



## Fitting and Forecasting Modified Arima Models to Poison Data

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### ABSTRACT

Certain models, like Autoregressive Moving Average (ARMA) model are commonly known in time series. This method takes into account that past information influences the variables of today. However, a problem occurs when Poisson data have to be modeled. The number of counts in a certain period can only be an integer and that is why the commonly used ARMA-model, which assumes stationarity, seems not very useful anymore, simply because there are some associated problems like outlier, excess zero and over dispersion that can be encountered in the Poisson data, which may actually lead to violation of stationarity assumption of the ARMA model. For this problem, Integrated Autoregressive Moving Average (ARIMA) model were studied on Poisson data. These models were used to capture different phenomena of Poisson data with different parameter. Data set were simulated in R statistical software with different sample sizes from Poisson process of  $\lambda_i, i = 5, 10, \text{ and } 20$ . The four models under study, namely: ARIMA (1,1,1), ARIMA (1,1,2), ARIMA (2,1,1) and ARIMA (2,1,2) were then fitted to the simulated data so as to examine the effect of the proportion of changes in  $\lambda$  and sample size on them. Their performances were then compared at different sample sizes and means of Poisson. Thereafter, the forecast performances of the four fitted models were examined at different steps ahead. All cases of simulation were randomized and replicated 1000 times each for the respective selected sample sizes. ARIMA (2,1,2) models are obviously preferred to be the best model that captured the Poisson data with different parameter at lower sample sizes using the selected criteria while ARIMA (1,1,2) is chosen as the best for the data with higher sample sizes.

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### INTRODUCTION

The primary objective of time series modeling is to study techniques and measures for drawing inferences from past data. The models can be employed to describe and analyze the sample data and make forecasts for the future. The main advantage of time series models is that they can handle any persistent pattern in data (Abdullahi & Taylor, 2004).

For time series, certain models are commonly known, like an Autoregressive Moving Average (ARMA) model. This method takes into account that past information influences the variables of today. For example, the rate of

inflation of a few days ago influences the rate of inflation today. Or import and export data can be predicted by using information on how many is imported and exported in the past few months. Many examples can be given and these ARMA-models work pretty well in modeling the time series (Akeyede *et al* 2016).

The ARIMA model is recursive in nature and thus relies on past calculations. This recursive nature comes directly from the AR and MA equation terms that are added to the model. The  $p$  value, or AR part, essentially describes how reliant your data points are on past data points. If  $p=1$  then the model's output for a specific time relies directly

on what the output was for the time before. If  $p=2$ , then the output would rely on the outputs from the last two time periods. Similarly, the  $q$  value, or MA part, uses the same recursive concept. The difference is that  $q$  describes how related your current output is to its past error or noise calculations. So, if  $q=1$ , then your current output would rely on the past time period's noise calculation. For  $q=2$ , your output would rely on the noise from the last two time periods (Brockwell, & Davis, (2016). Introduction to time series and forecasting (3rd ed). New York, USA:

Turney, (2022). A Poisson distribution is a discrete probability distribution, meaning that it gives the probability of a discrete (countable) outcome. For Poisson distributions, the discrete outcome is the number of times an event occurs, represented by  $k$ . You can use a Poisson distribution to predict or explain the number of events occurring within a given interval of time or space. "Events" could be anything from disease cases to customer purchases to meteor strikes. The Poisson distribution is a probability model which can be used to find the probability of a single event occurring a given number of times in an interval of (usually) time. The occurrence of these events must be determined by chance alone which implies that information about the occurrence of any one event cannot be used to predict the occurrence of any other event. It is worth noting that only the occurrence of an event can be counted; the non-occurrence of an event cannot be counted. This contrasts with Bernoulli trials where we know the number of trials, the number of events occurring and therefore the number of events not occurring. The Poisson distribution has widespread applications in areas such as analyzing traffic flow, fault prediction in electric cables, defects occurring in manufactured objects such as castings, email messages arriving at a computer and in the prediction of randomly occurring events or accidents. One example of the use of a Poisson distribution where the events are not necessarily time related is in the prediction of fault occurrence along a long weld - faults may occur anywhere along the length of the weld.

## MATERIALS AND METHODS

Data set were simulated in R statistical software with sample sizes of 20, 40, ..., 200 from

Poisson process of  $\lambda_i, i = 5, 10, 15$  and 20. The four models under study, namely: ARIMA (1,d,1), ARIMA (1,d,2), ARIMA (2,d,1) and ARIMA (2,d,2) were then fitted to the simulated data so as to examine the effect of the proportion of changes in mean and sample size on them. Their performance was compared at different sample sizes and means of Poisson. Thereafter, the forecast performances of the four fitted models were also be examined at different steps ahead. All cases of simulation were randomized and replicated 1000 times each for the respective selected sample sizes. In simulation, we set our parameters to be  $\phi_1 = 1, \phi_2 = 1$  to ensure discrete nature of Poisson data generated.

$$\text{Model 1. AR}(2): Y_{ti} = Y_{ti-1} + Y_{ti-2} + e_t \dots 1$$

The response  $Y_{ti}$  in 3.1 was generated from Poisson distributions. The four models under study were considered to analyze how well each of the models fits the selected data sets.

Data were generated from linear second orders of autoregressive functions given as follows:

$$t = 10, 20, 30, 40, 50, 60, 70, 80, 90, 100.$$

$$i = 1, 2, \dots, 1000$$

Where  $Y_{ti}$  will be simulated from Poisson as follows:

The Poisson regression model which is based on the Poisson distribution with probability density function

$$\frac{\lambda^{y_i} e^{-\lambda_i}}{y_i!}, \text{ for } y_i = 0, 1, 2, \dots \dots 2$$

Thus, for the Poisson models  $E(y_i) = V(y_i) = \mu_i$ . The restrictive condition that the mean must equal the variance is often violated by overdispersed data (where variance exceeds the mean). As a result of that Poisson model is generally considered inappropriate for count data, which are usually highly skewed and over dispersed (Cameron and Trivedi, 2008)

Differencing is an important technique in transforming data, which attempts to de-trend to control autocorrelation and achieve stationary time series. The first difference is denoted as:

$$\nabla X_t = X_t - X_{t-1} = (1 - B)X_t, \dots 3$$

where  $B$  is the backshift operator. We may extend the notion further and define the differences of order  $d$  as:

$$\nabla^d X_t = (1 - B)^d X_t, \dots 4$$

Usually, single differencing is used to remove linear trends and double differencing is used to remove quadratic trend. We can eliminate seasonality and trend of period  $d$  by introducing the lag  $d$  difference operator  $\nabla_d$  :

$$\nabla_d X_t = X_t - X_{t-d} = (1 - B^d)X_t \dots 5$$

Normally, the correct amount of differencing is the lowest order of differencing that yields a time series which fluctuates around a well-defined mean value and whose autocorrelation function (ACF) plot decays rapidly to zero, either from above or below. Thus, at every stage of differencing, we check the plots of sample autocorrelation function (ACF) and the sample partial autocorrelation function (PACF) to see where the ACF/PACF "cuts off" the bounds  $\pm 1.96$

/n . It is desirable to find a sample ACF that decays fairly rapidly. We say that a series is stationary if the sample ACF has very few significant spikes at very small lags and then cuts off drastically or dies down very quickly. If the sample ACF dies slowly, the series still has some trend. If ACF has periodicity, the series has seasonality. We should do some more differencing of the data before continuing.

The ARIMA modeling procedure for fitting ARIMA models to time series, which was developed by Box and Jenkins (1976), consists of four iterative steps: model identification; parameter estimation, diagnostic checking, and the model use. The procedure is itemized as follows in the flowchart order below:

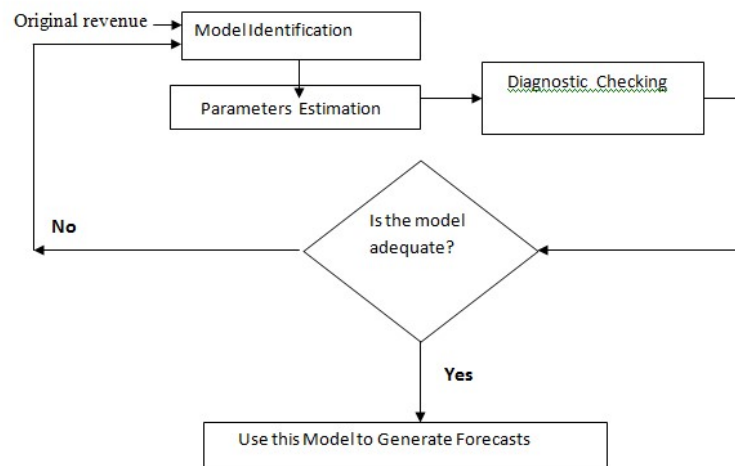


Figure 1: Flowchart of ARIMA modeling (Lee and Ko, 2011)

In the model identification (or specification), the classes of time series models are selected that may be appropriate for a given observed series. In this step we look at the time plot of the series, compute many different statistics from the data, and also apply any knowledge of the subject matter in which the data arises. In choosing a model, we shall attempt to adhere to the model used and it should require the smallest number of parameters that will adequately represent the time series. The model will inevitably involve one or more parameters

whose value must be estimated from the observed series. (Lee and Ko, 2011)

Model fitting (or parameter Estimation) consist of finding the best possible estimates of those unknown parameters within a given model. The primary objective of these parameters estimation is to minimize the forecasting error and determine both the model and its parameters. In this study, each ARIMA tentative model parameter will be tested using  $t$  values and  $p$  values.

Model diagnostic is concerned with assessing the quality of the model that have been specified and estimated. How well does the model

fit the data? Are the assumptions of the model reasonably well satisfied? If no inadequacies are found, the modeling may be assumed to be complete, and the model may be used, for example, to forecast future values. Otherwise, we choose another model in the light of the inadequacies found; that is, we return to the model specification step. In this way, we cycle through the three steps until, ideally, an acceptable model is found.

Because the computations required for each step in the model building are intensive, we rely on readily available statistical software (R software) to carry out the calculation and do the plotting. (Lee and Ko, 2011). For data to be tested using time series models, it has to be stationary. So, in this paper the stationarity of the data will be tested using Augmented Dickey – Fuller (ADF) Test, when a unit root is found in the data it will have to go through some transformation before fitting. The statistical summaries as well as distribution of the time series will be tested by means of coefficient of skewness. More so, the normality assumption will also be tested using Shapiro statistic under R statistical software.

To model the series, we check the structure of the data in order to obtain some preliminary knowledge about the stationarity of the series, whether there exist a trend or seasonal pattern. A time series is said to be stationary if both the mean and variance are constant over time. At one plot of the data is suggested to determine whether any differencing is needed before performing formal test. If the data is non-stationary, we do a logarithmic transformation or take the first order difference of the data series which may lead to a stationary time series. This process will be repeated until the data exhibit no apparent deviations from stationarity. The times of differencing of the data is indicated by the parameter d in the ARIMA (p,d,q) model. Then an Augmented Dickey fuller test (ADF test) is used to determine the stationarity of the data. (Box-Jenkins 1970). The ADF test is used to test for unit

root, the testing procedure for the ADF test is the same as for the Dickey-Fuller test but it is applied to the model. A random walk with drift and trend is represented as;

$$\Delta Y_t = \alpha + \beta t + \gamma Y_{t-1} + \sigma_1 \Delta Y_{t-1} + \dots + \sigma_{p-1} \Delta Y_{t-p} + \epsilon_t \quad \dots 6$$

Where  $\alpha$  is a constant,  $\beta$  the coefficient on a time trend and P the lag order of the autoregressive process. Imposing the constraints  $\alpha = 0$  and  $\beta = 0$  corresponds to modeling a random walk and using the walk with a drift.

The test statistic, value is calculated as follows:

$$t = \frac{\hat{\gamma}}{\sigma_{\hat{\gamma}}} \quad \dots 7$$

where  $\hat{\gamma}$  is the estimated coefficient and  $\sigma_{\hat{\gamma}}$  is the standard error in the coefficient estimate.

The null – hypothesis for an ADF test:  $H_0: \gamma = 0$ ,  
 Vs  $H_1: \gamma < 0$

Where  $H_0$ : is the null hypothesis (has unit root) and  $H_1$ : Does not have unit root. The test statistics value t is compared to the relevant critical value for the Dickey-Fuller test. If the test statistic is less than the critical value, we reject the null hypothesis and conclude that no unit – root is present. The ADF test does not directly test for stationarity but indirectly through the existence (or absence) of a unit – root.

In addition, by means of autocorrelation function (ACF) partial autocorrelation function (PACF) are applied. The correlogram examines the time series data by plotting the ACF and PACF in order to try and get the function forms of the data. ACF represent the degree of persistence over respective lags of variable at the  $X_t$  and  $X_{t+k}$ . PACF measures the amount of correlation between two variables which is not explained by their mutual correlation with a specified set of variables. ACF will be used to identify the order of MA process while PACF will identify the order of AR model. Primary distinguishing characteristics of theoretical. ACFs and PACFs for stationary processes is tabulated below: (O’Donovani; 1983)

**Table 1:** Distinguishing characteristics of ACF and PACF for stationary process

Process ACF	PACF
AR Tails off towards to Zero (exponentially decay or damped sine wave)	Cut off to Zero (After lag p)
MA cut off to Zero (after lag q)	Tails off towards zero (exponentially decay).

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If the original or different series comes out to be non-stationary some appropriate transformation will be made achieving stationarity. Otherwise, we will proceed to next phase where preliminary values of  $p$  and  $q$  are chosen. (Brockwell and Darries 1986)

## RESULTS AND DISCUSSION

The sample of data generated at different categories and sample sizes are presented in graphs as discussed in the subsequent sections. The results of the four ARIMA models [ARIMA (1,1,1), ARIMA (1,1,2),

ARIMA (2,1,1) and ARIMA (2,1,2)] presented are Table 2-9. In the first instance, model identification was carried out and test of stationarity done are presented in table 2-3. Furthermore, the data simulated were differenced. The experiment was repeated 1000 times for the four models and each sample size.

### Plot of Sample Simulated Data with $\lambda = 5$

Samples of data simulated from Poisson with parameter ( $\lambda = 5$ ), and sample size ( $n = 20$  and 200) are plotted and presented as follows

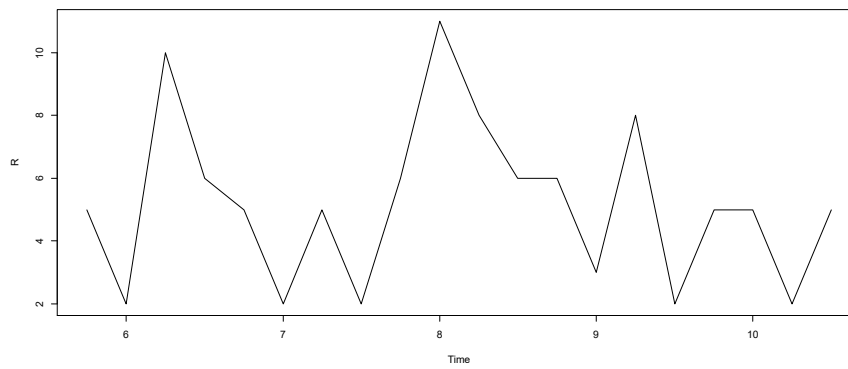


Figure 2a: Poisson Data with  $\lambda = 5$ , and sample size  $n = 20$

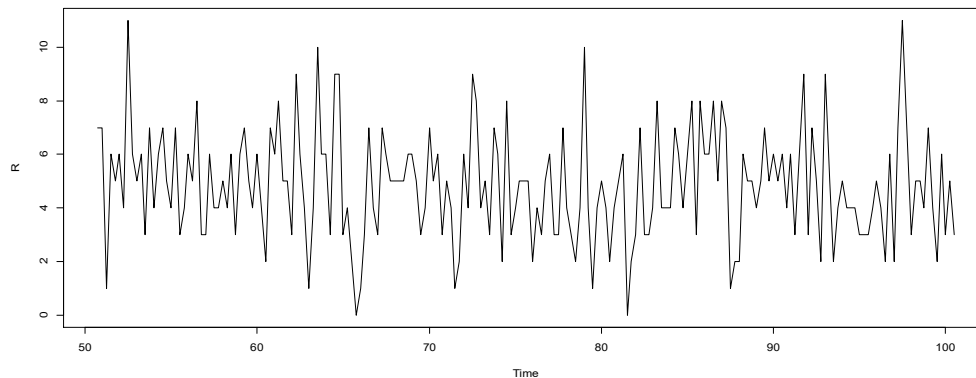


Figure 2b: Poisson Data with  $\lambda = 5$ , and sample size  $n = 200$

### Testing for Unit root and Stationarity in Poisson Data with $\lambda = 5$

The stationarity unit root test was carried out using Augmented Dickey Fuller (ADF),

Phillip. The statistic tests the null hypothesis that the data series has a unit root with the alternative that the data series is stationary. The results obtained are presented in table 2.

**Table 2:** ADF Test of Poisson Data with  $\lambda = 5$  at different Sample Sizes

Sample Size	Values	Lag order	P-value	Hypothesis ( $H_0$ )	Decision	Remark
20	-3.0077	2	0.1885	Unit root	Accept $H_0$	Not stationary
40	-2.2075	3	0.492	Unit root	Accept $H_0$	Not stationary
60	-2.8784	3	0.2197	Unit root	Accept $H_0$	Not stationary
80	-3.715	4	0.02884	Unit root	Reject $H_0$	Stationary
100	-5.2466	4	<0.01	Unit root	Reject $H_0$	Stationary
120	-4.398	4	<0.01	Unit root	Reject $H_0$	Stationary
140	-4.7182	5	<0.01	Unit root	Reject $H_0$	Stationary
160	-4.6407	5	<0.01	Unit root	Reject $H_0$	Stationary
180	-5.8542	5	<0.01	Unit root	Reject $H_0$	Stationary
200	-5.2129	5	<0.01	Unit root	Reject $H_0$	Stationary

Table 2 above shows ADF has the p-values which are greater than the critical value of 0.05 for lower sample sizes and we accept null hypothesis of having unit root series for the series. However, as sample size increases, the data series is stationary due to the p-values less than 5%. It clear that the time series plot of the Poisson data series and stationarity tests suggest that the data need to be transformed or differenced since

it is confirmed to have a unit root at lower sample sizes.

**Testing for Unit root and Stationarity in Differenced Poisson Data with  $\lambda = 5$**

The stationarity unit root test was carried out using Augmented Dickey Fuller (ADF), Phillip. The statistic tests the null hypothesis that the data series has a unit root with the alternative that the data series is stationary. The results obtained are presented in table 3.

**Table 3:** ADF Test of Differenced Poisson Data with  $\lambda = 5$  at different Sample Sizes

Sample Size	Values	Lag order	P-value	Hypothesis ( $H_0$ )	Decision	Remark
20	-2.8068	2	0.025	Unit root	Reject $H_0$	Stationary
40	-3.0384	3	0.016	Unit root	Reject $H_0$	Stationary
60	-4.3813	3	<0.01	Unit root	Reject $H_0$	Stationary
80	-3.8288	3	0.023	Unit root	Reject $H_0$	Stationary
100	-4.9996	4	<0.01	Unit root	Reject $H_0$	Stationary
120	-4.5907	4	<0.01	Unit root	Reject $H_0$	Stationary
140	-3.9346	4	0.014	Unit root	Reject $H_0$	Stationary
160	-4.157	5	<0.01	Unit root	Reject $H_0$	Stationary
180	-3.8714	5	0.0173	Unit root	Reject $H_0$	Stationary
200	-5.565	5	<0.01	Unit root	Reject $H_0$	Stationary

Table 3 presents the stationarity tests for the differenced Poisson data with parameter  $\lambda=5$  over the period of investigation with a null hypothesis of a unit root against alternative hypothesis of a level of stationarity. The p-values of 0.01 are less than the 5% level of significance while p-value which indicate that, the null hypothesis of having a unit root series should be rejected in favor of alternative of being stationary.

Indeed, the data is stationary after the first difference hence we can proceed to fitting and forecasting of the series.

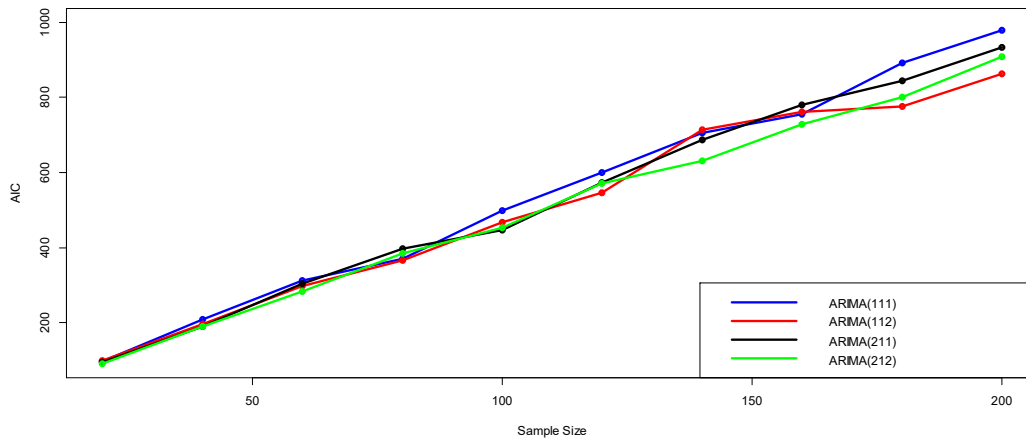
**Performance of ARIMA (p,1,q) Models on Poisson Data with Parameter ( $\lambda = 5$ )**

Table 4 below gives the relative performance of the ARIMA (p,1,,q) with parameter  $\lambda$

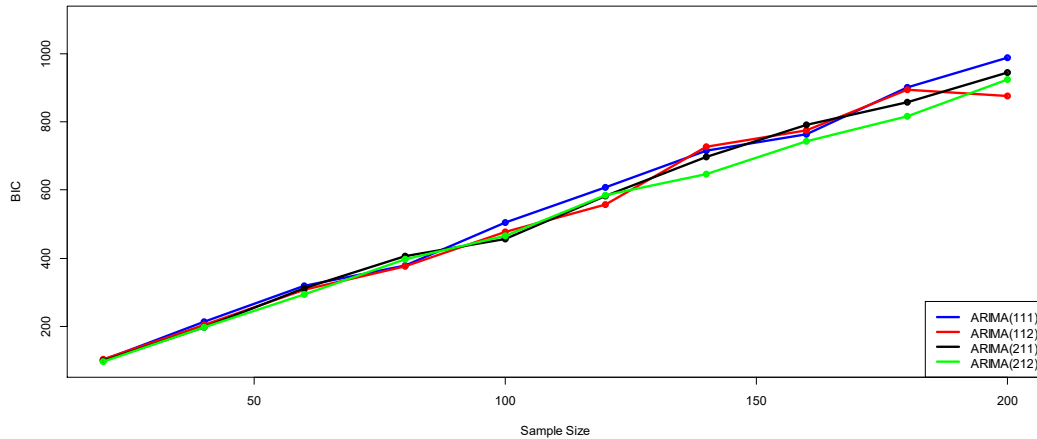
= 5 at different sample sizes based on the AIC and BIC criteria . The results were plotted on figure 3.

**Table 4:** AIC and BIC Values of ARIMA (p,1,q) of with Parameter ( $\lambda = 5$ )

Sample Sizes	ARIMA (1,1,1)	ARIMA (1,1,2)	ARIMA (2,1,1)	ARIMA (2,1,2)	ARIMA (1,1,1)	ARIMA (1,1,2)	ARIMA (2,1,1)	ARIMA (2,1,2)
20	98.048	99.4584	94.4932	91.259	100.720	103.019	98.0547	95.712
40	208.108	196.034	190.555	189.614	213.021	202.584	197.106	197.801
60	311.713	298.668	303.529	283.811	317.894	306.91	311.771	294.114
80	370.996	366.734	396.660	385.635	378.066	376.161	406.087	397.419
100	497.755	467.567	446.083	452.469	505.510	477.907	456.422	465.393
120	599.586	546.161	572.275	571.993	607.898	557.244	583.358	585.846
140	706.667	714.975	686.157	631.34	715.449	727.225	697.866	645.976
160	755.901	762.409	779.56	727.905	765.088	774.66	791.811	743.218
180	892.128	776.746	845.144	800.759	901.673	894.73	857.871	816.668
200	979.827	863.746	933.499	908.944	989.692	876.899	946.652	925.385



**Figure 3a:** AIC values of the Fitted models on Poisson Data with Parameter  $\lambda = 5$



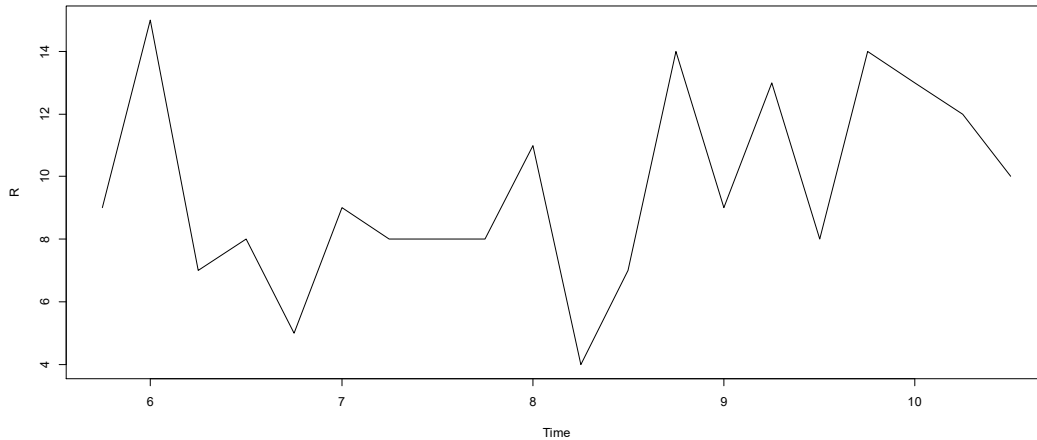
**Figure 3b:** BIC values of the Fitted models on Poisson Data with Parameter  $\lambda = 5$

Table 5 shows the fitted performances of the four ARIMA model to data simulated with underline Poisson distribution of parameter with the average values of AICs and BICs of each model at various sample sizes. The results obtained were plotted on the graphs as shown in Figure 3a and 3b respectively. The best fitted model based on AIC and BIC criteria is ARIMA (2,1,2) followed by ARIMA (1,1,2), especially, at

lower sample sizes below 180 while ARIMA (112) is the best from sample size of 180 and above. ARIMA (1,1,1) is the worst among fitted models across the periods and sample sizes

**Plot of Sample Simulated Data with  $\lambda = 10$**

Samples of data simulated from Poisson with parameter ( $\lambda = 10$ ), and sample size ( $n = 20$  and 200) are plotted and presented as follows



**Figure 4a:** Poisson Data with  $\lambda = 10$ , and sample size  $n = 20$



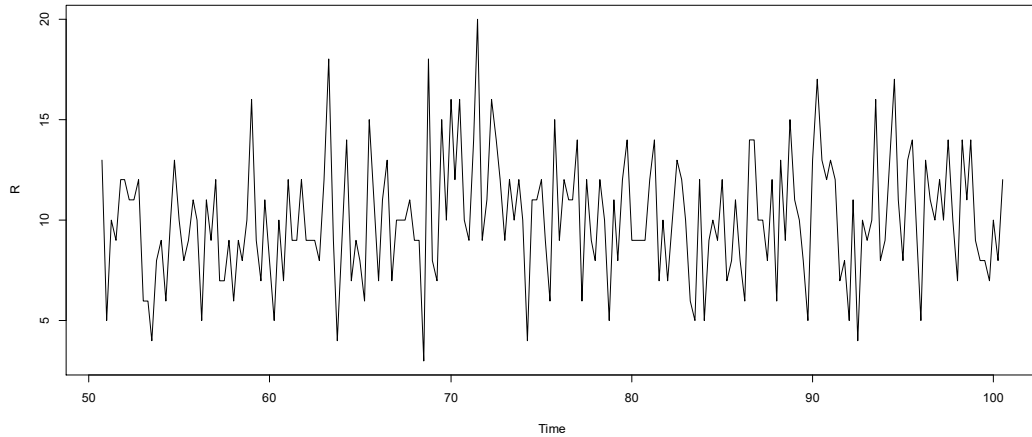


Figure 4b: Poisson Data with  $\lambda = 10$ , and sample size  $n = 200$

**Testing for Unit root and Stationarity in Poisson Data with  $\lambda = 10$**

The stationarity unit root test was carried out using Augmented Dickey Fuller (ADF),

Phillip. The statistic tests the null hypothesis that the data series has a unit root with the alternative that the data series is stationary. The results obtained are presented in table 5.

Table 5: ADF Test of Poisson Data with  $\lambda = 10$  at different Sample Sizes

Sample Size	Values	Lag order	P-value	Hypothesis ( $H_0$ )	Decision	Remark
20	-3.6268	2	0.04808	Unit root	Reject $H_0$	Stationary
40	-4.2136	3	0.01146	Unit root	Reject $H_0$	Stationary
60	-6.1385	3	<0.01	Unit root	Reject $H_0$	Stationary
80	-5.4582	4	<0.01	Unit root	Reject $H_0$	Stationary
100	-3.3743	4	0.06277	Unit root	Reject $H_0$	Stationary
120	-5.1814	4	<0.01	Unit root	Reject $H_0$	Stationary
140	-5.0753	5	<0.01	Unit root	Reject $H_0$	Stationary
160	-4.7945	5	<0.01	Unit root	Reject $H_0$	Stationary
180	-5.593	5	<0.01	Unit root	Reject $H_0$	Stationary
200	-5.7945	5	<0.01	Unit root	Reject $H_0$	Stationary

**Performance of ARIMA (p,1,q) Models on Poisson Data with Parameter ( $\lambda = 10$ )**

Table 6 below gives the relative performance of the ARIMA (p,1,q) with parameter

$\lambda = 10$  at different sample sizes based on the AIC and BIC criteria . The results were plotted on figure

Table 6: AIC and BIC Values of ARIMA (p,1,q) of with Parameter ( $\lambda = 10$ )

Sample Sizes	ARIMA (1,1,1)	ARIMA (1,1,2)	ARIMA (2,1,1)	ARIMA (2,1,2)	ARIMA (1,1,1)	ARIMA (1,1,2)	ARIMA (2,1,1)	ARIMA (2,1,2)
20	109.565	104.758	112.444	101.450	112.398	108.535	116.222	106.172
40	208.222	212.151	216.951	207.460	213.212	218.806	223.606	215.778
60	314.531	312.488	308.334	330.441	320.763	320.798	316.644	340.828
80	408.571	413.369	413.822	411.615	415.679	422.847	423.300	423.463

Sample Sizes	ARIMA (1,1,1)	ARIMA (1,1,2)	ARIMA (2,1,1)	ARIMA (2,1,2)	ARIMA (1,1,1)	ARIMA (1,1,2)	ARIMA (2,1,1)	ARIMA (2,1,2)
100	507.671	524.302	515.869	543.901	515.456	534.682	526.249	556.876
120	597.063	596.940	607.364	630.043	605.400	608.057	618.480	643.939
140	704.063	707.108	733.050	701.437	712.866	718.846	744.788	716.109
160	807.426	816.501	853.909	834.713	816.633	828.776	866.185	850.057
180	927.558	921.701	897.163	927.493	937.121	934.451	909.912	943.430
200	1034.70	1071.11	1052.56	1049.32	1044.58	1084.28	1065.73	1065.79

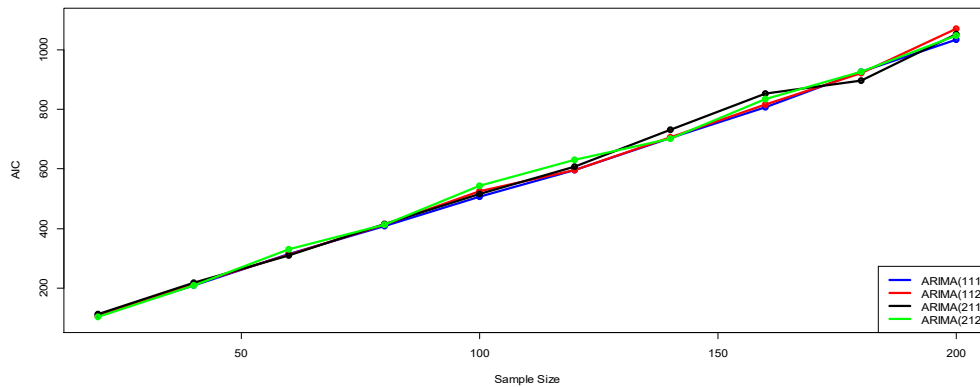


Figure 5a: AIC values of the Fitted models on Poisson Data with Parameter  $\lambda = 10$

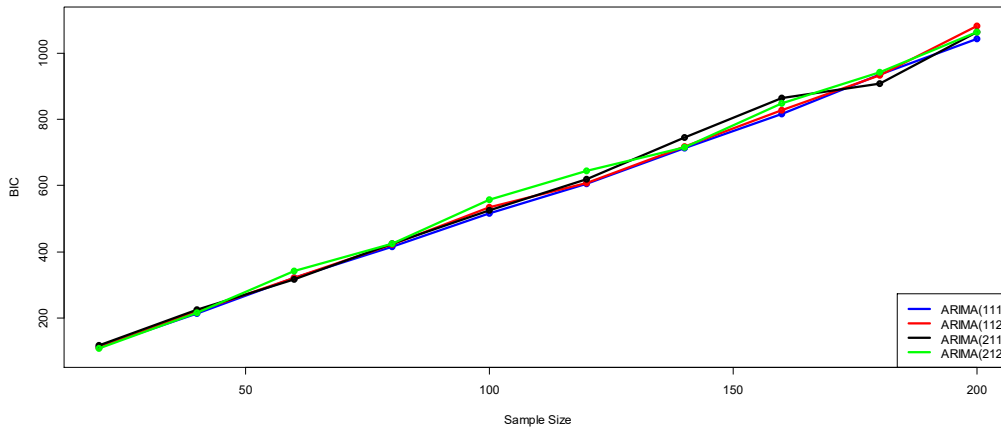


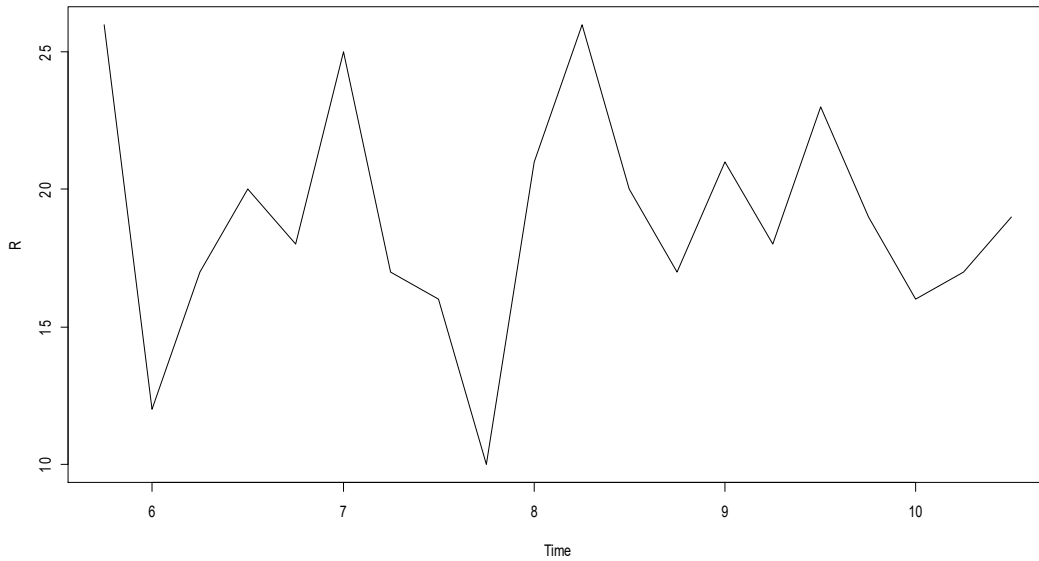
Figure 5b: BIC values of the Fitted models on Poisson Data with Parameter  $\lambda = 10$

Table 6 shows relative performance of the four fitted ARIMA models with the average values of AIC and BIC of each model at various sample sizes. The results obtained were plotted on the graphs as shown in Figure 45a and 5b respectively. The best fitted model on poison data with parameter  $\lambda = 10$  is ARIMA (111), especially at various sample sizes followed by ARIMA

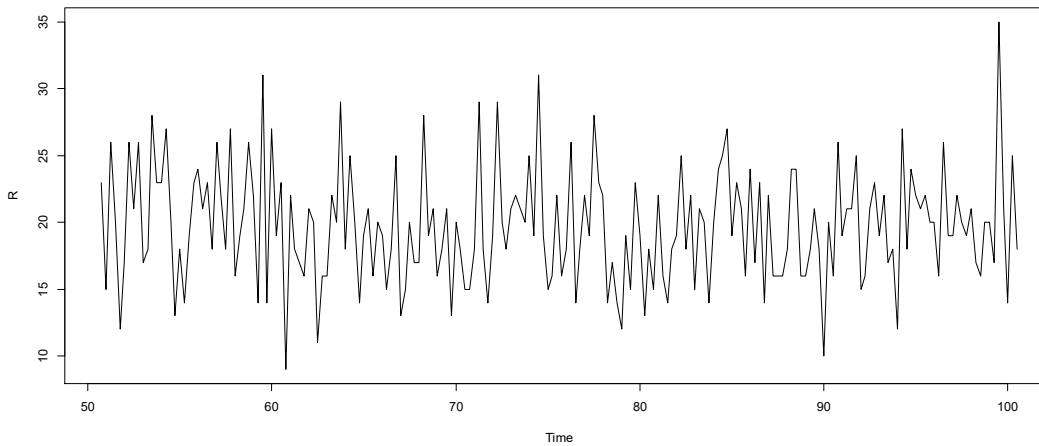
(2,1,2). ARIMA (2,1,2) out performed others as sample sizes increases based on AIC criteria.

**Plot of Sample Simulated Data with  $\lambda = 20$**

Samples of data simulated from Poisson with parameter ( $\lambda = 10$ ), and sample size ( $n = 20$  and 200) are plotted and presented as follows



**Figure 6a:** Poisson Data with  $\lambda = 20$ , and sample size  $n = 20$



**Figure 6b:** Poisson Data with  $\lambda = 20$ , and sample size  $n = 200$

**Testing for Unit root and Stationarity in Poisson Data with  $\lambda = 10$**

The stationarity unit root test was carried out using Augmented Dickey Fuller (ADF).

The statistic tests the null hypothesis that the data series has a unit root with the alternative that the data series is stationary. The results obtained are presented in table 7.

**Table 7:** ADF Test of Poisson Data with  $\lambda = 20$  at different Sample Sizes

Sample Size	Values	Lag order	P-value	Hypothesis ( $H_0$ )	Decision	Remark
20	-2.8515	2	0.248	Unit root	Reject $H_0$	Stationary
40	-3.946	3	0.02188	Unit root	Reject $H_0$	Stationary
60	-3.0749	3	0.1403	Unit root	Reject $H_0$	Stationary
80	-3.8491	4	0.02088	Unit root	Reject $H_0$	Stationary

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Sample Size	Values	Lag order	P-value	Hypothesis ( $H_0$ )	Decision	Remark
100	-4.333	4	<0.01	Unit root	Reject $H_0$	Stationary
120	-4.4322	4	<0.01	Unit root	Reject $H_0$	Stationary
140	-4.9275	5	<0.01	Unit root	Reject $H_0$	Stationary
160	-5.6941	5	<0.01	Unit root	Reject $H_0$	Stationary
180	-5.8183	5	<0.01	Unit root	Reject $H_0$	Stationary
200	-5.8718	5	<0.01	Unit root	Reject $H_0$	Stationary

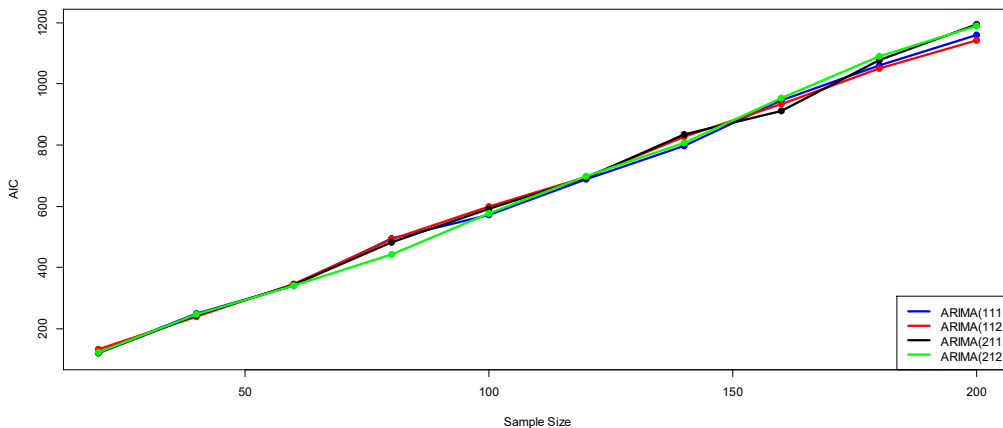
**Performance of ARIMA (p,1,q) Models on Poisson Data with Parameter ( $\lambda = 10$ )**

Table 8 below gives the relative performance of the ARIMA (p,1,q) with parameter

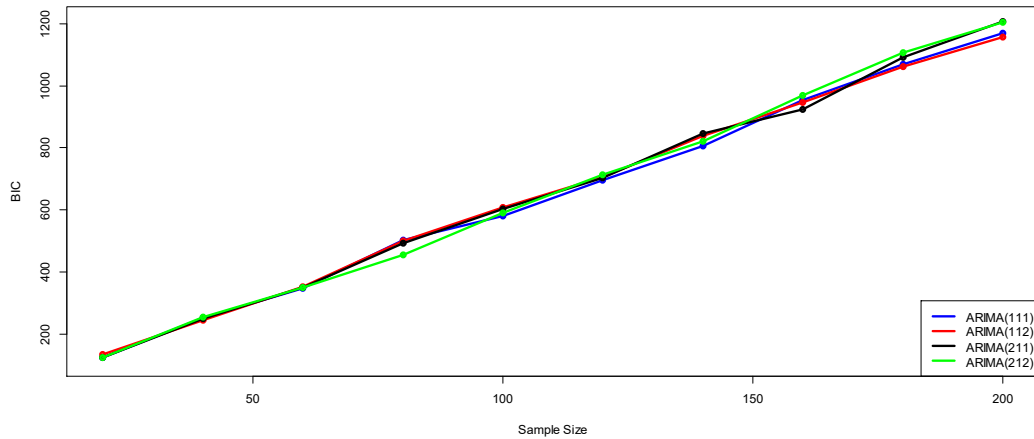
$\lambda = 10$  at different sample sizes based on the AIC and BIC criteria . The results were plotted on figure 7

**Table 8:** AIC and BIC Values of ARIMA (p,1,q) of with Parameter ( $\lambda = 20$ )

Sample Sizes	ARIMA (1,1,1)	ARIMA (1,1,2)	ARIMA (2,1,1)	ARIMA (2,1,2)	ARIMA (1,1,1)	ARIMA (1,1,2)	ARIMA (2,1,1)	ARIMA (2,1,2)
20	121.293	131.669	120.510	120.556	124.127	135.447	124.288	126.278
40	248.819	239.329	244.383	235.889	253.809	245.983	251.037	254.207
60	341.452	345.023	342.613	339.917	347.684	353.333	350.924	350.305
80	495.409	491.951	483.401	443.677	502.517	501.429	492.878	455.524
100	572.069	599.048	591.894	576.839	579.854	609.428	602.275	589.814
120	687.362	695.285	692.641	698.687	695.699	706.401	703.758	712.582
140	797.261	828.101	835.319	806.991	806.065	839.838	847.057	821.663
160	945.708	933.620	912.399	953.358	954.915	945.896	924.675	968.702
180	1059.77	1050.19	1078.32	1091.31	1069.34	1062.94	1091.07	1107.25
200	1159.28	1142.56	1194.43	1188.49	1169.16	1155.74	1207.60	1204.96



**Figure 7a:** AIC values of the Fitted models on Poisson Data with Parameter  $\lambda = 20$



**Figure 7b:** BIC values of the Fitted models on Poisson Data with Parameter  $\lambda = 20$

Table 8 shows the fitted performances of the four ARIMA models to data simulated under poison data with  $\lambda = 20$ . The average values of AIC and BIC of each model at various sample sizes recorded. The results obtained were plotted on the graphs as shown in Figure 7a and 7b respectively from figure 4.6a and 4.6b; the four models have close performances on the basis of AIC and BIC. However, it can be seen that the ARIMA (2,1,2) model performs more absolutely well than other models at sample sizes below 150 while ARIMA (1,1,2) performs best at higher sample sizes due to their lowest value of AIC. However, the performance of the ARIMA (1,1,1) models supersedes others at the moderate sample size of 150.

**Relative Forecast Performance of the Models when Sample Size is small with  $\lambda = 5$**

In this section, data were simulated under the condition of the Poisson with  $\lambda = 5$ , and then fitted to ARIMA (1,1,1), ARIMA (1,1,2), ARIMA (2,1,1) and ARIMA (2,1,2) models which are the best models under this category. Thereafter, the fitted models were used to predict h-steps ahead based on the observations using small sample size of 20. The Theil U Statistic computed from 1000 iterations from each model at different combinations of steps ahead and sample sizes were recorded below:

**Table 9:** Forecast Performance of the Models using Theil U Statistic

Steps Ahead	ARIMA (1,1,1)	ARIMA (1,1,2)	ARIMA (2,1,1)	ARIMA (2,1,2)
5	1.33013	2.84114	2.57219	3.1075
10	1.30324	2.81424	2.54529	3.09641
15	0.27634	2.78735	2.5184	3.0435
20	0.24944	2.76045	1.4915	3.0286
25	0.22255	2.73356	1.46461	3.0026
30	0.19565	2.70666	1.43771	2.9785
35	0.16876	2.67977	1.41082	2.94872
40	0.14186	2.65287	1.38392	2.92182
45	0.11497	2.62598	1.35703	2.89493
50	0.08807	2.59908	1.33013	2.86803

Based on the Theils Analysis in table 9 above, the ARIMA (2,1,2) has the highest forecasting power due to their values greater than

1 and also greater than other values of the models across the steps ahead; this is followed by ARIMA (1,1,2) and ARIMA (2,1,1). However, the Theil



values of ARIMA (1,1,1), at higher steps ahead are close to zero, hence it is not as good as other models in forecasting. Indeed, the forecasting ability of all the models decrease as steps ahead increase.

The test of stationarity/ unit root of the data when  $\lambda = 5$  shows that the data series is not stationary at lower sample sizes. However, as sample size increases the data seems to be stationary. After the first difference, the data became stationary as the test p-values are less than 5% which indicate the rejection of the null hypothesis of unit root. In Poisson data with  $\lambda = 10$  and  $\lambda = 20$ , the data generated were stationary at all levels of sample sizes.

Table 4 shows relative performance of the four fitted ARIMA models with the average values of AIC and BIC of each model at various sample sizes. The results obtained were plotted on the graphs as shown in Figure 3a and 3b respectively. The best fitted model on poison data with parameter  $\lambda = 10$  is ARIMA (111), especially at various sample sizes followed by ARIMA (2,1,2). ARIMA (2,1,2) out performed others as sample sizes increases based on AIC criteria

Table 6 shows relative performance of the four fitted ARIMA models with the average values of AIC and BIC of each model at various sample sizes. The results obtained were plotted on the graphs as shown in Figure 5a and 5b respectively. The best fitted model on poison data with parameter  $\lambda = 10$  is ARIMA (111), especially at various sample sizes followed by ARIMA (2,1,2). ARIMA (2,1,2) out performed others as sample sizes increases based on AIC criteria.

Table 8 shows the fitted performances of the four ARIMA models to data simulated under Poisson data with  $\lambda = 20$ . The average values of AIC and BIC of each model at various sample sizes recorded. The results obtained were plotted on the graphs as shown in Figure 7a and 7b respectively. From Figure 7a and 7b; the four models have close performances on the basis of AIC and BIC. However, it can be seen that the ARIMA (2,1,2) model performs more absolutely well than other models at sample sizes below 150 while ARIMA (1,1,2) performs best at higher sample sizes due to their lowest value of AIC. However, the performance of the ARIMA (1,1,1)

models supersedes others at the moderate sample size of 150.

Based on the Theils Analysis in table 9 above, the ARIMA (2,1,2) has the highest forecasting power due to their values greater than 1 and also greater than other values of the models across the steps ahead; this is followed by ARIMA (1,1,2) and ARIMA (1,1,1). However, the Theil values of ARIMA (2,1,1), at higher steps ahead are close to zero, hence it is not as good as other models in forecasting. Indeed, the forecasting ability of all the models decrease as steps ahead increase.

## CONCLUSION

In this paper, comparative performance of the ARIMA models of different orders was carried out on R simulated data. In the comparative performances of the models ARIMA (2,1,2) models are obviously preferred to be the best model that captured the Poisson data with different parameter at lower sample sizes using the selected criteria while ARIMA (1,1,2) is chosen as the best for the data with higher sample sizes. However, the best performance of all the fitted models is experienced at lower sample sizes. Based on the forecasting ability of the models, the ARIMA (2,1,2) has the highest forecasting power due to their values greater than 1 and also greater than other values of the models across the steps ahead; this is followed by ARIMA (1,1,2) and ARIMA (1,1,1). However, the Theil values of ARIMA (2,1,1), at higher steps ahead are close to zero, hence it is not as good as other models in forecasting. Indeed, the forecasting ability of all the models decrease as steps ahead increase.

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