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Adomian Decomposition Method and Variational Iteration Method for Solving Fourth Order Differential Equation with Initial Conditions

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ABSTRACT
This paper discusses the numerical solutions of fourth order differential equation with initial value problem. The numerical methods consider to generate the sequence of improving approximate solution of the problems considered in both Adomian decomposition method (ADM) and Variational Iteration Method (VIM). The methods are applied on three differential examples of fourth order ordinary differential equation whose exact solutions are known. It is established that the VIM gives better solution as its series solution converges faster when compared to ADM.

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INTRODUCTION

Physics is not the only science in which differential equations play a prominent role. There are many areas where differential equations are used as a model for the problem at hand. To name a few examples: the reaction and diffusion of chemicals, the dynamics of populations in biology, the development and treatment of diseases in medicine, or the flow of a fluid or gas, which has applications ranging from fundamental astronomy to meteorology to industrial engineering. In this paper, the examples we considered are classical fourth order differential equations in which they have applications in Beam-Column theory which is a useful tool for modelling and studying naturally occurring phenomena. Such as determining when a uniform cross section beam may break, as well as predicting future outcomes. Researchers has considered the two methods ADM and VIM separately to solve many problems and compare them on some certain problems. (Agom et al., 2015), (Agom et al., 2016) did numerical solution of Fourth Order Linear Differential Equations by Adomian decomposition method. (Abdul-Majid and Wazwaz, 2007) did comparative study between the variational iteration method and Adomian decomposition method by investigating the homogeneous and the nonhomogeneous advection problems. (Hamood Ahmed et. al., 2014) researched a comparison between Adomian decomposition method and variational iteration method for solving delay differential equation with Initial Condition.

In this paper we outline a reliable comparison between two powerful methods. The first is the variational iteration method (VIM). The second is Adomian decomposition method (ADM). The two methods give rapidly convergent series with specific significant features for each scheme. The main advantage of the two methods is that it can be applied directly for all types of differential and integral equations, homogeneous or inhomogeneous. Another important advantage is that the methods are capable of greatly reducing the size of computational work while still maintaining high accuracy of the numerical solution. The effectiveness and the usefulness of both methods are demonstrated by finding exact solutions to the models that will be investigated. The application of the two methods can be found in (Peter, et. al., 2018), (Peter, et. al., 2019), (Peter, et. al., 2020), (Peter, O. J., Yusuf., A. et. al., 2021), (Peter, et. al., 2020), (Adebisi, et. al., 2018), (Adebisi, et. al.,2019), (Adebisi, et. al., 2021), other applications of numerical methods

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can be found in (Peter et. al., 2021), (Ishola et. al., 2022), (Uwaheren, 2021), (Abioye, et al., 2018), (Abioye, et al., 2020), (Ayoade et. al., 2020). (Oyedepo, et al., 2018) and (Oyedepo, et al., 2021). However, fourth order homogeneous linear differential equation with initial conditions will be considered which has general form of:

$$u^{iv}(x) = f(x, u(x), u'(x), u''(x), u'''(x)) \quad (1)$$

$$u(x) = a, u'(x) = b, \qquad u''(x) = c, \quad u'''(x) = d \quad (2)$$

METHODOLOGY

Thus, in this paper, fourth order linear differential equation is solved using the two methods; Adomian decomposition method and Variation iteration method.

Adomian Decomposition Method

The Adomian decomposition method (ADM) is applied in solving a wide class of linear and nonlinear ordinary differential equations, partial differential equations, algebraic equations, difference equations, integral equations and integro-differential equation.

This method was introduced and developed by George Adomian. A considerable amount of research work has been invested in different types of equation as stated above.

Consider the following equation:

$$Lu + Nu + R = g \quad (3)$$

Where L is the linear operator. N is nonlinear operator and R is the remaining part. Solving for Lu we have.

$$Lu = g - Ru - Nu \quad (4)$$

L is invertible and in this paper L^{-1} is a fourfold integrations operator and is defined as a definite integration from *0 to x;* i.e

$$L^{-1} = \int_0^x \int_0^x \int_0^x \int_0^x (\cdot) dxdxdxdx \quad (5)$$

For the operator $L = \frac{d^4}{dx^4}$ we have;

$$L^{-1}L = u(0) + xu'(0) + \frac{1}{2!}x^2u''(0) + \frac{1}{3!}x^3u'''(0) \quad (6)$$

Operating on both sides of (2) with L^{-1} we have

$$L^{-1}L = L^{-1}g - L^{-1}Ru - L^{-1}Nu \quad (7)$$

Combining (6) and (7) we have

$$u(x) = u(0) + xu'(0) + \frac{1}{2!}x^2u''(0) + \frac{1}{3!}x^3u'''(0) + L^{-1}g - L^{-1}Ru - L^{-1}Nu \quad (8)$$

$u(x)$ is decomposed into a series as given in (7) below with $u_0(x)$ identified as the first five terms on the right-hand side of (6). The nonlinear term which is decomposed into Adomian polynomial which is considered as zero in this paper. Thus, decomposition series is written as

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \quad (9)$$

Or equivalently, $u(x) = u_0(x) + u_1(x) + u_2(x) + \cdots$

Therefore,

$$\sum_{n=0}^{\infty} u_n(x) = L^{-1}g - L^{-1}\left(\sum_{n=0}^{\infty} Ru_n(x)\right) \quad (10)$$

Thus, we have the recurrence algorithm to be

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$$u_0 = \quad u(0) + xu'(0) + \frac{1}{2!}x^2u''(0) + \frac{1}{3!}x^3u'''(0) + L^{-1}g$$
$$u_1 = -L^{-1}(Ru_0)$$
$$u_2 = -L^{-1}(Ru_1)$$
$$\vdots$$
$$u_{n+1} = -L^{-1}(Ru_n) \qquad (11)$$

This converges when nth partial sum $u_n(x) = \sum_{k=0}^{n-1} u_k$ will be approximate solution.

Variational Iteration Method

The variational iteration method (VIM) was first introduced by (He, 1999). VIM is a system that in many instances gives rapid convergent successive approximation of the exact solution if such a solution exists. If convergence is assured, the obtained approximation by this technique are of high accuracy level even if some iteration is used which is one of the aims of this paper.

Consider the equation:

$$Lu + Nu = \qquad (12)$$

Where *L* is linear operator, *N* is nonlinear operator and *g(x)* is analytical function. Furthermore, we can construct a correctional functional as follows:

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda(s)\left(Lu_n(s) + Nu_n(s) - g(s)\right)ds, n \geq 0 \qquad (13)$$

Where $\lambda(s)$ is General Lagrange Multiplier which can be identified optimally by variational theory. u_n is the approximate solution and $\tilde{u}_n$ is a restricted variation, which means $\delta\tilde{u}_n = 0$

It is clear that the main steps of He's variation iteration method is to determine the Lagrange multiplier $\lambda(s)$by using integration by parts. The successive approximation u_{n+1} of the solution $u(x)$ will be readily obtained upon using selective function $u_0(x)$ should be selected by using the initial conditions as follows.

$$u_0(x) = \quad u(0) + xu'(0) + \frac{1}{2!}x^2u''(0) + \frac{1}{3!}x^3u'''(0)$$

Consequently, the solution is given by

$$u = \lim_{n\to\infty} u_n \qquad (14)$$

Thus, the iteration formula for 4th order IVP and the Lagrange multiplier $\lambda(s)$ is

$$u_{n+1}(x) = u_n(x)$$
$$-\frac{1}{6}\int_0^x (s-x)^3\,[u_n^4(s) + f(u_n, u'_n, u''_n, u'''_n)]ds \qquad (15)$$

Numerical Examples

For comparison purposes, we used two analytical methods to solve the fourth order homogenous differential equations; firstly, we start with Adomian Decomposition Method. Secondly, we apply Variational Iteration Method (VIM). The main objective here is to solve these examples using the ADM and VIM and compare our results with the presented results.

Example 1:

$$u^{iv} - 10u'' + 9u = 0$$

$u(0) = 5, u'(0) = -1, u''(0) = 21, u'''(0) = -49$

Exact Solution: $u(x) = 2e^{-3x} - e^{-x} + 4e^x$

Discussion:

Series Solution: $5 - x + \frac{21x^2}{2} - \frac{49x^3}{6} + \frac{55x^4}{8} - \frac{481x^5}{120} + \cdots$ (16)

Adomian Decomposition Method Solution

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$$u^{iv} = 10u'' - 9u$$

Applying ADM to (1) we have

$$Lu = 10u'' - 9u$$

$$L^{-1}(\cdot) = \int_0^x\int_0^x\int_0^x\int_0^x (\cdot)dtdtdtdt$$

Applying L^{-1} to both sides of (2)

$$L^{-1}Lu = L^{-1}(10u'' - 9u)$$

$$u = u(0) + xu'(0) + \frac{1}{2!}x^2u''(0) + \frac{1}{3!}x^3u'''(0) + L^{-1}u$$

Applying initial conditions, we have

$$u = 5 - x + \frac{21}{2!}x^2 - \frac{49}{3!}x^3 + L^{-1}u$$

Adomian series is said to be

$$u = \sum_{n=0}^{\infty} u_n$$

$\sum_{n=0}^{\infty} u_n = 5 - x + \frac{21}{2}x^2 - \frac{49}{6}x^3 + L^{-1}(\sum_{n=0}^{\infty} u_n)$

Thus, we have that

$$u_0 = 5 - x + \frac{21}{2!}x^2 - \frac{49}{3!}x^3$$

$$u_{n+1} = L^{-1}\left(\sum_{n=0}^{\infty} u_n\right)$$

When $n = 0$

$$u_1 = L^{-1}\left(\sum_{n=0}^{\infty} u_0\right) = 5x^4 - \frac{1}{2}x^5 + \frac{7}{2}x^6 - \frac{49}{6}x^7 \qquad (20)$$

When $n = 1$

$$u_2 = L^{-1}(\textstyle\sum_{n=0}^{\infty} u_1) = x^8 - \frac{1}{12}x^9 + \frac{1}{2}x^{10} - \frac{49}{192}x^{11} \qquad (21)$$

When $n = 2$

$$u_3 = L^{-1}\left(\sum_{n=0}^{\infty} u_2\right) \qquad (17)$$

$$= \frac{x^{12}}{9} - \frac{1}{120}x^{13} + \frac{1}{22}x^{14} - \frac{49}{2304}x^{15} \qquad (18)$$

$$\therefore u(x) = 5 - x + \frac{21}{2}x^2 - \frac{49}{6}x^3 + 5x^4 - \frac{1}{2}x^5 + \frac{7}{2}x^6 - \frac{49}{6}x^7 + x^8 - \frac{1}{12}x^9 + \frac{1}{2}x^{10} - \frac{49}{192}x^{11} + \frac{x^{12}}{9} - \frac{1}{120}x^{13} + \frac{1}{22}x^{14} - \frac{49}{2304}x^{15} + \cdots$$

VARIATIONAL ITERATION METHOD SOLUTION

Using the correctional function

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda(s)\,[u_n^{iv}(s) - 10u'''_n(s) + 9u_n(s)]ds$$

where

$$\lambda(s) = \frac{(s-x)^3}{3!} \qquad (19)$$

$$u_0(x) = 5 - x + \frac{21}{2}x^2 - \frac{49}{6}x^3 \qquad (23)$$

$$u_1(x) = 5 - x + \frac{21}{2!}x^2 - \frac{49}{3!}x^3 + \int_0^x \frac{(s-x)^3}{3!}\left[u_0^{iv}(s) - 10u'''_0(s) + 9u_0(s)\right]ds$$

$$= 5 - x + \frac{21}{2}x^2 - \frac{49}{6}x^3 + \frac{15}{8}x^4 - \frac{3}{8}x^5 + \frac{63}{10}x^6 - \frac{49}{16}x^7 \qquad (24)$$

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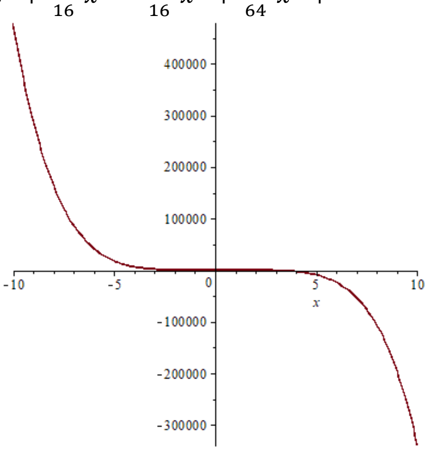
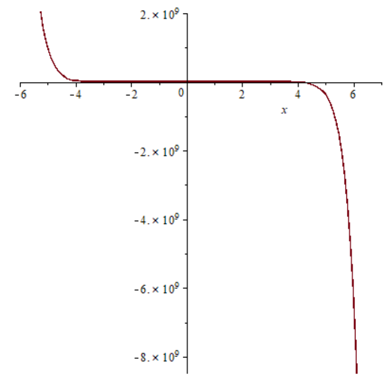
$$u_2(x) = 5 - x + \frac{21}{2}x^2 - \frac{49}{6}x^3 + \frac{15}{8}x^4 - \frac{3}{8}x^5 + \frac{63}{10}x^6 - \frac{49}{16}x^7 + \int_0^x \frac{(s-x)^3}{3!}\left[u_1^{iv}(s) - 10u'''_1(s) + 9u_1(s)\right]ds$$

$$= 5 - x + \frac{21}{2}x^2 - \frac{49}{6}x^3 + \frac{15}{8}x^4 - \frac{3}{8}x^5 + \frac{63}{10}x^6 - \frac{49}{16}x^7 + \frac{45}{64}x^8 - \frac{9}{64}x^9 + \frac{189}{128}x^{10} - \frac{147}{128}x^{11}$$

$$u_3(x) = u_2(x) + \int_0^x \frac{(s-x)^3}{3!}\left[u_2^{iv}(s) - 10u'''_2(s) + 9u_2(s)\right]ds$$

$$= 5 - x + \frac{21}{2}x^2 - \frac{49}{6}x^3 + \frac{45}{8}x^4 - \frac{9}{8}x^5 + \frac{189}{16}x^6 - \frac{147}{16}x^7 + \frac{135}{64}x^8 + \cdots$$





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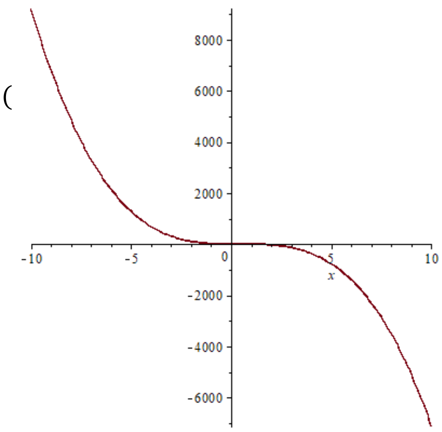

Fig. 1. Exact Solution, ADM and VIM Solution

DISCUSSION

Example 2:

$$u^{iv} - 18u'' + 81u = 0$$

$$u(0) = 0, u'(0) = -1, u''(0) = 0, u'''(0) = 0$$

Exact Solution: $u(x) = \frac{1}{4}[e^{-3x}(1+x) + e^{-3x}(1-x)]$

Series Solution: $-x + \frac{27x^5}{40} + \frac{81x^7}{280} + \frac{243x^9}{4480} + \frac{729x^{11}}{123200} + \cdots$

ADOMIAN DECOMPOSITION METHOD SOLUTION

$$u^{iv} = 18u'' - 81u$$

Applying ADM to (1) we have

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$$Lu = 18u'' - 81u$$

$$L^{-1}(\cdot) = \int_0^x \int_0^x \int_0^x \int_0^x (\cdot)dtdtdtdt$$

Applying L^{-1} to both sides of (2)

$$L^{-1}Lu = L^{-1}(18u'' - 81u)$$

$$u = u(0) + xu'(0) + \frac{1}{2!}x^2u''(0) + \frac{1}{3!}x^3u'''(0) + L^{-1}u$$

Applying initial conditions, we have

$$u = -x + L^{-1}u$$

Adomian series is said to be

$$u = \sum_{n=0}^{\infty} u_n$$

$$\sum_{n=0}^{\infty} u_n = -x + L^{-1}\left(\sum_{n=0}^{\infty} u_n\right)$$

Thus, we have that

$$u_0 = -x$$

$$u_{n+1} = L^{-1}\left(\sum_{n=0}^{\infty} u_n\right)$$

When $n = 0$

$$u_1 = L^{-1}\left(\sum_{n=0}^{\infty} u_0\right) = -\frac{x^5}{120}$$

When $n = 1$

$$u_2 = L^{-1}\left(\sum_{n=0}^{\infty} u_1\right) = -\frac{x^9}{322560}$$

When $n = 2$

$$u_3 = L^{-1}\left(\sum_{n=0}^{\infty} u_2\right) = -\frac{x^{13}}{5535129600}$$

$$\therefore u(x) = -x - \frac{x^5}{120} - \frac{x^9}{322560} - \frac{x^{13}}{5535129600} + \cdots$$

Variational Iteration Method

Using the correctional function

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda(s)\,[u_n^{iv}(s) - 18u'''_n(s) + 81u_n(s)]ds$$

$$\lambda(s) = \frac{(s-x)^3}{3!}$$

$$u_0(x) = u(0) + xu'(0) + \frac{1}{2!}x^2u''(0) + \frac{1(27)}{3!}x^3u'''(0)$$

$$u_0(x) = -x$$

$$u_1(x) = -x + \int_0^x \frac{(s-x)^3}{3!}\left[u_0^{iv}(s) - 18u'''_0(s) + 81u_0(s)\right]ds$$

$$= -x + \int_0^x \frac{(s-x)^3}{3!}[-81x]ds$$

$$= -x + \frac{27x^5}{40}$$

$$u_2(x) = u_1(x) + \int_0^x \frac{(s-x)^3}{3!}\left[u_1^{iv}(s) - 18u'''_1(s) + 81u_1(s)\right]ds$$

$$= -x + \frac{27x^5}{40} + \int_0^x \frac{(s-x)^3}{3!}\left[81s - 729s^2 - 81s + \frac{2187s^5}{40}\right]ds$$

$$= -x + \frac{27x^5}{40} + \frac{81x^6}{40} + \frac{81^9}{4480}$$

$$u_3(x) = u_2(x) + \int_0^x \frac{(s-x)^3}{3!}\left[u_2^{iv}(s) - 18u'''_2(s) + 81u_2(s)\right]ds$$

$$u_3(x) = -x + \frac{27x^5}{40} + \frac{81x^6}{40} - \frac{81x^9}{4480} + \frac{247813x^{10}}{400} + \frac{365229x^{13}}{25625600}$$

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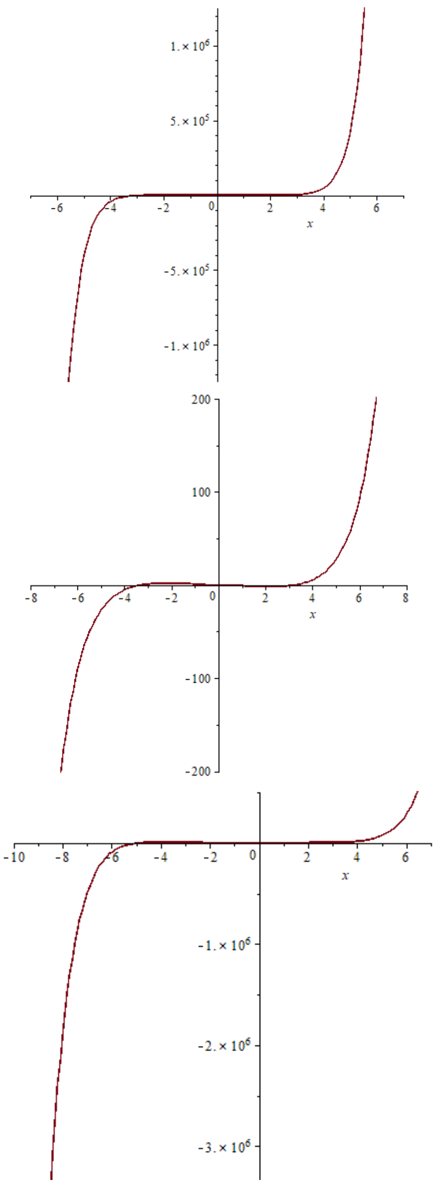

Fig. 2. Exact Solution, ADM and VIM Solution

Example 3

$$u^{iv} - 3u''' - 4u = 0$$

$$u(0) = 1, u'(0) = \frac{1}{3}, u''(0) = 0, u'''(0) = 0$$

Exact Solution: $\frac{1}{12}\left(\frac{7}{5}e^{2x} + e^{-2x}\right) + \frac{4}{5}\left(\frac{1}{3}sinx + cosx\right)$

Series Solution: $1 + \frac{x}{3} + \frac{x^4}{6} + \frac{x^5}{90} + \frac{x^6}{60} + \frac{x^7}{1260} + \frac{13x^8}{10080} + \cdots$

Adomian Decomposition Method Solution:

$$u^{iv} - 3u''' - 4u = 0$$

Applying ADM to (1) we have

$$Lu = 3u''' - 4u$$

$$L^{-1}(.) = \int_0^x \int_0^x \int_0^x \int_0^x (.)dtdtdtdt$$

Applying $L^{-1}$to both sides of (2) we have

$$u = u(0) + xu'(0) + \frac{1}{2!}x^2u''(0) + \frac{1}{3!}u'''(0) + L^{-1}(u)$$

Applying the initial conditions

$$u = 1 + \frac{x}{3} + L^{-1}u$$

Using Adomian Decomposition Method Series

$$u_n(x) = \sum_{n=0}^{\infty} u_n(x)$$

$$\sum_{n=0}^{\infty} u_n(x) = 1 + \frac{x}{3} + L^{-1}\left(\sum_{n=0}^{\infty} u_n(x)\right)$$

$$u_0(x) = 1 + \frac{x}{3}$$

$$u_{n+1}(x) = L^{-1}\left(\sum_{n=0}^{\infty} u_n(x)\right)$$

Therefore, we have

$$u_1(x) = \frac{x^4}{24} + \frac{x^5}{360};$$

$$u_2(x) = \frac{x^5}{360} + \frac{x^8}{40320}$$

$$u_3(x) = \frac{x^9}{1088640} + \frac{x^{12}}{479001600} + \frac{x^{13}}{18681062400}$$

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$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \cdots$$

$$u(x) = 1 + \frac{x}{3} + \frac{x^4}{24} + \frac{x^5}{360} + \frac{x^8}{40320} + \frac{x^9}{1088640} + \frac{x^{12}}{479001600} + \frac{x^{13}}{18681062400} + \cdots$$

Variational Iteration Method Using the correctional function

$$u_{n+1}(x) = u_n(x) + \int_0^x \lambda(s)\,[u_n^{iv}(s) - 3u'''_n(s) + 4u_n(s)]ds$$

Where

$$\lambda(s) = \frac{(s-x)^3}{3!}$$

$$u_0(x) = u(0) + xu'(0) + \frac{1}{2!}x^2u''(0) + \frac{1}{3!}x^3u'''(0)$$

Using the initial conditions

$$u_0(x) = 1 + \frac{x}{3}$$

When n =1

$$u_1(x) = u_0(x) + \int_0^x \frac{(s-x)^3}{3!}\big[u_0^{iv}(s) - 3u'''_0(s) - 4u_0(s)\big]ds$$

$$u_1(x) = 1 + \frac{x}{3} + \int_0^x \frac{(s-x)^3}{3!}\big[u_0^{iv}(s) - 3u'''_0(s) - 4u_0(s)\big]ds$$

$$= 1 + \frac{x}{3} + \int_0^x \frac{(s-x)^3}{3!}\left[-4 - \frac{4s}{3}\right]ds$$

$$= 1 + \frac{x}{3} + \frac{2x^2}{9} + \frac{x^4}{6} + \frac{7x^5}{30}$$

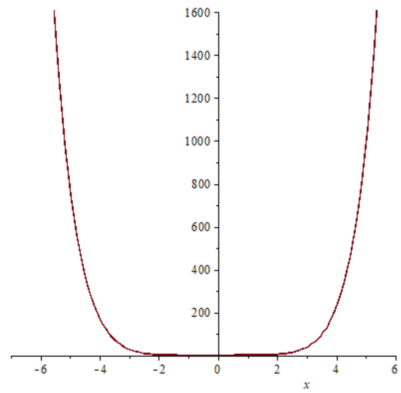
$$u_2(x) = u_1(x) + \int_0^x \frac{(s-x)^3}{3!}\big[u_1^{iv}(s) - 3u'''_1(s) - 4u_1(s)\big]ds$$

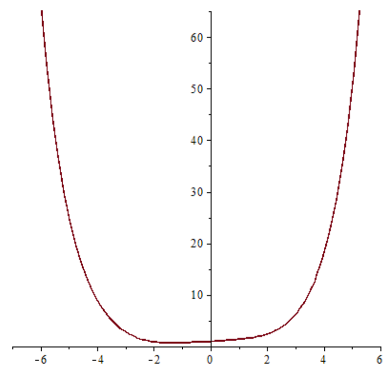
$$= 1 + \frac{x}{3} + \frac{2x^2}{9} + \int_0^x \frac{(s-x)^3}{3!}\big[u_1^{iv}(s) - 3u'''_1(s) - 4u_1(s)\big]ds$$

$$= 1 + \frac{x}{3} + \frac{2x^2}{9} + \frac{x^4}{6} + \frac{x^9}{9} + \frac{37x^6}{324} + \frac{x^8}{2520} + \frac{x^9}{3240}$$

$$u_3(x) = u_2(x) + \int_0^x \frac{(s-x)^3}{3!}\big[u_2^{iv}(s) - 3u'''_2(s) - 4u_2(s)\big]ds$$

$$= 1 + \frac{x}{3} + \frac{2x^2}{9} + \frac{x^4}{6} + \frac{x^9}{9} + \frac{37x^6}{324} + \frac{x^8}{2520} + \frac{x^9}{3240} + \cdots$$





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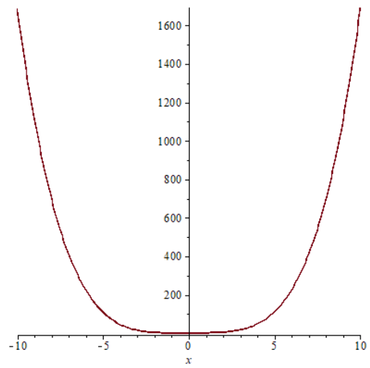

Exact Solution ADM Solution VIM Solution

CONCLUSION AND DISCUSSION OF RESULTS

The main objective of this work is to conduct a comparative study between variational iteration method and the Adomian decomposition method in order to know the approximate series solution that converges faster. The two methods are powerful and efficient methods that both give approximations of higher accuracy. An important conclusion can made here. variational iteration method gives several successive approximations through using the iteration of the correctional function. However, Adomian decomposition method provides the components of the exact solution, where these components should follow the summation given in (7). Moreover, the VIM requires the evaluation of the Lagrangian multiplier λ, whereas ADM requires the evaluation of the Adomian polynomials that mostly require tedious algebraic calculations. It is interesting to point out that unlike the successive approximations obtained by the VIM, the ADM provides the solution in successive components that will be added to get the series solution. More importantly, the VIM reduces the volume of calculations by not requiring the Adomian polynomials, hence the iteration is direct, straightforward and converges faster than ADM

REFERENCE

Abdelrazee A. and Ahmed H.M. (2008) Adomian decomposition method, convergence analysis and numerical approximations. Msc, McMaster University.

Abdul-Majid Wazwaz (2009) *Partial Differential Equations and Solitary Wave Theory*. Higher Education Press. Beijing.

Abdul-Majid Wazwaz, A. (2007). Comparison between the variational iteration method and Adomian decomposition method. Journal of Computational and Applied Mathematics. Pages 129-136.

Abioye, A. I., Ibrahim, M. O., Peter, O. J., Amadiegwu, S. and Oguntolu, F. A. Differential Transform Method for Solving Mathematical Model of SEIR and SEI Spread of Malaria. International Journal of Science: Basic and Applied Research (IJSBAR) 40(1):197-219, (2018).

Abioye, A. I., Peter, O. J., Ayoade, A. A., Uwaheren, O. A., and Ibrahim, M. O., (2020). Application of Adomian Decomposition Method on a Mathematical Model of Malaria. Advances in Mathematics: Scientific Journal, 9(1), 417–435.

Adebisi A. F., Peter O. J., Ayoola T. A., Oguntolu F. A., Ishola C. Y. (2018). Approximate Solution of Typhoid Fever Model by Variational Iteration Method ATBU, Journal of Science, *Journal of Science, Technology & Education (JOSTE);* 6 (3), 254- 265.

Adebisi, A. F., Peter, O. J., Ayoola, T. A., Ayoade, A. A., Faniyi, O. E and Ganiyu, A. B. Semi Analytic Method for Solving Infectious Disease Model. Science World Journal. 14(1), 88-91 (2019).

Adebisi, A.F., Ojurongbe, T.A., Okunlola, K.A., and Peter, O. J. (2021). Application of Chebyshev polynomial basis function on the solution of volterra integro-differential equations using Galerkin method Mathematics and Computational Sciences, 2(1): 41-51.

Adewale T. A. (2002) Numerical comparism of methods for solving second order ordinary initial value problem. *Journal of Applied Mathematical Modelling*. 31:292 – 301.

Agom E. U, Ogunfiditimi, F. O., Alhaji, T. (2016). Numerical Solution of Fourth Order Linear Differential Equations by

Corresponding author: Bello, K. A. ✉ bello.ak@unilorin.edu.ng Department of Mathematics, University of Ilorin, Kwara State.

Adomian decomposition method. *British Journal of Mathematics & Computer Science 17* (4): 1-8.

Agom EU, Ogunfiditimi FO. (2015). Modified Adomian polynomial for nonlinear functional with integer exponent. International Organization of Scientific Research - *Journal of Mathematics. 11*(6), 40-45.

Ayoade, A. A., Peter, O. J., Abioye, A. I., Aminu, T. F., and Uwaheren, O. A. (2020). Application of Homotopy Perturbation Method to an SIR Mumps Model. *Advances in Mathematics: Scientific Journal, 9(*3), 329–1340.

He, J. H (1999), Variational iteration method a kind of nonlinear analytical technique: Some examples*, Int. J. Nonlinear Mech., 34*, Pg 669-708.

Ishola, C. Y., Taiwo, O. A., Adebisi, A. F., and Peter, O. J. (2022). Numerical solution of two-dimensional Fredholm integro-differential equations by Chebyshev integral operational matrix method. *Journal of Applied Mathematics and Computational Mechanics. 20*(4), 67-79.

Khuri S. A. and Sayfy A. (2014) Variational iteration method: Green's functions and fixed point iterations perspective. *Applied Mathematics Letters. 32*, 24 – 34.

Oyedepo, T., Adebisi, A. F., Tayo, R. M., Adedeji, J. A., Ayinde, M. A., and Peter, O. J. (2021). Perturbed least squares technique for solving volterra fractional integro-differential equations based on constructed orthogonal polynomials. *Journal of Mathematical and Computational Science. 11*, 203-218.

Oyedepo, T., Uwaheren, O. A., Okperhie E. P. and Peter, O. J. Solution of Fractional Integro-Differential Equation Using Modified Homotopy Perturbation Technique and Constructed Orthogonal Polynomials as Basis Functions. *Journal of Science Technology and Education 7*(3), 157-164 (2019).

Peter, O. J., Akinduko, O. B., Ishola C. Y. and Afolabi, O. A. Series Solution of Typhoid Fever Model using Differential Transform Method. *Malaysian Journal of Computing, 3*(1), 67-80 (2018).

Peter, O. J and M. O. Ibrahim. Application of Variational Iteration Method in Solving Typhoid Fever Model, "2019 Big Data. Knowledge and Control Systems Engineering (BdKCSE), Sofia, Bulgaria. 2019. Pp. 1-5.

Peter, O. J. and Awoniran, A, F. "Homotopy Perturbation Method for Solving SIR Infectious Disease Model by Incorporating Vaccination". *The Pacific Journal of Science and Technology. 19*(1), 133-140 (2018).

Peter, O. J. (2020) Transmission Dynamics of Fractional Order Brucellosis Model Using Caputo–Fabrizio Operator. *International Journal of Differential Equations*, 1-11. Article ID 2791380, https://doi.org/10.1155/2020/2791380

Peter, O. J., Shaikh, A. S., Ibrahim, M. O., Nisar, K. S., Baleanu, D., Khan, I., and Abioye, A. I., (2021). Analysis and Dynamics of Fractional Order Mathematical Model of COVID-19 in Nigeria Using Atangana-Baleanu Operator. *Computers, Materials and Continua. 66*(2), 1823-1848.

Peter, O. J., Yusuf, A., Oshinubi, A., Oguntolu, F. A., Ibrahim, A. A., Abioye, A. I., and Ayoola, T. A. (2021). Fractional Order of Pneumococcal Pneumonia Infection Model with Caputo Fabrizio Operator. *Results in Physics. 29* 104581.

Uwaheren, O. A., Adebisi, A. F., Olotu, O. T., Etuk, M. O., and Peter, O. J. (2021). Legendre Galerkin Method for Solving Fractional Integro-Differential Equations of Fredholm Type. *The Aligarh Bulletin of Mathematics, 40*(1), 1-13.

Wazwaz A. M and Khuri S. A. (2014). A variational approach for a class of nonlocal elliptic boundary value problem. *J Math Chem. 52*, 1324 – 13

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