
COMPARISON BETWEEN ALTITUDE AND HOUR ANGLE METHODS OF AZIMUTH DETERMINATION

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Abstract

Azimuth determination is a principal activity in the surveying profession that helps in creating accurate and retraceable reference. This is achieved through the observation of celestial objects mainly the sun and the stars. There are two methods used for azimuth determination, the altitude method and the hour angle method. However, some surveyors and technicians often choose any method at random, without necessarily weighing the advantages and disadvantages of each before going to the field. This paper assesses the advantages one method may have against the other, providing clear preference for field astronomers. Results show that the altitude method is preferred when it comes to accuracy. The hour angle method is faster, requires less training for proficiency, has fewer restrictions on time of the day and geographic location and has more versatility than the altitude method. Both methods can be used interchangeably in the tropics.

Keywords: Comparison, Altitude, Hour Angle, Azimuth determination

Introduction

Azimuth is an angle measured clockwise from a reference meridian (north-south) direction to a line. It is one of the rudimentary stages in the field of surveying which form the basis for orienting different survey activities. At some occasions, a surveyor must determine the direction of the true meridian (the astronomical north) in an area where no usefully located station monuments exists. In this case, he has to rely on the astronomical observations taken on one of the celestial bodies that include the sun and the stars. Therefore, understanding the various methods of determining azimuth through astronomical and trigonometric principles is essential. The applications of azimuth falls within the realms of all surveying activities that include land surveys, cadastral surveys and mapping of various kinds (Chandra, 2006). It has also being used to coordinate geodetic surveys and other large-scale surveys such as pipeline laying and coastal applications.

Even before the advent of electronic and sophisticated surveying instruments, azimuth has been conventionally obtained from the observation of different celestial bodies. The most popular celestial body being observed is the sun, and then the stars. However, the unique characteristic of the sun and Polaris (also called the northern star) for places in the Northern Hemisphere made them the more popular choices in astronomic observations (Till, 1990).

Solar observation, as compared with stellar observation provides the surveyor with a more convenient and economical method of azimuth determination. Little time is spent during solar observation, and when reasonable care is exercised and proper equipment is used, accuracy within 10 sec can be obtained (Till, 1990).

There are several types of azimuth. One of which is the Astronomic azimuth defined as the angle measured in the horizontal plane between the astronomic meridian and the vertical plane. This value is measurable, and it is what is measured when the celestial body is observed. This is achieved with the aid of an instrument capable of measuring with respect to the local plumb line such as the theodolite.

A key to understanding astronomic azimuth is the astronomic triangle. It is a part of the celestial sphere. The celestial sphere is a unit sphere with the earth at the centre, considered as a point. The stars that are so far away can be considered as points located at infinite radius. The astronomic triangle is a spherical triangle formed by the intersection of the three directions on a spherical surface centered on the earth centre. The three principal directions noted are the celestial body's centre, the direction from the centre of the earth through the observer's station and the direction from the centre of the earth through the North Pole. The most convenient, accurate and retraceable reference is the astronomical north. There are several methods of determining azimuth, but the one permitting both left and right face observations should be preferred (Duggal, 2004).

Generally, determination of azimuth of a line is very important and common operation by surveyors. Surveys usually commence from a previously established point which help in getting the initial bearing to be carried forward. However, in a situation where such established points are not available or their reliability is doubtful, azimuth of such point to another station is required. Another situation that warrants the need for azimuth observation is surveys in isolated areas where no reference stations exist. Hence, azimuth

observation has to be carried out to get the initial bearing of the survey.

Azimuth observations are also required when traverse network is too large. In this case, the surveyor needs to determine azimuth on at least one of traverse legs or every 25 stations, in order to check for accumulated directional errors. Similarly, in a situation where recent cadastral surveyed maps are sparse and there may be only one existing control in the vicinity, azimuth observation is needed.

This paper aims to compare the altitude and hour angle methods of azimuth determination by observational means focusing on accuracies obtainable and length of time of observation.

This would be achieved through the following objectives.

1. To carry out solar observations at varying times of the day, so as to detect whether positional variations of the sun has any effect on the two methods.
2. To compare the two methods using the observed data.
3. To ensure accuracy of the computed results, latitude and longitude of the astronomical station is derived through astronomical means.

Area of Study

The study area is within the main campus of Modibbo Adama University of Technology (MAUTECH) Yola Nigeria. The study area lies between latitude ($9^{\circ} 20' 31''$ - $9^{\circ} 22' 36''$) and longitude ($12^{\circ} 51' 53''$ - $12^{\circ} 51' 54''$). The azimuth station (AZ1) used is one of the three azimuth stations established within the main campus and is located behind the University's administrative buildings. The reference object is the Adamawa State Television (ATV) mast, which is visible from the observation station. The length of the azimuth line was estimated to be about 300m. This length of the line is long enough to minimize errors that might arise due to mis-centering of the theodolite, reference mark signal due to faulty bisection of reference mark and this length will obviate the need to change focus between sighting the reference mark and the sun as a source of error and delay.

Both terminals of the line have a clear view of morning and evening sun's optimal altitude range. On the other hand both terminals of the survey line were permanently indicated prior to azimuth observation. At times, it may be required that the azimuth line should form an integral part of the survey being controlled. The line could be one leg of the traverse or one side of the triangulation station. However, this is not always applicable as the case in this work.

Time is the key to an accurate determination of astronomical azimuth. The Coordinated Universal Time (UTC) is time scale available from broadcast time signals. It is a uniform time scale kept by atomic clock around the world, based on mean solar time corrected by leap seconds periodically.

The basic difference between these two methods is that, the altitude method requires an accurate vertical angle and the approximate time, whereas the hour angle requires a very accurate time but no vertical angle (Mackie, 1985). In the past, the altitude method was a better choice primarily because of the difficulty of obtaining accurate time in the field. The development of time signals and accurate timepieces particularly digital watches with split time features and time modules for calculation, has eliminated this obstacle to the extent that the hour angle method is now preferred.

Observations are made by pointing to the celestial body to determine the direction that it moves at the instance of pointing. In order to be of any practical use, the azimuth must be transferred to a line on the earth surface between the observation station and some other fixed station called the reference mark. The azimuth is transferred by measuring the horizontal angle between the line to the celestial body and the line to the reference mark. It is desirable that the reference mark be located at some distance from observation station (Punmia, 2005 and Chandra, 2006). For example, a minimum distance of 200m is designated to be used in accordance with Surveyors Council of Nigeria (SURCON) specification for large-scale, cadastral and engineering surveys (SURCON, 2004).

However, in order to record the results of observation for the azimuth, a diagram is required to show the relative positions of the reference mark, celestial body and the meridian plane. The diagram will help to remove any doubt and difficulty arising from the application of included angles correctly between the celestial body and the reference mark, that are used in finding the azimuth later.

Method

Observations to the sun and Reference Object (RO) are done to provide data relevant for the computations and comparison of the two methods of azimuth determination. Instruments used are i) Theodolite (Wild T2) and Tripod, ii) Thermometer, iii) Barometer, iv) Stopwatch and v) Reolof prism.

The data set used in this study is observed field data, which comes from solar observation, longitude/latitude observations and time. Both temporary and permanent adjustments were carried out on the theodolite before use. The two methods of azimuth determination are done as follows:

Azimuth by Altitude Method

In order to make use of the altitude of the sun's method, the horizontal circle readings to the reference object and simultaneous horizontal and vertical circle readings to the sun are taken (Oliver and Glendenning, 1978; Stephenson, 1995). The azimuth (A) of the sun is then computed from the altitude (h) deduced from the vertical circle readings. The horizontal angle from the RO to sun is deduced from the horizontal circle readings. The latitude of the azimuth station computed and the declination (d) of the sun, extracted from the star almanac (Nautical Almanac, 2009).

To compute the azimuth of the sun by the altitude of the sun's method, the following of equation is used

$$Z = \cos^{-1} \frac{\sin d - \sin q \sinh}{\cos q \cosh}$$

1.1

Where AZ=Z (when sun is east of the local meridian)

AZ = 360 – Z (when sun is west of local meridian)

d = Declination of sun

Q = Latitude of observer.

h = vertical angle of the sun corrected for parallax and refraction.

To apply the hour angle method, the horizontal angle from a survey line to the sun is noted. With the knowledge of accurate time of observation and position (latitude and longitude), the azimuth of the sun is computed. This azimuth is then combined with the horizontal angle to yield the azimuth of the line.

Azimuth by Hour Angle Method

To compute the azimuth of the sun by the hour angle method, the following equation is used.

$$Z = \tan^{-1} \frac{\sin LHA}{\sin f \cos LHA - \cos f \tan d}$$

1.2

Where

Z = azimuth of the sun measured clockwise from North

LHA = local hour angle of the sun

d = declination of the sun

f = latitude of the observer

In order to calculate the LHA of the apparent sun at the instance of observation, accurate time that takes in to account the rotation of the earth was noted. The time obtained by adding the correction factor to the Greenwich Mean Time.

Details of method used are shown in appendices A to C.

Results and Discussions

Results for the altitude and hour angle azimuths are shown in table 1. These were obtained by averaging azimuths for West Sun and East Sun for eight (8) observations for each Altitude and Hour Angle Methods. Residuals were calculated based on the respective mean values for each method and variances are also shown. The residuals are plotted (figure 1) to show co-variability between the two methods. Both methods show higher residuals of the order of ±15" on an arc. The altitude method shows high residual for the second observation with about 18" of an arc.

Table 1: Residuals and Variances for Altitude and Hour Angle Azimuth

S/N	Altitude Azimuth			Hour Angle Azimuth		
	Azimuth	Residual $v = \bar{x} - x_i$	v^2	Azimuth	Residual $v = \bar{x} - x_i$	v^2
1	213° 42' 45"	-05	25	213° 42' 42"	-10	100
2	213 42 46	-06	36	213 42 44	- 18	324
3	213 42 37	03	09	213 42 27	05	25
4	213 42 24	16	256	213 42 31	01	01
5	213 42 29	11	121	213 42 31	01	01
6	213 42 42	-02	04	213 42 17	15	225
7	213 42 43	- 03	09	213 42 26	06	36
8	213 42 55	-15	225	213 42 35	- 03	09
Mean	213 42 40			213 42 55		
Sum		01	685 (Variance)		-03	721 (Variance)

We can determine variables that are most logically related to each other and to determine the type of data that will have to be collected in order to

construct their models (Cooper, 2010). Therefore, the observed data are tested to verify their potency and also compare the two methods.

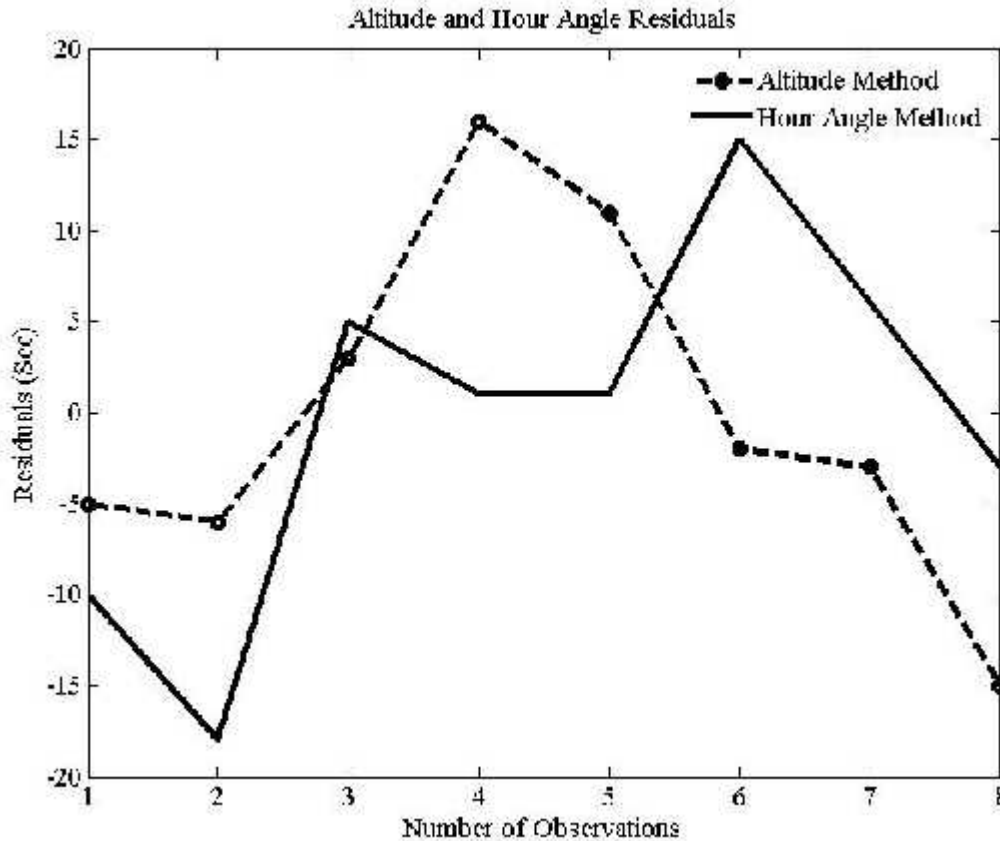


Figure 1: Residual plot of Altitude and Hour Angle azimuths.

Given sample of n observation, then sample average is calculated as,

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \quad 1.3$$

Where x_i represent the individual observation and

\bar{x} is the sample average showing the central tendency of the underlying random variable. The residuals are therefore, calculated as

$v = \bar{x} - x_i$, while the variance is calculated from the residuals following:

$$S^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \quad 1.4$$

Comparative Analysis

The primary difference between the hour angle and the altitude method is that the hour angle method requires extraordinary care in the determination of time while the altitude method requires extra care and skill in determination of the altitude.

The hour angle method is critical for time. It is possible to perform the hour angle observation with

the advent of accurate digital clocks. The primary difficulty of the altitude method is the difficulty in obtaining simultaneous vertical and horizontal sun pointing (Russell and Brinker, 1995). Another relevant factor in the final altitude used is the proper determination of parallax and refraction correction. The ability to point on the vertical limbs of the sun is not quite easy.

The observed vertical angle to the sun must be corrected for atmospheric refraction as well as parallax. The parallax correction is a small offset and may be easily and accurately computed. Determination of refraction is somewhat more uncertain. Refraction correction represent the amount of bending in the vertical light from the sun enters and passes through the varying density of air in the earth's atmosphere. This bending results in the sun's image appearing at different and higher vertical angle than its true direction. This bending increase as the sun appears nearer to the horizon, since light rays are passing through more atmospheres and at a more oblique angle.

Reference on geodetic astronomy indicates that the formulae and tables used to determine refraction connection may be accurate around 3" for vertical

angles of 15 degrees or greater. The accuracy estimates given in this context should be considered with some skeptics, hence geodetic astronomy is primarily performed at night when atmospheric conditions affecting refraction would normally be more stable (Adeyemi, 1983; Chandra, 2006).

The refraction models used to derive most formulae assumed an ideal atmosphere, with density varying as some function of elevation to fit the actual refraction data. Then, an important phase of statistical investigation is the construction of conceptual or mathematical model to be used.

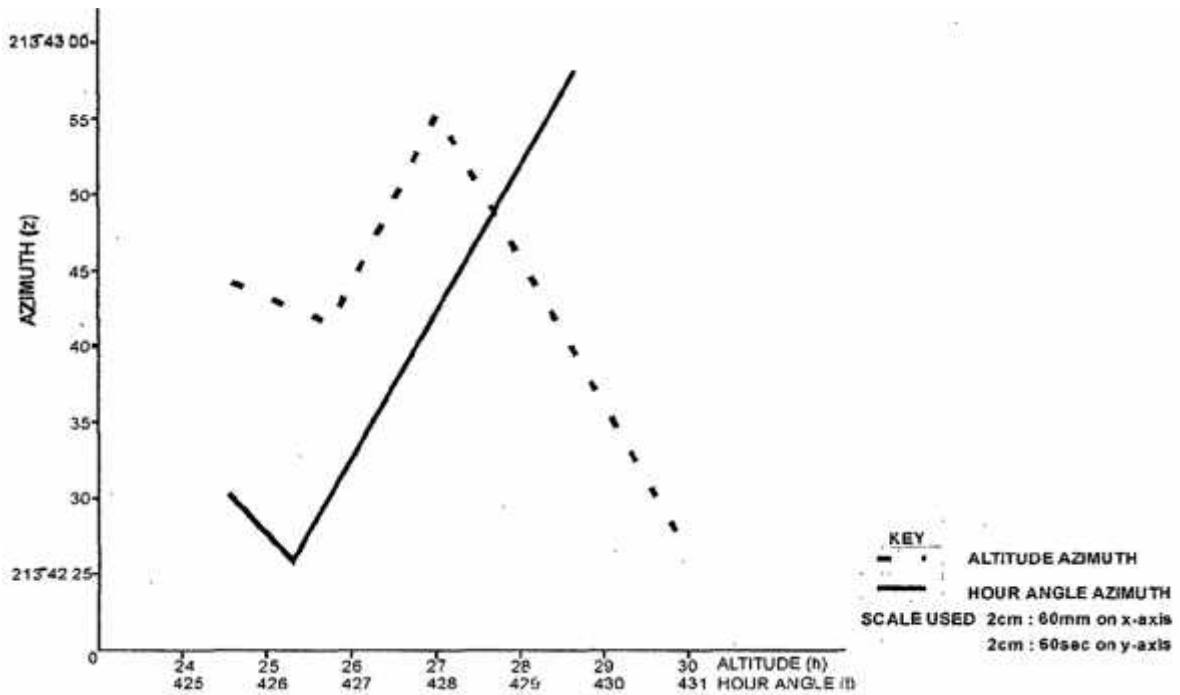


Figure 2: A graph of hour angle (t) and, altitude (h) against azimuth (z)

Two graphs are plotted (figure 2 & 3) for the two methods of azimuth with the view of discovering of similarities and dis-similarities between the two methods. There was no significant relationship between the two methods as shown by the shape of the graphs. Rather, both methods are independent

of each other. We hypothesized that both methods are the same. After testing the hypothesis using t-test distribution at 5% confidence level for the null hypotheses H_0 , we could not rejected the null hypothesis, as it shows that both methods are of equal potency.

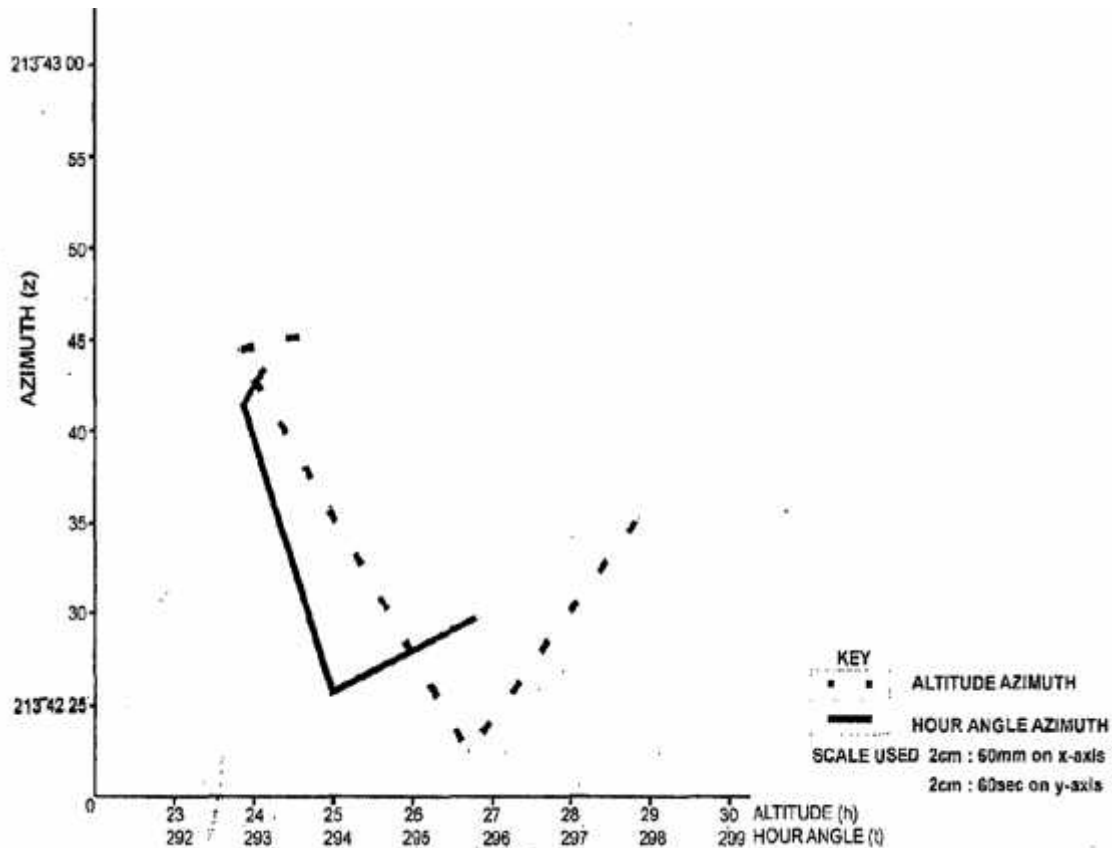


Figure 3: A Graph of hour angle (t), altitude, against azimuth (z) sun west

After a thorough investigation and analysis of the data derived from both methods of azimuth determination, the following conclusions were drawn. The altitude method provides a strong check to a good hour angle observation due to its lack of criticality for time and longitude. It was much less practical method prior to the ready availability of accurate electronic time piece since 1978.

In the altitude method, the average of the adjusted data would not have any measurable effects on the final azimuth. Whereas in the hour angle the average of the adjusted data will give errors due to the fact that the sun travels in an apparent curved path and the semi-diameter correction would be affected by the change in altitude.

In the hour angle method, precise time is being noted when the considered celestial body is being bisected. The observation time is used to derive the hour and declination of the celestial body at the instant of observation. In terms of time spent on the field during observations, the hour angle method takes shorter time to accomplish than the altitude method; hence concentration is only made on one limb of the sun and not both as in the case of the altitude method.

While the sum of the variance achieved from altitude method was 685 and that of hour angle method was 721, one could deduce that, the altitude

method was more accurate and reliable than the hour angle method.

Similarly, from the analysis of the estimate of the standard error of the difference between means was ± 1.74 . Equally one may assume that there is no much significant difference between the two methods both are equally reliable and could be adopted here within the tropics.

While undertaking the field procedure, much time was spent on altitude's method in order to achieve the simultaneous tangencies of the sun's limb than in the hour angle method. Besides, refraction corrections have to be applied to the altitude of sun's method of azimuth determination. No vertical angle is measured in the case of the hour angle.

Conclusion and Recommendations

One may conclude after analysis of the two methods, that the difference is insignificant. Although, there are other factors which may likely influence the selection of the most suitable method, both methods are reliable and may be interchangeably used. However, on the basis of time for observational procedure, the method of hour angle is far a better option than the altitude's method, because it does not require the difficult procedure of simultaneous tangencies of both horizontal and vertical hairs, which is the chief

source of error of the method. The hour angle method could be a preference to this respect. The altitude method however is preferred when it comes to accuracy as it provides least sum of

variances as seen in table 1. Both methods can work well in the tropics but one should weigh the difficulty in attaining higher accuracy for each method before making a choice.

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Appendix A

DATA SHEET AND COMPUTATIONS

Sun Observation on 19 August 2012

Observer: Mohammed Chubado

Booker: Ibrahim Muhammed

Timekeeper: Ezra Enoch Alhamdu

Morning Observation (East Sun)

Altitude Method

Measured vertical angle: $34^{\circ} 09' 07''$

Corrected (h) = $34^{\circ} 06' 52.64''$

LMT = $08^{\text{h}} 24^{\text{m}} 45^{\text{s}} - 1\text{h}$

UT = $07^{\text{h}} 24^{\text{m}} 45^{\text{s}}$

Declination of UT 6h = $12^{\circ} 53.1'$

Declination at $1^{\text{h}} 24^{\text{m}} 45^{\text{s}}$ = $0^{\circ} 1' 9.21''$

Declination at $07^{\text{h}} 24^{\text{m}} 45^{\text{s}}$ = $12^{\circ} 54' 15.21''$

Latitude of FUTY (θ) = $09^{\circ} 20' 35.5''$

Using $\cos Z = \frac{\sin \delta - \sin \theta \sin h}{\cos \theta \cos h}$

$$= \frac{\sin 12^{\circ} 54' 15.21'' - \sin 09^{\circ} 20' 35.5'' \sin 34^{\circ} 06' 52.64''}{\cos 09^{\circ} 20' 35.5'' \cos 34^{\circ} 06' 52.64''}$$

$$= 0.16190936$$

$$= \cos^{-1} 0.16190936$$

$$Z = 80^{\circ} 40' 56.14''$$

$$\text{H.A.} = 227^{\circ} 21' 14''$$

$$\text{Bearing of the line} = 360^{\circ} - (\text{H. A.} + Z)$$

$$= 360^{\circ} - (227^{\circ} 21' 16'' + 80^{\circ} 40' 56.14'')$$

$$= 213^{\circ} 42' 35''$$

Appendix B

DATA SHEET AND COMPUTATIONS

Sun Observation on 19 August 2012

Observer: *Mohammed Chubado*

Booker: *Ezra Enoch Alhamdu*

Timekeeper: *Ibrahim Muhammed*

Morning Observation (*East Sun*)

Hour Angle Method

$$t = Ut + E + \lambda$$

$$L.M.T. = 8^h 24^m 40^s$$

$$UT = 07^h 24^m 40^s \times 15^\circ = 111^\circ 10' 0''$$

$$E \text{ at } Ut = 11^h 56^m 11.0^s$$

$$\begin{aligned} E \text{ at } Ut 1^h 24^m 40^s &= 11^h 56^m 58.98^s \times 15^\circ \\ &= 179^\circ 14' 44.6'' \\ &= 12^\circ 30' 07.1'' \end{aligned}$$

$$\theta = 09^\circ 20' 35.5''$$

$$\delta = 12^\circ 54' 15.14''$$

$$t = Ut + E + \lambda$$

$$= 111^\circ 10' + 179^\circ 14' 44.6'' + 12^\circ 30' 07.1''$$

$$= 302^\circ 54' 51.7''$$

$$= 360^\circ - 302^\circ 54' 51.7''$$

$$= 57^\circ 05' 8.3''$$

$$\text{Tan } Z = \text{tant} \cdot \cos B \cdot \text{cosec}(B - \theta),$$

where,

$$B = \tan^{-1}(\tan \delta \sec t)$$

$$= \tan^{-1}(\tan 12^\circ 54' 15.14'' \times 1/\cos 57^\circ 05' 8.3'')$$

$$= 22^\circ 51' 42.44''$$

$$\tan Z = \tan 57^\circ 05' 8.3'' \times \cos 22^\circ 51' 42.44'' \times 1/\sin 13^\circ 31' 6.94''$$

$$z = \tan^{-1}(6.089786794)$$

$$= 80.67472014$$

$$= 80^\circ 40' 28.99''$$

$$\text{Bearing of the line} = 360 - (H. A. + Z)$$

$$= 360^\circ - 227^\circ 21' 16.8'' + 80^\circ 40' 28.99''$$

$$= 213^\circ 42' 16.3''$$

Appendix C

Morning Observation (East Sun)

S/N	Horizontal Angle	Vertical Angle	Azimuth	Mean LMT
1	227°21'16''	34°09'07''	213° 42' 40.1''	08 ^h 25 ^m 45 ^s
2	227°21'30''	34°09'03''	213° 42' 46''	08 ^h 24 ^m 22 ^s
3	227°21'54''	34°10'07''	213° 42' 43''	08 ^h 25 ^m 20.8 ^s
4	227°21'45''	34°10'42''	213° 42' 12.7''	08 ^h 25 ^m 4.7 ^s
5	227°21' 49''	34°10'07''	213° 42' 26''	08 ^h 25 ^m 16.4 ^s
6	227°21'18.3''	34°10'30''	213° 42' 34''	08 ^h 25 ^m 35.2 ^s
7	227°21'17.6''	34°09'07''	213° 42' 14''	08 ^h 25 ^m 55 ^s
8	227°21'47''	34°09'57''	213° 42' 42.2''	08 ^h 25 ^m 15 ^s

Figure A: Sample of observation table for East Sun (Morning) and West Sun (evening) as applied for both Altitude and Hour Angle methods.