

Control Policy and Sustainability for Decision-Making in Asset Management

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ABSTRACT

The essential factor determining decision-making in private investors and public corporations is controlled asset management. In a developed asset allocation and in a time of debt crisis, an important issue on which investors base their decisions is when and how to more or fewer investments such that expected net values of assets are maximized with minimal risks for sustainability in the management of capital assets. The acknowledgement of this enables investors to make rational decisions. The aim of this paper is to provide control policy to help investors make necessary decisions for sustainability in asset management. This is done by applying an optimal control model to a real-life problem asset-liability, thus derive the control policy.

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INTRODUCTION

Asset management is the systematic process of making. definition of the optimal control models of stochastic, deploying, operating, upgrading and disposing of assets c

Optimal, risk-based decision-making is an indispensable component supporting effective asset management. This includes finding the correct trade-off between competing interests, such as asset performance/ utilization versus asset maintenance, capital investment cost versus working expenditures, or short-term benefits versus long-term manageability. In this work, the concept of portfolio optimization is utilized where the expected value of utility function considered.

Accordingly, of more noteworthy utilization of methods and results in science, optimal control theory has accomplished excellent advancements and had been connected to many fields like engineering, medicine, economics and finance in creating models. Examples include optimal control for analysis of human postural balance; electric power generating system and the dynamic portfolio management (Arthur, 1995; Bamigbola & Aderinto, 2009; Ralf, 1996).

Merton concentrated on stochastic optimal control for finance which made the study of optimal control for finance attracts numerous research works as in the connection of the investigations of optimal control in finance; the presentation of the Stochastic Linear Quadratic control as a general system to analyse the continuous-time mean-variance and hedging problem; the communication of the utilization of dynamic programming in optimization over Ito's procedure; concentration stochastic optimal on stopping problem in risk theory, introduction of the idea of optimal fuzzy control which is based on the fuzzy set theory,

proposition of solution to debt crisis model by utilizing stochastic optimal control. (Cairns, 2000; Dixit & Pindyck, 1994; Jensen, 1998; Karatzas, 1989; Komolov *et al.*, 1979; Merton, 1971; Stein, 2003; Zhou & Li, 2000)

In any case, the occasions confronted in life are made uncertain in a few routes because of the multifaceted nature on the planet. A review case is that of a business which must have enough finances to balance its financial obligations at a particular point in time to ensure continuity of the business operations.

Meanwhile, since Merton developed a lifetime portfolio selection under uncertainty where the case of continuous-time for risk and risk-free assets, the portfolio theories have been adapted into capital asset management which has also been used interchangeably in some cases. Although some researchers argued about portfolio management is a subset of asset management. In either way, it is feasible to apply the underlying assumptions of both processes to decision making in capital asset management.

Keeping in mind the aim to oversee such a case, an optimal control approach is connected to the case to show what ratio of investor's business capital assets can be utilized for current financial commitments. Understanding this control tool would help investors keep their organizations from confronting serious issues that may be brought about by surprising occasions and sudden changes in the business condition.

MATERIALS AND METHODS

Asset Management model

In asset management, one of the most serious problems investors are faced with is the decision on how to invest and the understanding of his net worth, τ , must he/she choose to be incurred on liability of the business such that the expected present value of the utility of asset, $J(\psi)$, is optimized?

In considering the issue involved generally in risky capital assets, the following problem statement is considered.
 An individual invests his/her wealth in capital assets, $A(t)$, of business for a time, t , ranging from t_0 to t_f . Suppose he/she starts with a known initial net worth $X_0(t)$. At time t , what fraction of his/her net worth, ψ , must he/she

In addressing the issues raised above, some parameters used in the model formulation are presented and described in Tables 1 and 2.

Table 1: Definition of Parameters of the Objective function to the model

Parameter	Description
U	Utility function
η	subjective discount rate, e.g., $\frac{A}{\eta+1}$ = Presentvalue
λ	degree of relative risk, where $(1 - \lambda)$ is the risk aversion
ψ	Capital asset ratio (control) $\psi \in \mathfrak{R}$
$X(t)$	Net worth at a time t (state variable)

Table 2: Definition of Parameters of the constraint to the model

Parameter	Description
$X(t)$	Net worth at a time t (state variable)
$\tau(t)$	Liability ratio (control) at a time t , $\tau \in \mathfrak{R}$
$\sigma_\tau(t)$	Diffusion volatility of liability (with variance σ_τ^2 per unit time)
ψ	Capital asset ratio (control) $\psi \in \mathfrak{R}$
$\sigma_b(t)$	Diffusion volatility of asset (with variance σ_b^2 per unit time)
$\kappa(t)$	Capital gain on asset due to inflation at a time t
$\sigma_p(t)$	Diffusion volatility on asset price (with variance σ_p^2 per unit time)
$\beta(t)$	Mean rate of return on asset
ω	Mean interest rate of liability
$C(t)$	Liu canonical process at a time t
$\mu(t)$	Consumption level at a time t
$j(t)$	Tax ratio at a time t
$g(t)$	Depreciation ratio at a time t
$h(t)$	Asset supplies ratio at a time t

Measurements

The measurements considered in obtaining data in Tables 1 and 2 are described below. The values of parameters in the tables are measured in monetary units (e.g US Dollars) except the following parameters: $C(t)$ - measures the uncertainty process which exists in the interval $0 < C(t) < 1$; λ - is used to measure risk which exists in the interval $0 < \lambda < 1$; and η - measures discount rate which exists in the interval $0 < \eta < 1$. The Debt of a country stands as a proxy to liability in this case while the exchange rate at 31st December of each year is used.

An optimal control model of asset management based on the preliminary of Uncertainty theory was formulated as

$$J(\psi) = \text{opt}_\psi E_C \left[\int_{t_0}^{t_f} \frac{1}{\lambda} e^{-\eta t} (\psi X(t))^\lambda dt \right]$$

subject to

$$X(t) = [(\kappa + \beta)\psi - (\omega(\psi - 1) + \mu + h - j - g)]X(t)dt$$

$$+ [\psi\sigma_p + \psi\sigma_b - \sigma_\tau(\psi - 1)]X(t)d(t) \tag{2}$$

(Liu, 2007; Latunde & Bamigbola 2016; 2018)

Application and Solution of the Model to Debt Crisis

Debt simply means a sum of money that is owed or due. In international finance, debt is categorized as an example of liability. Debt is as old as economy. It was the principal type of trade (barter system) recorded in mankind's history and existed around 2,900 years before the development of coinage. Today there are numerous cases of moneylenders of fiscal debt that incorporate countries, banks, credit card companies, organizations, individuals, and so forth, who in numerous cases subject their borrowers to legally binding terms that assign the sum and timing of reimbursements of the obligation and that habitually incorporate the installment of principal and interest. National debt (also known as

government debt, public debt and sovereign debt) refers to the total amount owed by the federal government of the country. This may also refer to the sum total of external/ foreign and internal/ domestic (state and local government) debts. Debt crisis is the general term used to describe a massive public debt relative to tax revenues. This simply means the combination of accumulated debt stock and problems in servicing debt.

However, public debt which is either denominated in internal or external currency must be repaid at maturity in the case of short-term debt or serviced regularly in the case of long-term debt. To accomplish that and prevent recurrence of debt crisis, an uncertainty optimal control approach is presented such that there exists some sources of uncertainty when dealing with debt: return on capital, price of capital and interest rate of liability (debt) such that they are positively or negatively correlated.

Nigeria Debt Crisis

Nigeria's debt crisis is selected as a case study to the model of risky capital asset because it could stand as structure could stand as a study case for both the perspective of an individual and government as an investor of capital assets.

The early 1980s, the difficulty in debt servicing started in Nigeria when the foreign exchange earnings rose as a result of the collapse of prices in the international oil market and external obligations began to be procured indiscriminately. Since then, the GDP growth rate, Gross Fixed Capital Formation (GFCF) rate or investment rate and export earnings has been affected thus resulting in a drastic rise of interest rates $r(t)$ leading to difficulties in servicing debt, thus, resulting to a declining economic growth and development of the nation.

GDP is the monetary value of all the finished goods and services produced within a country's borders in a specific period meanwhile the GFCF (formerly gross domestic fixed investment) which includes land improvements (fences, ditches, drains, and so on); plant, machinery, and equipment purchases; and the construction of roads, railways, and the like, including schools, offices, hospitals, private residential dwellings, and commercial and industrial buildings of the nation.

Despite the Paris Club debt forgiveness in October 2005 where Nigeria and the Paris Club announced a final agreement for debt relief worth 18 billion dollars and an overall reduction of Nigeria's debt stock by 30 billion dollars. The arrangement was concluded on April 21, 2006, when Nigeria made its last installment, and its books were cleared of any Paris Club debt. It was expected to solve the nation's debt crisis and accelerate the growth and development of the economy. Rather, it didn't put an end to the nation's debt crisis. Thus, it is necessary to be aware, predict and control the period of the debt crisis as a signal warning.

Utilizing our model in international finance, we take the revenue to be the Gross Domestic Product of a nation (GDP) or value added the risky capital asset as the capital investment or the Gross Fixed Capital Formation, Consumption as Household and Government Consumption, Depreciation as the Consumption of Fixed capital. We also take debt as a study case of liability to be considered.

However, it is necessary for investors to be able to predict and control when to make more or fewer capital investments to provide a sustainable debt balance. Thus, model is analysed using the available real-life data in order to provide control policy to solving the problem of the study case and also to showing satisfaction of optimality criteria.

Table 3: Nigeria Net worth Profile

Year	GDP	Debt	GFCF	Net worth	Consumption	Indirect Tax	Depreciation	Supplies
2007	166.451	22.330	15.396	-6.934	149.152	2.553	3.738	5.089
2008	208.065	21.399	17.318	-4.081	161.035	3.436	3.853	30.988
2009	169.481	25.817	20.487	-5.330	147.601	3.180	2.952	0.445
2010	369.062	40.100	61.099	21.860	293.507	5.623	16.079	28.662
2011	411.744	47.898	63.960	16.062	323.540	4.516	18.815	38.719
2012	460.953	48.496	65.283	16.787	348.597	5.686	24.260	86.210
2013	514.966	64.510	72.964	8.454	453.699	7.929	23.857	26.280
2014	568.499	67.726	85.737	18.011	464.696	6.857	25.272	32.499
2015	481.066	65.429	71.329	5.900	417.560	5.362	23.097	0.000
2016	405.083	57.392	73.261	15.869	206.414	3.188	10.332	-3.341

Source:

Columns 1 and 3. The world bank (<http://data.worldbank.org/indicator>)

Column 2. Debt Management Office of Nigeria (<https://www.dmo.gov.ng/>).

Columns 5, 6, 7, 8 and 9. National Bureau of Statistics (<http://nigerianstat.gov.ng/>)

Table 4: Parameters for the Nigeria Net worth Profile

Year	κ	β	ω	σ_p	σ_b	σ_r	σ	μ	h	j	g
2007	-7.25	-51.72	-6.10	-3.78	-21.47	-2.46	-22.79	-21.51	-0.73	-0.37	-0.54
2008	-12.32	-87.88	-10.36	-6.42	-36.48	-4.17	-38.73	-39.46	-7.59	-0.77	-0.94
2009	-9.43	-67.29	-7.93	-4.92	-27.93	-3.20	-29.65	-27.69	0.08	-0.60	-0.55
2010	2.30	16.41	1.93	1.20	6.81	0.78	7.23	13.43	1.31	0.26	0.74
20122	3.13	22.33	2.63	1.63	9.27	1.06	9.84	20.14	2.14	0.28	1.17
2012	2.30	21.37	2.52	1.56	8.87	1.02	9.41	20.77	5.14	0.34	1.45
2013	5.95	42.42	5.00	3.10	17.61	2.02	18.69	53.67	3.11	0.94	2.82
2014	2.79	19.91	1.43	1.46	8.27	0.95	8.78	25.80	1.92	0.37	1.40
2015	8.52	60.79	4.37	4.44	25.23	2.89	26.78	70.77	0.00	0.91	3.91
2016	3.17	22.60	1.63	1.65	9.38	1.07	9.96	13.01	-0.21	0.20	0.65

Algorithm

The following algorithm is formulated based on the Runge-Kutta method for solving uncertain differential equations, (Ya

Step 1. Given time interval $t, [a, b]$, iteration number N , step length $h = \frac{b-a}{N}$. Set $t_i = a + ih, i = 0, 1, \dots, N$ and $\alpha > 0$.

Step 2. Compute the corresponding differential equation

$$d(t)^\alpha = [(\kappa + \beta)\psi - (\omega(\psi - 1) + \mu + \eta)]X(t)^\alpha + [\psi\sigma_p + \psi\sigma_b - \sigma_r(\psi - 1)]X(t)^\alpha \ln \frac{d}{dt}$$

$X_0^\alpha = X_0$, with the Runge-Kutta method for solving uncertain differential equations.

Step 3. Set $i = i + 1$, repeat Step 2 and step 3 for N times, then $X(t)^\alpha$ is derived. Go back to step 1 until $t_i = b$,

$X(t)^\alpha$ for every α -path can be derived using the Algorithm. A computer program written in Maple 18 is used to run the fourth-order Runge-Kutta method for the uncertain differential equation on the model.

By using the available data in Tables 3 and 4 above, the numerical results derived for the asset management problem was presented in Table 5 and discussed using Figures 1 - 4.

All the values of parameters are measured in Billion US Dollars except the following parameters: $C(t)$ - measures the uncertainty process which exists in the interval $0 < C(t) < 1$; λ - is used to measure risk which exists in the interval $0 \leq \lambda < 1$; and η - measures discount rate which exists in the interval $0 < \eta < 1$. The debt is calculated as, the total debt of the nation by summing the external debt stock (federal government and state) and Domestic debt (federal government and state) together. The net worth is also calculated by deducting the debt from the Gross Fixed Capital Formation (GFCF). The Central Bank of Nigeria (CBN) Official Exchange rate of U at 31st December of each year is used while current market prices from the national account are used in the computations.

Furthermore, using the available data of net worth for the year 2007 to 2016, the behaviour of the optimal control in each year is studied.

Table 5: Results of Solution to Nigeria Debt Crisis with $h = 0.05, \psi > 0, a = 0, b = 1$.

Year	ψ_0	X_0	X_e	X_a	Rel. Error(X)	J_e	J_a	Rel. Error of J
2007	-4.09	-6.934	-1.104×10^9	-1.083×10^9	1.90%	60.899	60.782	0.19%
2008	-5.88	-4.081	-2.718×10^1	-2.100×10^1	22.7%	275.097	268.091	2.55%
2009	-5.09	-5.330	-4.197×10^1	-2.388×10^1	43.20%	141.935	134.156	5.48%
2010	0.67	21.860	23.860	23.823	0.16%	8.700	8.698	0.03%
2011	1.42	16.062	33.098	33.851	2.28%	9.690	9.656	3.23%
2012	1.22	16.787	23.728	23.256	1.99%	9.232	9.213	0.21%
2013	5.59	8.454	9.011×10^0	8.219×10^0	8.79%	38.846	38.490	0.92%
2014	2.01	18.011	43.282	42.956	0.76%	10.306	10.298	0.02%
2015	7.88	5.900	2.167×10^1	2.087×10^1	3.83%	276.930	275.881	0.38%

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Year	ψ_0	X_0	X_e	X_a	Rel. Error(X)	J_e	J_a	Rel. Error of J
2016	0.74	15.869	23.801	23.914	0.47%	8.784	8.789	0.06%

Note that the optimal control ψ_0 for each year was derived analytically using the equation of optimality for uncertain optimal control problem, (Zhu, 2010) where

$$\psi_0 = \frac{(\mu + j + g + \omega - h)\lambda - \eta}{(1 - \lambda)(\kappa + \beta - \omega)}$$

and the actual control is

$$\psi_A = \frac{A(t)}{X(t)}$$

RESULTS AND DISCUSSION

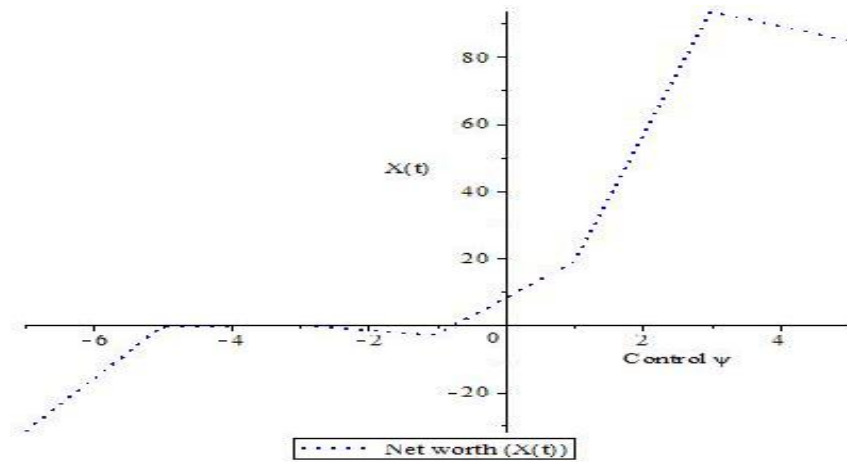


Figure 1: Graph of Net worth $X(t)$ versus control ψ

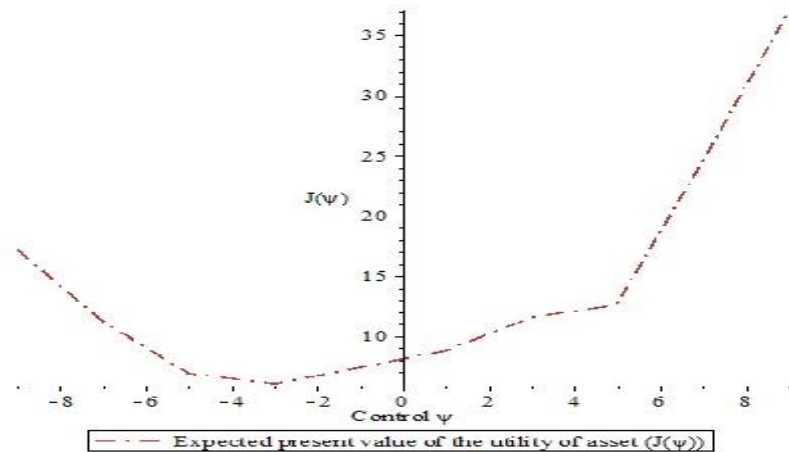


Figure 2: Graph of the Expected present value of the utility of asset $J(\psi)$ versus the control ψ

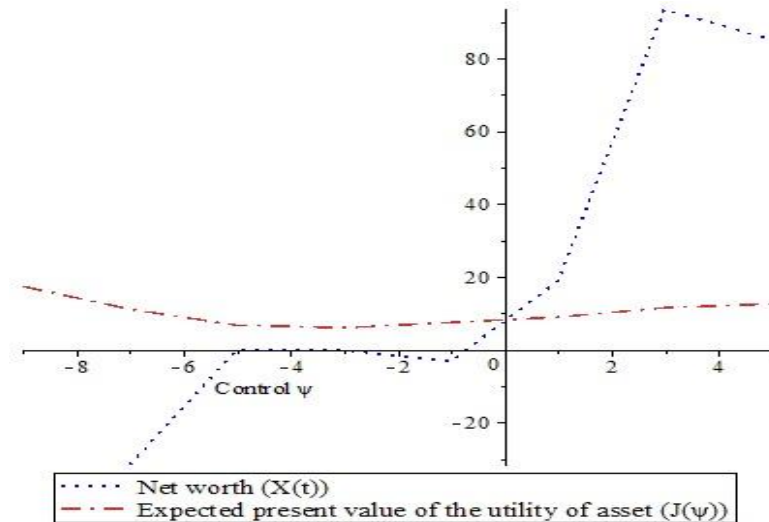


Figure 3: Graph of Net worth X and the Expected present value of the utility of asset $J(\psi)$ versus control ψ

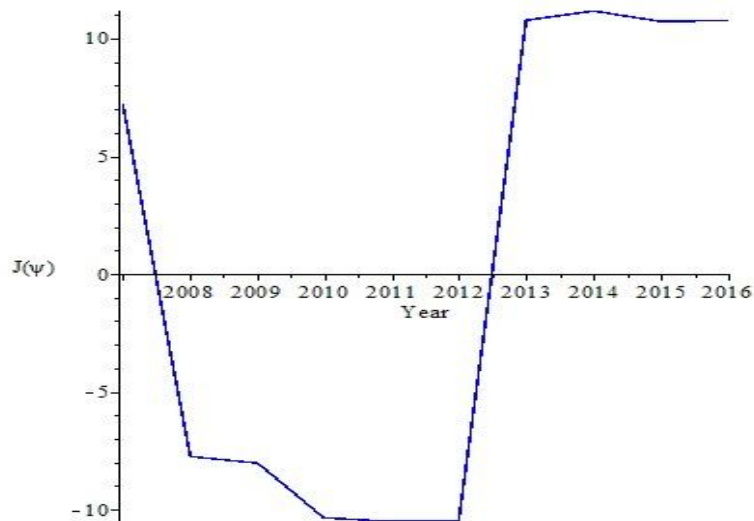


Figure 4: Graph of the Expected present value of the utility of asset $J(\psi)$ with year

As a measure of precision, for the case examined, the relative error of the results $A(t)$ and $X(t)$ is examined. From Figure 1, it is observed generally that when the control variable ψ is negative, there is a negative net worth $X(t)$ and vice versa. Meanwhile, a negative net worth indicates that the liability (e.g debt) is more than the total assets.

Also from Figure 2, it is observed generally that when the control variable ψ is negative, there is a negative expected value of the utility of asset $J(\psi)$ and vice versa which implies that the more there is reduction in the investment of capital assets $A(t)$ with negative net worth $X(t)$, the more there is a reduction in the expected present value of the utility of asset $J(\psi)$ which implies that the more there is investment in capital assets with negative net worth $X(t)$, the more the expected present value of the utility of asset $J(\psi)$. Likewise, the more there is

net worth $X(t)$, the more there is increased in the expected present value of the utility of asset $J(\psi)$ which implies that reduction in the investments made on capital assets with positive net worth $X(t)$ would lead to a reduction in the expected present value of the utility of asset $J(\psi)$.

Figure 3 shows the difference in the net worth $X(t)$ and the expected value of the utility of asset $J(\psi)$.

Figure 4 shows the expected present value of the utility of asset $J(\psi)$ with year after the problem is controlled.

It is also observed that optimal asset ratio, ψ_0 is less than the actual asset ratio, ψ_A , thus, making it obvious that the optimal control model for management of capital assets deals with the asset-

liability problem in the Nigeria economy by deriving the optimal controls which are lower than the actual controls whereas leading to greater solvency and availability of more funds to meeting the current obligations. However, the difference between the two forms of controls serves as the boundary in which investors are advised to utilize, thus representing the performance profile of the capital asset.

Finally, investors are advised to invest more on capital assets no matter the nature of the business' net worth (negative or positive), for the expected present value of the utility of asset to be maximized. This can be used as a control policy in overcoming the problem of asset management to aid decision-making thus enhancing the sustainability of assets.

CONCLUSION

The requirement of making good asset management decisions to Benignizing Sustainability in the presence of liability is that Asset management as a real-life endeavour needs to be studied as a multidisciplinary subject making use of contributions from the relevant fields. In this work, the notion of mathematical modelling has been utilized to study a problem of asset management with a view to proposing a sustainable solution by deriving control policy to evaluate and monitor the asset management problem. The application of optimal control concept to the mathematical model provides a better understanding of the problem.

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