371 (0.1216)	1.5200 (0.2570)	2.0075 (0.4481)	2.4875 (0.6944)	
	GEL a=b=.5	0.2670 (0.0088)	0.5291 (0.0329)	1.0287 (0.1206)	1.5009 (0.2682)	1.9566 (0.4973)	2.3851 (0.8280)	
	K=-1	0.2621 (0.0084)	0.5225 (0.0325)	1.0303 (0.1220)	1.5258 (0.2596)	2.0023 (0.4468)	2.4724 (0.6750)	
	GEL a=b=.5	0.2676 (0.0089)	0.5280 (0.0335)	1.0289 (0.1227)	1.5065 (0.2680)	1.9545 (0.4956)	2.3842 (0.8168)	
	GEL a=b=1	0.2676 (0.0089)	0.5280 (0.0335)	1.0289 (0.1227)	1.5065 (0.2680)	1.9545 (0.4956)	2.3842 (0.8168)	
	GPL a=b=.5	0.2738 (0.0101)	0.5449 (0.0385)	1.0732 (0.1390)	1.5939 (0.3041)	2.0950 (0.5187)	2.5831 (0.7995)	
	GPL a=b=1	0.2776 (0.0104)	0.5516 (0.0395)	1.0749 (0.1437)	1.5660 (0.3157)	2.0328 (0.5694)	2.4934 (0.9723)	
	30	GSL a=b=.5	0.2616 (0.0055)	0.5229 (0.0211)	1.0331 (0.0789)	1.5368 (0.1707)	2.0303 (0.3011)	2.5233 (0.4590)
		GSL a=b=1	0.2638 (0.0056)	0.5247 (0.0211)	1.0347 (0.0800)	1.5191 (0.1714)	1.9924 (0.3044)	2.4523 (0.4908)
		K=1	0.2584 (0.0051)	0.5161 (0.0200)	1.0203 (0.0753)	1.5164 (0.1638)	2.0098 (0.2827)	2.4845 (0.4362)
GEL a=b=.5		0.2618 (0.0054)	0.5187 (0.0200)	1.0190 (0.0750)	1.5083 (0.1676)	1.9781 (0.2936)	2.4301 (0.4774)	
K=-1		0.2572 (0.0052)	0.5150 (0.0196)	1.0205 (0.0766)	1.5192 (0.1671)	1.9983 (0.2813)	2.4854 (0.4446)	
GEL a=b=.5		0.2600 (0.0054)	0.5181 (0.0198)	1.0208 (0.0752)	1.5009 (0.1628)	1.9732 (0.2987)	2.4204 (0.4726)	
GEL a=b=1		0.2600 (0.0054)	0.5181 (0.0198)	1.0208 (0.0752)	1.5009 (0.1628)	1.9732 (0.2987)	2.4204 (0.4726)	
GPL a=b=.5		0.2654 (0.0058)	0.5306 (0.0224)	1.0536 (0.0844)	1.5616 (0.1827)	2.0644 (0.3158)	2.5569 (0.4845)	
GPL a=b=1		0.2689 (0.0060)	0.5350 (0.0231)	1.0480 (0.0842)	1.5465 (0.1819)	2.0218 (0.3192)	2.4952 (0.5247)	



**Table 3:** Average estimates and corresponding MSEs (within parenthesis) for  $\varphi$

n	method	$\xi = 0.25$	$\xi = 0.5$	$\xi = 1.0$	$\xi = 1.5$	$\xi = 2$	$\xi = 2.5$	
50	GSL a=b=.5	0.2562 (0.0032)	0.5134 (0.0115)	1.0207 (0.0437)	1.5224 (0.0981)	2.0178 (0.1718)	2.5072 (0.2626)	
	GSL a=b=1	0.2584 (0.0032)	0.5148 (0.0115)	1.0173 (0.0436)	1.5126 (0.0969)	1.9977 (0.1715)	2.4682 (0.2660)	
	K=1	0.2540 (0.0030)	0.5077 (0.0109)	1.0144 (0.0432)	1.5101 (0.0945)	2.0023 (0.1654)	2.4872 (0.2527)	
	GEL a=b=.5	0.2562 (0.0032)	0.5116 (0.0110)	1.0118 (0.0433)	1.5051 (0.0952)	1.9833 (0.1647)	2.4514 (0.2648)	
	GEL a=b=1	0.2540 (0.0030)	0.5086 (0.0109)	1.0104 (0.0427)	1.5120 (0.0948)	2.0040 (0.1652)	2.4909 (0.2590)	
	K=-1	0.2540 (0.0030)	0.5086 (0.0109)	1.0104 (0.0427)	1.5120 (0.0948)	2.0040 (0.1652)	2.4909 (0.2590)	
	GEL a=b=.5	0.2562 (0.0032)	0.5116 (0.0110)	1.0118 (0.0433)	1.5051 (0.0952)	1.9833 (0.1647)	2.4514 (0.2648)	
	GEL a=b=1	0.2540 (0.0030)	0.5086 (0.0109)	1.0104 (0.0427)	1.5120 (0.0948)	2.0040 (0.1652)	2.4909 (0.2590)	
	GPL a=b=.5	0.2588 (0.0033)	0.5174 (0.0120)	1.0291 (0.0462)	1.5400 (0.1025)	2.0348 (0.1754)	2.5361 (0.2710)	
	GPL a=b=1	0.2604 (0.0035)	0.5197 (0.0120)	1.0312 (0.0462)	1.5286 (0.1003)	2.0198 (0.1793)	2.5048 (0.2811)	
	100	GSL a=b=.5	0.2513 (0.0019)	0.5065 (0.0054)	1.0097 (0.0211)	1.5117 (0.0471)	2.0071 (0.0823)	2.5022 (0.1284)
		GSL a=b=1	0.2523 (0.0019)	0.5076 (0.0054)	1.0132 (0.0214)	1.5056 (0.0463)	1.9985 (0.0824)	2.4852 (0.1287)
K=1		0.2511 (0.0018)	0.5061 (0.0054)	1.0053 (0.0205)	1.5059 (0.0465)	2.0022 (0.0808)	2.4947 (0.1264)	
GEL a=b=.5		0.2511 (0.0019)	0.5055 (0.0053)	1.0069 (0.0210)	1.5022 (0.0463)	1.9936 (0.0821)	2.4743 (0.1282)	
GEL a=b=1		0.2511 (0.0019)	0.5055 (0.0053)	1.0069 (0.0210)	1.5022 (0.0463)	1.9936 (0.0821)	2.4743 (0.1282)	
K=-1		0.2505 (0.0018)	0.50499 (0.0053)	1.0050 (0.0206)	1.5051 (0.0457)	2.0002 (0.0812)	2.4933 (0.1263)	
GEL a=b=.5		0.2512 (0.0018)	0.5053 (0.0052)	1.0060 (0.0206)	1.5014 (0.0451)	1.9892 (0.0817)	2.4765 (0.1278)	
GEL $\alpha=1$		0.2512 (0.0018)	0.5053 (0.0052)	1.0060 (0.0206)	1.5014 (0.0451)	1.9892 (0.0817)	2.4765 (0.1278)	
GPL a=b=.5		0.2524 (0.0020)	0.5085 (0.0054)	1.0152 (0.0216)	1.5207 (0.0484)	2.0216 (0.0840)	2.5207 (0.1293)	
GPL a=b=1		0.2536 (0.0019)	0.5101 (0.0055)	1.0161 (0.0217)	1.5127 (0.0469)	2.0117 (0.0840)	2.5002 (0.1303)	

The squared error loss function and when  $n \leq 20$ , the estimates are also more stable than that of the entropy loss function. It is further observed that the

estimates under the entropy loss function are slightly better than the estimates under the squared error loss function. Hence, the Bayesian estimates under the

asymmetric loss function should be used for estimating the shape parameter of GRD when the scale parameter is known.

## CONCLUSION

In this paper, Bayesian estimation of the shape parameter of a generalized Rayleigh distribution is considered. A gamma prior on the shape parameter is assumed and the Bayes estimators under squared error, Entropy and Precautionary loss functions are provided. Comparison between the performances of the different Bayes estimators were carried out and it was observed that Bayes estimators under the asymmetric loss functions are better than the Bayes estimator under symmetric loss function.

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