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Least-squares Bernstein Method for Solving Fractional Integro-differential Equations

[1]Oyedepo, T., [2]Ishola, C. Y., [3]Uwaheren, O. A., [4]Olaosebikan, M. L., [5]Ajisope, M. O., [3]Victor, A. A.
[1]Federal College of Dental Technology and Therapy, Enugu State, Nigeria
[2]Department of Mathematics, National Open University of Nigeria, Jabi, Abuja, Nigeria.
[3]Department of Mathematics, University of Ilorin, Kwara State, Nigeria.
[4]Department of Mathematical Sciences, Osun State University, Osogbo, Osun State, Nigeria
[5]Department of Mathematics, Federal University, Oye-Ekiti, Ekiti State, Nigeria.

ABSTRACT
This study gears towards finding a simple numerical algorithm for the solution to fractional integro-differential equations. The technique involves the application of Caputo properties and the properties of Bernstein polynomials to reduce the problem to system of linear algebraic equations and then solved using MAPLE 18. To demonstrate the accuracy and applicability of the presented method some numerical examples are given. Numerical results show that the method is easy to implement and compares favorably with the exact results. The graphical solution of the method is displayed.

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INTRODUCTION

Fractional integro-differential equations has played a significant role in modelling of real world physical problems e.g the modeling of earthquake, reducing the spread of virus, control the memory behavior of electric socket and many others. Fractional calculus is a field dealing with integral and derivatives of arbitrary orders, and their applications in science, engineering and other fields. The idea is from the ordinary calculus. According to (Caputo 1967; Adam 2004 & Awadeh *et al.* 2011). It was discovered by Leibniz in the year 1695 few years after he discovered ordinary calculus but later forgotten due to the complexity of the formula. Since most Fractional Integro-Differential Equations (FIDEs) cannot be solved analytically, approximation and numerical techniques, therefore, they are used extensively.

Numerical solution to FIDEs in different fields has been a point of attraction for researchers in recent times. (Dilkel & Aysegul 2018 & Ma & Huang 2013a; 2014b) applied collocation techniques for solving FIDEs using different basis functions. (Amr *et al*. 2018) applied Sumudu transform method and Hermite Spectral collocation method for solving FIDEs. (Alkan & Hatipoglu 2017) introduced approximate solutions of Volterra- Fredholm integro-differential equations of fractional order. (Mohammed 2014 & Mahdy & Mohamed 2016) used Least - Squares method for the solution of FIDEs. (Mohammed 2014; Yi & Huang 2015 & Nemati *et al*. 2016 introduced numerical solution of fractional singular integro-differential equations by using Taylor series expansion and Galerkin method and a fast numerical algorithm based on the second kind of Chebyshev polynomials. (Setia *et al*. 2014) applied numerical solution of Fredholm-Volterra fractional integro-differential equation with nonlocal boundary conditions. (Oyedepo *et al*. 2021a) employed Bernstein modified homotopy perturbation method for the Solution of Volterra fractional integro-differential equations. Also, several numerical methods to solve the FIDEs have been found in (Nawaz 2011; Emiroglu 2015; Meng *et al*. 2015; Chatterjee 2021; Wang *et al*. 2021 & Oyedepo *et al*. 2021b). The application of the two methods can be found in (Peter, et. al., 2018), (Peter, et. al., 2019), (Peter, et. al., 2020), (Peter, O. J., Yusuf, A. et. al., 2021), (Peter, et. al., 2020), (Adebisi, et. al., 2018), (Adebisi, et. al., 2019), (Adebisi, et. al., 2021), other applications of numerical methods can be found in (Peter et. al.,

Corresponding author: Oyedepo, T. oyedepotaiye@yahoo.com *Department of Mathematics, University of Ilorin, Kwara State.*

2021), (Ishola et. al., 2022), (Uwaheren, 2021), (Abioye, et al., 2018), (Abioye, et al., 2020), (Ayoade et. al., 2020). (Oyedepo, et al., 2018) and (Oyedepo, et al., 2021).

The objective of the present paper is to extend the application of the least squares and Bernstein properties with different weight functions to provide an approximate solution for fractional integro-differential equations, which is presented for the first time in the literature for better accuracy .

The general form of the class of problem considered in this work is given as:

$$D^{\alpha}u(x) = p(x)u(x) + f(x) + \int_0^x k(x,t)u(x)dt, \quad o \le x,t \le 1, \tag{1}$$

With the following supplementary conditions:

$$u^{(j)}(0) = \delta_j j = 0,1,2,\dots,m-1,\ m \quad -1 < \alpha \le m,\ m \in, N \tag{2}$$

Where $D^{\alpha}u(x)$ indicates the $\propto th$ Caputo fractional derivative of $u(x)$; $p(x)$, $f(x)$, $K(x,t)$ are given smooth functions, δ_j are real constant, x and t are real variables varying [0, 1] and $u(x)$ is the unknown function to be determined.

Some Relevant Basic Definitions Fractional

There are various definitions of fractional integration and derivatives. The widely used definition of a fractional integration is the Riemann-Liouville definition and the definition of a fractional derivative is the Caputo definition.

Definition 2.1: Riemann – Liouville fractional integral is defined as (Edwards 2018):

$$J^{\alpha}f(x) = \frac{1}{\Gamma(\alpha)}\int_0^x \frac{f(x)}{(x-t)^{1-\alpha}}dt,\ \alpha > 0, x > 0, \tag{3}$$

J^{α} denotes the fractional integral of order $\propto$

Definition 2.2: The Caputor Factional Derivative is defined as (Edwards 2018):

$$D^{\alpha}f(x) = \frac{1}{\Gamma(n-\propto)}\int_0^x (x-s)^{n-\propto-1}f^m(s)ds \tag{4}$$

Where m is a positive integer with the property that $n-1 <\propto< n$

For example if $0 <\propto< 1$ the Caputo fractional derivative is

$$D^{\alpha}f(x) = \frac{1}{\Gamma(1-\propto)}\int_0^x (x-s)^{-\propto}f^1(s)ds \tag{5}$$

Hence, we have the following properties:

(1) $J^{\alpha}\ J^{v}\ f = j^{\alpha+v}\ f,\ \alpha, v > 0, f \in C_{\mu}, \mu > 0$

(2) $J^{\alpha}x^{\gamma} = \frac{\Gamma(\lambda+1)}{\Gamma(\alpha+\gamma+1)}x^{\alpha+\gamma,}\ \alpha > 0, \gamma > -1, x > 0$

(3) $J^{\alpha}\ D^{\alpha}\ f(x) = f(x) - \sum_{k=0}^{n-1} f^k(0)\frac{x^k}{k!}, \qquad x > 0, n-1 < \alpha \le n$

(4) $D^{\alpha}\ J^{\alpha}\ f(x) = f(x), \qquad x > 0, n-1 < \alpha \le n,$

(5) $D^{\alpha}C = 0, C$ is the constant,

(6) $\begin{cases} 0, & \beta \in N_0, \beta < [\alpha], \\ D^{\alpha}x^{\beta} = \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)}x^{\beta-\alpha}, & \beta \in N_0, \beta \ge [\alpha], \end{cases}$

Where $[\alpha]$ denoted the smallest integer greater than or equal to α and $N_0 = \{0,1.2,\dots\}$

Definition 2.3: Bernstein basis polynomials: A Bernstein polynomial (Grant, 2014) of degree N is defined by

$$B_{i,m}(x) = \binom{m}{i} x^i(1-x)^{m-i} \qquad i = 0,1\dots n, \tag{6}$$

Corresponding author: Oyedepo, T. oyedepotaiye@yahoo.com *Department of Mathematics, University of Ilorin, Kwara State.*

where,

$$\binom{m}{i} = \frac{m!}{i!(m-1)!} \quad (7)$$

Often, for mathematical convenience, we set $B_{i,m}(x) = 0$ if < 0 or $j > m$

Definition 2.4: Bernstein polynomials: A linear combination Bernstein (Grant, 2014) basis polynomials

$$u_m(x) = \sum_j^m a_j u_j(x) \quad (8)$$

the Bernstein polynomial of degree n where $a_{j,}$ $j = 0,1,2, \dots \dots$ are constants.

Demonstration of Least Squares Bernstein Method (LSBM)

The Least Squares Bernstein Method is based on approximating the unknown function $u(x)$ in Eq. (1) by assuming an approximate solution of the form defined in Eq. (8). Consider Eq. (1) operating with $J^{\propto}$ on both sides as follows:

$$J^{\propto} D^{\alpha} u(x) = J^{\propto} f(x) + J^{\propto}(\int_0^x k(x,t)u(t)dt) \quad (9)$$

$$u(x) = \sum_{k=0}^{n-1} u^k(0)\frac{x^k}{k!} + J^{\propto} f(x) + J^{\propto}[\int_0^x k(x,t)u(t)dt] \quad (10)$$

$$a_{j,} \quad j = 0,1,2, \dots \dots \quad (11)$$

Substituting Eq. (8) into Eq. (10)

$$\sum_j^m a_j u_j(x) = \sum_{k=0}^{n-1} u^k(0)\frac{x^k}{k!} + J^{\propto} f(x) + J^{\propto}[\int_0^x k(x,t)\sum_j^m a_j u_j(t)\, dt] \quad (12)$$

Hence, the residual equation is obtained as

$$R(a_0, a_1, \dots, a_n) = \sum_j^m a_j u_j(x) - \{\sum_{k=0}^{n-1} u^k(0)\frac{x^k}{k!} + J^{\propto} f(x) + J^{\propto}[\int_0^x k(x,t)\sum_{j=0}^m a_j u_j(t)\, dt]\}$$

Let (13)

$$S(a_{0,}a_1, \dots, a_m) = \int_0^1 [R(a_{0,}a_1, \dots, a_m)]^2 w(x)dx \quad (14)$$

Where $w(x)$ is the positive weight function defined in the interval, $[a, b]$. In this work, we take $w(x) = \sqrt{c + dx^i}$, $i = 2,\ c = 1$ and $d = -1$. Thus,

$$S(a_{0,}a_1, \dots, a_m) = \int_0^1 \left\{\sum_j^m a_j u_j(x) - \{\sum_{k=0}^{m-1} u^k(0)\frac{x^k}{k!} + J^{\propto} f(x) + [\int_0^x k(x,t)\sum_j^m a_j u_j(t)\, dt]\} \right\}^2 dx \quad (15)$$

In order to minimize equation Eq. (15) , we obtained the values of a_j $(j \geq 0)$ by finding the minimum value of S as:

$$\frac{\partial s}{\partial a_j} = 0, j = 0,1,2 \dots, m \quad (16)$$

Applying (16) on (15), we have

$$\int_0^1 \left\{\sum_{j=0}^m a_j u_j(x) - \{\sum_{k=0}^{n-1} u^k(0)\frac{x^k}{k!} + J^{\propto} f(x) + J^{\propto}[\int_0^x k(x,t)\sum_{i=0}^m a_j u_j(t)\, dt]\} \right\} dx$$

$$\times \int_0^1 \{u_j{}^*(x) - J^{\propto}(\int_0^x k(x,t)u_j(t)dt)\} dx \quad (17)$$

Thus, Eq. (17) is then simplified for $j = 0,1, \dots n$ to obtain $(m+1)$ algebraic system of equations in $(m+1)$ unknown a'_i s which are then put in matrix form as follow:

$A =$

Corresponding author: Oyedepo, T. oyedepotaiye@yahoo.com Department of Mathematics, University of Ilorin, Kwara State.

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$$\begin{pmatrix} \int_0^1 R(x,a_0)h_0dx & \int_0^1 R(x,a_1)h_0dx \cdots & \int_0^1 R(x,a_m)h_0dx \\ \int_0^1 R(x,a_0)h_1dx & \int_0^1 R(x,a_1)h_1dx \cdots & \int_0^1 R(x,a_m)h_1dx \\ \vdots & \vdots & \ddots \\ \int_0^1 R(x,a_0)h_mdx & \int_0^1 R(x,a_1)h_mdx \ldots & \int_0^1 R(x,a_m)h_mdx \end{pmatrix} \tag{18}$$

$$B = \begin{pmatrix} \int_0^1 \left[J^{\propto}f(x) + \sum_{k=0}^{n-1} u^k(0)\frac{x^k}{k!}\right] h_0dx \\ \int_0^1 \left[J^{\propto}f(x) + \sum_{k=0}^{n-1} u^k(0)\frac{x^k}{k!}\right] h_1dx \\ \vdots \\ \int_0^1 \left[J^{\propto}f(x) + \sum_{k=0}^{n-1} u^k(0)\frac{x^k}{k!}\right] h_mdx \end{pmatrix} \tag{19}$$

Where

$$h_j \ = u_j{}^*(x) - J^{\propto}[\int_0^x k(x,t)u_j(t)dt], \qquad j = 0,1,\ldots,m$$

$$R(x,a_j) = \sum_{i=0}^{m} a_i u_j(t) - \ J^{\propto}[\int_0^x k(x,t)\sum_{i=0}^{m} a_i u_j(t)\,dt], \qquad j = 0,1,\ldots,m$$

The $(\mathrm{m}+1)$ linear equations are then solved to obtain the unknown constants $a_j(j = 0(1)m)$, which are then substituted back into the assumed approximate solution to give the required approximation solution.

4. Numerical Examples

In this section, the above technique is implemented on some problems. The problems are then solved via the Bernstein polynomials as basic functions. The problems are then solved to illustrate the computational cost accuracy and efficiency of the proposed method using Maple 18.

Example 4.1: Consider the following fractional Integro-differential (Amr *et al*. 2018)

$$D^{\frac{1}{3}}u(x) = f(x) + \frac{1}{4}\int_0^1 x(1-t)(u(t))^2dt \tag{20}$$

where

$$f(x) = \frac{9}{5\Gamma(\frac{2}{3})}x^{\frac{5}{3}} - \frac{7}{40}x \tag{21}$$

Subject to $u(0) = 0$

Here, Eq. (20) is solved by applying $J^{\frac{1}{3}}$ on both sides of Eq. (20) and substituting Eq. (8) into Eq. (20) to have

$$J^{\frac{1}{3}}[D^{\frac{1}{3}}\sum_j^m a_j u_j(x)] = J^{\frac{1}{3}}[f(x)] + J^{\frac{1}{3}}[\frac{1}{4}\int_0^1 x(1-t)\left(\sum_j^m a_j u_j(x)\right)^2dt] \tag{22}$$

Applying fractional integral operator on Eq. (22) and simplifying further, where $m = 2$ gives

$$a_0(1-2x+x^2) + a_1(2x-2x^2) + a_2x^2 - 1 + 0.1469798559x^{1.333333333} - 0.9999999998x^2 - 0.03499520380{a_0}^2x^{1.333333333} - 0.02799616304a_0a_1x^{1.333333333} - 0.006999040759a_0a_2x^{1.333333333} - 0.01399808152{a_1}^2x^{1.333333333} - 0.01399808152a_1a_2x^{1.333333333} - 0.006999040759{a_2}^2x^{1.333333333} = 0 \tag{23}$$

Hence, the residual equation is defined as

Corresponding author: Oyedepo, T. oyedepotaiye@yahoo.com *Department of Mathematics, University of Ilorin, Kwara State.*

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$R(a_{0,}a_1, a_{2)} = a_0(1-2x+x^2) + a_1(2x-2x^2) + a_2x^2 - 1 + 0.1469798559x^{1.333333333} - 0.9999999998x^2 - 0.03499520380{a_0}^2x^{1.333333333} - 0.02799616304a_0a_1x^{1.333333333} - 0.006999040759a_0a_2x^{1.333333333} - 0.01399808152{a_1}^2x^{1.333333333} - 0.01399808152a_1a_2x^{1.333333333} - 0.006999040759{a_2}^2x^{1.333333333}$ (24)

Let

$$S(a_{0,}a_1, a_m) = \int_0^1 \left[R(a_{0,}a_1, \dots\dots, a_m)\right]^2 w(x)dx \quad (25)$$

Where $w(x)$ is has been defined above. Thus,

$S(a_{0,}a_1, a_m) = a_0(1-2x+x^2) + a_1(2x-2x^2) + a_2x^2 - 1 + 0.1469798559x^{1.333333333} - 0.9999999998x^2 - 0.03499520380{a_0}^2x^{1.333333333} - 0.02799616304a_0a_1x^{1.333333333} - 0.006999040759a_0a_2x^{1.333333333} - 0.01399808152{a_1}^2x^{1.333333333} - 0.01399808152a_1a_2x^{1.333333333} - 0.006999040759{a_2}^2x^{1.333333333}$ (26)

$\int_0^1 [a_0(1-2x+x^2) + a_1(2x-2x^2) + a_2x^2 - 1 + 0.1469798559x^{1.333333333} - 0.9999999998x^2 - 0.03499520380{a_0}^2x^{1.333333333} - 0.02799616304a_0a_1x^{1.333333333} - 0.006999040759a_0a_2x^{1.333333333} - 0.01399808152{a_1}^2x^{1.333333333} - 0.01399808152a_1a_2x^{1.333333333} - 0.006999040759{a_2}^2x^{1.333333333}]^2(\sqrt{1-x^2})dx$ (27)

In order to minimize Eq. (27), applying Eq. (16) on Eq. (27) and integrating with respect to x over the interval [0,1] to give three system of equations with three unknown constants $a_i(i = 0,1,2)$. Solving these equations, the following constants were obtained as: $a_0 = 1, a_1 = 1$ and $a_2 = 2$. Substituting the values back into Eq. (8) to get the approximate solution as: $1 + x^2$

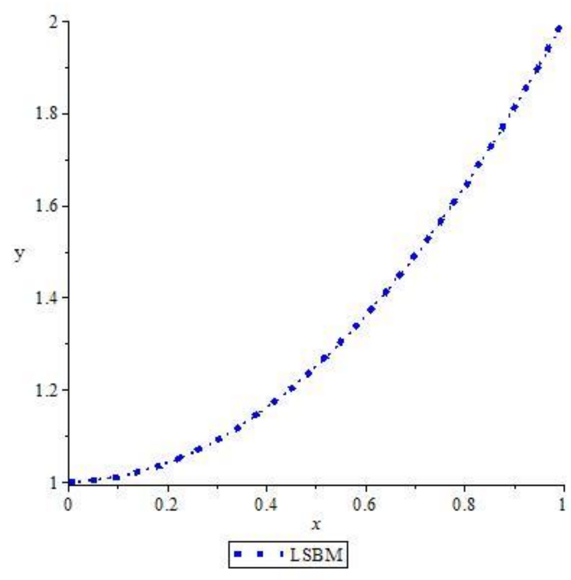

Figure 1: Showing the graph of approximation solution and exact of example 1

Corresponding author: Oyedepo, T. oyedepotaiye@yahoo.com Department of Mathematics, University of Ilorin, Kwara State.

Example 5.2: Consider the following fractional Integro-differential (Dilkel & Aysegul, 2018)

$$D^{\propto}u(x) = f(x) + \int_0^x \cos(x-t)\, u(t)dt \tag{28}$$

where

$$f(x) = \frac{1}{\Gamma(3-\sqrt{3})}x^{2-\sqrt{3}} + 2\sin x - 2x \tag{29}$$

Subject to $u(0) = 0,\ u'(0) = 0$ with exact solution $U(x) = x^2$. Solving Eq. (28), following the same procedure above, we take $\propto = \sqrt{3}$ and $m = 2$, the following constants as: $a_0 = 0, a_1 = 0$ and $a_2 = 1$. Substituting the values back into the assumed approximate solution, we obtain the approximate solution which is the same as the exact solution.

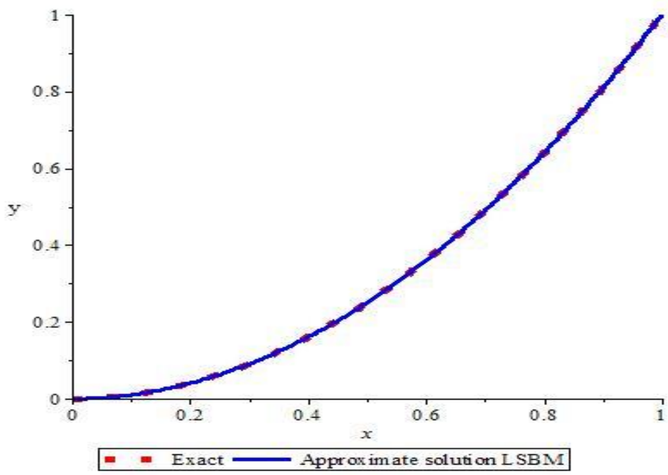

Figure 2: Showing the graph of approximation solution and exact of example 2

Example 5.3: Consider the following fractional Integro-differential (Saleh *et al*. 2015)

$$D^{\propto}u_1(x) = \frac{3x^{\frac{1}{3}}}{2}\frac{\sqrt{3}\ \Gamma(\frac{2}{3})}{\pi} - \frac{1}{6}x + \int_0^1 2xt\big(u_1(t) + u_2(t)\big)dt, \tag{30}$$

$$D^{\propto}u_1(x) = \frac{9x^{\frac{4}{3}}}{4}\frac{\sqrt{3}\ \Gamma(\frac{2}{3})}{\pi} - \frac{5}{6}x^3 + \int_0^1 x^3\big(u_1(t) - u_2(t)\big)dt \tag{31}$$

Subject to $u_1(0) = -1,\ u_2(0) = 0$ with the exact solution $u_1(x) = x - 1,\ u_2(x) = x^2$

Similarly, solving example 3, following the same procedure above, we take $\propto = \frac{2}{3}$ and $m = 2$. The following constants were obtained as: $a_0 = -1, a_1 = 0.5, a_2 = 0$ for Eq. (30) and $a_0 = 0, a_1 = 0, a_2 = 1$ for Eq. (31). Substituting the values back into the assumed approximate solution, we obtain the approximate solution as the exact solution. Comparing the result obtained by (Saleh *et al*. 2015) with our method, it tends to be said that the proposed method performed more accurately since the exact solution is found.

Corresponding author: Oyedepo, T. ✉ *oyedepotaiye@yahoo.com Department of Mathematics, University of Ilorin, Kwara State.*

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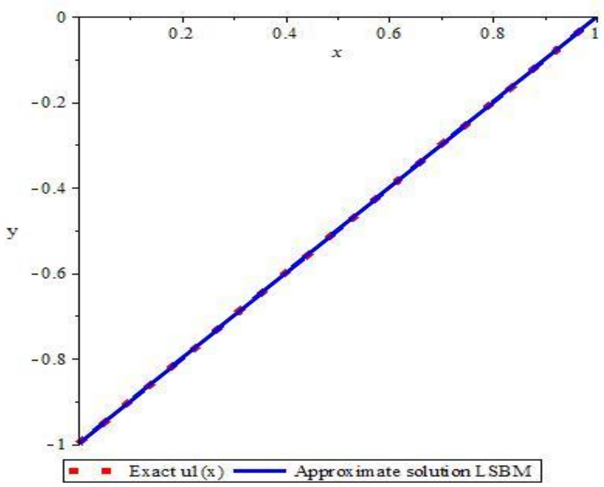

Figure 3: Showing the graph of approximation solution $u_1(x)$and exact of example 3

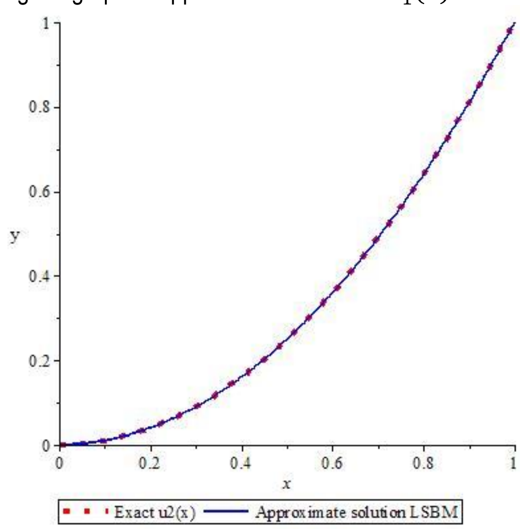

Figure 4: Showing the graph of approximation solution $u_2(x)$and exact of example 3

CONCLUSION

In this study, the properties of Bernstein polynomials together with caputo properties with different weight functions are used to find the solution of linear and non-linear FIDEs. There is a high rate of convergence of the approximate solutions to the exact solutions. Specifically, the performance of the proposed method with Bernstein polynomials was compared with some existing results in the literature and are found to be more efficient in the terms of timing and accuracy. Also, it is pleasing to note that the Bernstein polynomials and the method used are accurate in finding the solution of linear and non-linear fractional integro-differential equations and do not require the difficult computation for finding the Adomian polynomials and also using Sumudu transform method It is also observed that the number of iterations needed in solving the problems using the proposed method is few and with lower values of M (the degree of the approximant).

Corresponding author: Oyedepo, T. oyedepotaiye@yahoo.com Department of Mathematics, University of Ilorin, Kwara State.

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Corresponding author: Oyedepo, T. ✉ *oyedepotaiye@yahoo.com Department of Mathematics, University of Ilorin, Kwara State.*

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Corresponding author: Oyedepo, T. oyedepotaiye@yahoo.com Department of Mathematics, University of Ilorin, Kwara State.

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Corresponding author: Oyedepo, T. ✉ oyedepotaiye@yahoo.com Department of Mathematics, University of Ilorin, Kwara State.