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Application of Differential Transform Method for Solving Drug Abuse Model

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ABSTRACT

This study focused on establishing the positivity, stability analysis and numerical solution in modelling substance abuse. In this research work, a deterministic mathematical model of substance abuse using Prosecution approach is constructed from compartmentalized model. Building on the insights from epidemiology, our model entails for controlling the spread of abusers' ideologies in the society. We introduce a simple compartmental model suitable to describe abusers' group. The population N(t) is divided into six compartments: S(t), L(t), H(t), P(t), R(t) and C(t) denoting the Susceptible, Light substance users, Heavy substance users, Prosecuted compartment, Rehabilitated class and Punishment populations respectively. The motive is basically to examine whether the substance abusers population will persist or on the other hand cease to exist with the introduction of the compartment C control class. Next Generation Matrix method and algorithm for calculating the basic reproduction number R_0 was explicitly enumerated. The Differential Transform Method was employed as numerical solution of the system hence plot-visibility of each compartment behaviour; conclusively showing a significant decline in the P(t) and C(t) abusers' control groups.

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INTRODUCTION

Conditions and/or circumstances of living under the influence of stimulants and charging substances have in general been directly associated with societal menace. Excessive use of psychoactive substances such as alcohol, pain medications or illegal drugs can lead to physical, social or emotional harm(It is however just ain't about illegal substance usage alone but also employing meds, alcohol and other legal substances the wrong way can also harm the health) (Joah et al 2018). The phenomenon of substance addiction are usually kept in check by the advanced and developed nations, these are usually enforced by strict drug use evaluation and monitoring mechanisms. Unfortunately, a number of developing nations have a very high number of its inhabitants that keeps abusing several substances with impunity.

Substance abuse is simply the use of a drug in amounts or by methods which are harmful to the individual or others. It is a form of substance related disorder. Different definitions of substance abuse are used in Public Health, Medical and Criminal Justice contexts. In some cases, criminal or antisocial behaviour occurs when the individual is under the influence of a substance, and long-term personality changes in individuals may also occur. In addition to possible physical, social and psychological harm, the use of some drugs may also lead to criminal penalties, although these vary widely depending on the local jurisdiction.

Muhammad *et al.* (2019) inferred that criminal law gives a significant impact to decrease the number of mild drug addicts and also a number of abusers population of substance addicts. Drugs such as tramadol, legally and legitimately prescribed by doctors for pain relief, are abused

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by millions in search of a fix or a release from poverty, unemployment and lack of opportunity. According to a survey in 2018 by the National Bureau of Statistics (NBS) and the Centre for Research and Information on Substance Abuse (CRISA) with technical support from the United Nations Office on Drugs and Crime (UNODC 2019), the prevalence of any drug use in Nigeria is estimated at 14.4% , corresponding to 14.3 million people aged 15-64 years who had used a psychoactive substance in the past year for non-medical purposes. The highest levels of drug use were among those aged 25-39 years which we can regard as youths.

Also, up to 67% of adolescents in urban Nigeria have used substances. Factors including experimental curiosity, peer pressure, poor socio-economic conditions at homes and the need for extra energy for daily activities have been identified as contributors to illicit drug use. These are precursors that are being magnified by COVID-19, with an increasing number of the Nigerian youth at risk of turning to substance abuse. In a world drug report by the UNODC in 2019, people worldwide who had used opioids – a class of drugs naturally found in the opium poppy plant – had surged by 56% and this was attributed to improved knowledge of the extent of drug use from new surveys conducted in two highly populated countries, namely India and Nigeria.

Millennials – people born in the 1980s, 1990s or early 2000s – the architects of the Nigerian future are full of adventure and willing to try new things, however, this same fearless adventurism makes them the perfect target for hard drug experimentation and eventual dependency. Aggravating the situation are the manners and genres of music that influence the young, and have a negative effect on their behaviour, schoolwork, social interactions, and mood. Music lyrics that glorify drugs, sex, and violence have become the order of the day. The perceptions and effects of music-video messages are important as exposure to violence, sexual messages and stereotypes, and use of substances of abuse in music videos might produce significant changes in behaviours and attitudes of young viewers. The low investment in treatment and care for people with drug use disorders complicates matters for those carried away by this perception. Furthermore, there is a strong correlation between drug addiction, rape and crime. Combating drug addiction and trafficking in Nigeria is a good first step to ending the menace of rapes and crimes in the country.

A study by Carl *et al.* (2001) on Epidemic increases in Cocaine and Opiate use by trauma center patients revealed that marijuana is the common illegal substance found in impaired drivers, victims of crashed accidents and serious injured drivers of the transport sector. Moreover, some of other substances are Opium, Amphetamine, Cocaine, Alcohol, Ayahausca, Central Nervous System Depressants, DMT, GHB, Hallucinogens, Heroin, Inhalants, Netamine, Khat, Kratim, LSD, MDMA (Ecstacy/Mocky), Mescaline (Peyote), Methanphetamine, Over - The - Counter Medicines (Dextromethorphan, Loperamide), PCP, Prescription Opioids, Prescription Stimulants, Psilocybin, Rohypnol (Fluintrazepam), Salvia, Steroids (Anabolic), Synthetic Cathinones (Bat salts), Tobacco; NIDA(2013). In Nigeria, the most common types of abused substances as cited by Haladu (2003), Haladu (2003).

A study conducted by World Bank collection of development indicators placed smoking prevalence among adult male in Nigeria at 17.4% and alcohol consumption study conducted by the World Health Organisation (WHO) placed 56% of Nigerians over the age of 15 consume alcohol. The study also reveals that 50% of Nigerians in urban area consume alcohol. Eguda et al (2022) formulated a 5 sub-class model to analyze the population dynamics of drug addiction among youths and the effect of addiction on their development and productivity towards contributing their quota to the human population. The model was developed using ordinary differential equations approach and the basic reproduction number was obtained using the next generation operator method. Stability analysis was carried out and it was discovered that drug-free equilibrium exists and is locally and globally asymptotically stable whenever effective basic reproduction number is less than one; and

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unstable whenever effective basic reproduction number exceeds one. It concluded that drug abuse persisted among youths and are both locally and globally asymptotically stable whenever effective basic reproduction number is greater than one and unstable whenever effective basic reproduction number is less than one.

Katelyn et al. (2021) examined young adults' changes in cigarette, e-cigarette, marijuana, and alcohol use from pre- to during COVID-19(given the potential for increased substance use during COVID-19); and related risk/protective factors. They examined risk/protective factors (i.e. adverse childhood experiences [ACEs], depressive symptoms, resilience) in relation to changes in past 30-day substance use frequency. In their sample (N = 1084, Mage=24.76, SD = 4.70; 51.8% female; 73.6% White; 12.5% Hispanic), W3/W4 past 30-day use prevalence was: 29.1% cigarettes (19.4% increased/26.4% decreased), 36.5% e-cigarettes (23.2% increased/28.6% decreased), 49.4% marijuana (27.2% increased/21.2% decreased), and 84.8% alcohol (32.9% increased/20.7% decreased). Multivariate regressions indicated that, greater increases were predicted by: for e-cigarettes, greater ACEs; and for alcohol, greater depression. Among those with low resilience, predictors included: for e-cigarettes, greater depression; and for marijuana, greater ACEs.

Concluding that Interventions to reduce substance use during societal stressors should target both risk and protective factors, particularly resilience. Fikiri and Dmitry (2021) employed numerical simulation to establish that the dynamical behavior of drug abuse model, their result shows the significant contribution by the rate of adequate contact between the susceptible individual and the drug user, and the rate of recovery of drug user after rehabilitation as the major factors or players that dictate the dynamics of the drug user population in the society. The model also analyzed the dynamical behavior of drug abuse when control is applied, whose result shows the drug users are minimized significantly. Inferences from the work signifies the importance of early control of the drug abuse problem through the establishment of strict laws that will narrow the possibility of the bad practice in societies

Sara et al. (2020) analysed that the response to treatment may have varied as a function of six empirically-based baseline moderators and predictors: biological sex, age, race/ethnicity, mental health problems, parent-adolescent communication, and peer deviance. They used data from the parent trial randomizing 102 parents to either the FCU (n = 51) or PE (n = 51) interventions were re-analyzed across four time points (baseline, 3-, 6-, and 12-months). Moderators and predictors were tested via a series of hierarchical linear models; giving result that Parent-adolescent communication and peer deviance emerged as significant predictors of adolescent treatment response. And specifying that low-levels of parent-adolescent communication or peer deviance were associated with worse treatment response.

Abubakar et al. (2021) conducted a systematic search of the literature on PubMed to identify information on drug abuse and drug laws in Nigeria from the inception of the database to March 2020. Using Additional information from Google Scholar, a manual search of included articles, discussion with experts on the subject matter, and gray literature. Their Study selection was performed using the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) statements. Information from gray literature was assessed for quality and accuracy using the AACODS checklist (authority, accuracy, coverage, objectively, date, significance).

A prevalence of 20 – 40% and 20.9% of drug abuse was reported among students and youths, respectively. Commonly abused drugs include cannabis, cocaine, amphetamine, heroin, diazepam, codeine, cough syrup and tramadol. Sources where abusers obtained drugs, were pharmacies/patent medicine shops, open drug markets, drug hawkers, fellow drug abusers, friends, and drug pushers. Drug abuse was common among undergraduates and secondary school students, youths, commercial bus drivers, farmers, and sex workers. Reason for use included to increase physical performance, stress and to derive pleasure. Poor socioeconomic

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factors and low educational background were the common risk factors associated with drug abuse. They identified several drug laws and policies that were established under government agencies such as the National Drug Law Enforcement Agency (NDLEA), National Agency for Foods and Drugs Administration and Control (NAFDAC), Pharmacists Council of Nigeria (PCN) and a Presidential Advisory Committee. They concluded that the burden of drug abuse is still high despite the existing drug laws, policies, and strategies for prevention. Measures to reduce the burden should involve the community, government, and religious bodies. Preventive measures should target the youths, the students, identified sources of the drugs, reasons and risk factors associated with drug abuse in Nigeria.

Victoria *et al.* (2022) conducted a cross-sectional study on a total of 9742 high school, college and university students. They used tools to document socio-demographic characteristics, economic indicators, drug and alcohol use and related perceptions and Diagnostic and Statistical Manual of Mental Disorders, Fourth Edition (DSM-IV) related psychiatric disorders. Basic descriptive statistics (means and standard deviations for numerical variables and frequencies for nominal and ordinal variables) were done. Logistic regression models were used to assess the association and odds ratios between the use of a given substance and the use of the other substances, as well as associations with the various available socio-demographic factors and economic indicators.

Chi-squared tests were used in socio-economic characteristics disaggregated by current alcohol use. They found a wide range of different drugs of abuse. Alcohol abuse was the commonest and inhalants were the least, with different perceptions.Both alcohol and drug abuse were associated with various economic indicators and various mental disorders. Their study established that Kenya the multifaceted associations and predictors of alcohol and drug abuse in a cross-sectional student population ranging from high school to college and university levels. Their study contributes to global data on the subject; calling for an integrated and multifaceted approach in addressing alcohol and substance abuse. This approach should take into account various associations and predictors as part of holistic approach in both public awareness and clinical interventions

The work by Xiaohan *et al.* (2019) inferred that drug courts solve problems, namely, reducing drug use and recidivism. Legal professionals and scholars have recognized that there are multiple internal and external constraints that could undermine the drug court operation. At this point, the mechanism(s) by which threaten factors affect drug court program fidelity has been neither theoretically modeled nor empirically tested. They found that using a national sample and the Structural Regression Analysis (SRA), collaboration and judicial decision making are the most important factors for maintaining drug court program fidelity. This is because collaboration and judicial decision making can mediate the constraints and challenges from many quarters, including the lack of information sharing, evaluation and treatment.

Subasish *et al.* (2020) highlighted the landscape of marijuana regulation which is changing now rapidly. Several countries have already legalized marijuana in some form, and marijuana for recreational purposes will soon become legal for a quarter of the U.S. population. George and Patrick (2019) explained the role of physical and psychological health problems in drug use treatment process. Less worries and compliance with normal driving ethics should usually not necessitate addiction to a substance before every wheel drive. Research findings by Rebecca (2011) examined the social and historical significance of coerced drug treatment within the criminal justice system. Drug courts, the most prominent example of her approach, served as a case study to explore how seemingly contradictory perspectives on substance use—therapeutic and punitive—are merged to justify increased criminal justice oversight of defendants in the name of facilitating recovery.

Drawing on an analysis of drug court organizational documents and interviews with key advocates, she examined the punitive, therapeutic, and medical knowledge drug court

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advocates draw on and construct to justify an increased role for the courts in solving the problem of addiction, and links these theories historically to broader discussions about the causes of crime and the courts' role in solving social problems. Overall, she established connection on how scientific theories are fused with moral considerations in the name of an "enlightened" criminal justice approach to complex social problems.

Muketo *et al.* (2021) developed a SEIGWR model which is a mathematical model of drug abuse as an infectious disease with six compartments; susceptible, exposed, infected, guided and counseled, rehabilitated and recovered. The quantitative analysis of the study found out that the local stability existed when R_0 < 1. Numerical analysis of the drug abuse model determined that guidance and counseling affects the dynamics of the model and established that an increase in guidance and counseling leads to a decrease in the number of infections. The work suggested need for all the relevant stakeholders in the secondary schools to increase the rate at which guidance and counseling is offered in the schools that guidance and counseling would serve as a control strategy if given at an early age when the students are still in the primary schools. This work serves as an extension and an improvement to the existing work by the inclusion of a prosecution compartment and thereby leading to jailing before the abuser gets rehabilitation. A different mathematical concept is also employed (see Fig. 2). In the next section, the model of substance abuse as a disease will be developed and analyzed with the effect of prosecution facility in the compartment.

METHODOLOGY

This work will divide the societal populace into six compartments depending on their substance use status, determine the common abused substances and their supposed assumption behind them, formulation of the mathematical model of substance abuse with control strategies, establish the existence and positivity, use the Differential Transform Method to solve the model.

The sum total of the entire population, $N(t)$, takes form;

$$N(t) = S(t) + L(t) + H(t) + P(t) + R(t) + C(t)$$

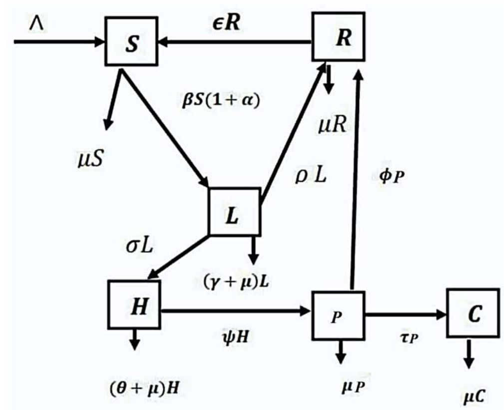

Figure 1: Model Diagram

$S \Rightarrow$ Susceptible Class
$L \Rightarrow$ Light Substance Users
$H \Rightarrow$ Heavy Substance Users
$P \Rightarrow$ Prosecution Class
$R \Rightarrow$ Rehab Compartment
$C \Rightarrow$ Punishment

$$\frac{dS(t)}{dt} = \Lambda + \epsilon R - \beta S(1 + \alpha L) - \mu S$$
$$\frac{dL(t)}{dt} = \beta S(1 + \alpha L) - (\sigma + \gamma + \mu + \rho)L$$
(1)
$$\frac{dH(t)}{dt} = \sigma L - (\psi + \theta + \mu)H$$
$$\frac{dP(t)}{dt} = \psi H - \mu P + (\phi - \tau)P$$
$$\frac{dR(t)}{dt} = \rho L + \phi P - (\epsilon + \mu)R$$
$$\frac{dC(t)}{dt} = \tau P - \mu C$$

The substance invariant region

The population size N can be determined by

$$N(t) = S(t) + L(t) + H(t) + P(t) + R(t) + C(t),$$

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The solution of the system remains positive at any point in time if the initial values of all the variables are positive.

Theorem 1
Consid$\Omega = \{S(t), L(t), H(t), P(t), R(t), C(t) \in \mathbb{R}^6, S(0) > 0, L(0) > 0, H(0) > 0, P(0) > 0, R(0) > 0, C(0) > 0\}$, then, the solution of $\{S(t), L(t), H(t), P(t), R(t), C(t)\}$ are positive for $t \geq 0$.

Proof :
Boundedness refers to the region in which solutions of the model or system is uniformly bounded in the proper subset $\Omega \subset \mathbb{R}_+^6$. Looking at the total population at any time (t):

$$N(t) = S(t) + L(t) + H(t) + P(t) + R(t) + C(t),$$

$$\frac{dN(t)}{dt} = \Lambda - \mu S,$$

There exists no Light and Heavy users(L and H), Prosecution(P), Recovery(R) and Punishment(C) in the absence of drug. Thus $L = 0, H = 0, P = 0, R = 0, C = 0$.
The equation thus becomes

$$\frac{dN}{dt} = \Lambda - \mu S.$$

If the total population N is equal to the number of susceptible S, it implies that $N = S$, such that,

$$\frac{dN}{dt} = \Lambda - \mu N.$$

On integrating both sides of the equation

$$\int \frac{dN}{\Lambda - \mu N} \leq \int dt$$

$$-\frac{In(\Lambda - \mu N)}{\mu} \leq t + C$$

$$In(\Lambda - \mu N) \leq t + C$$

$$In(\Lambda - \mu N) \geq -\mu(t + C)$$

$$(\Lambda - \mu N) \geq e^{-\mu(t+C)}$$

$$(\Lambda - \mu N) \geq e^{-\mu t} + e^{-\mu C}$$

where $e^{-\mu C} = K$. Then $\Lambda - \mu N \geq K e^{-\mu t}$. Utilizing the condition at $N(0) = S(0)$, $\Lambda - \mu N(0) = K$.

Therefore, $\Lambda - \mu N = (\Lambda - \mu N(0))e^{-\mu(t)}$. Further simplification yields

$$N \leq \frac{\Lambda}{\mu} - (\frac{\Lambda - \mu N(0)}{\mu})e^{-\mu(t)}.$$

As time tends to infinite, i.e. $t \to \infty$, then the side of population $N \to \frac{\Lambda}{\mu}$.
Deductively, $0 \leq N \leq \frac{\Lambda}{\mu}$ and $N(t) \leq \frac{\Lambda}{\mu}$. Hence, $\Omega = \{(S, L, H, P, R, C) \in \mathbb{R}_+^6 : S + L + H + P + R + C \leq \frac{\Lambda}{\mu}\}$.

Existence and Positivity of Solution

The below results guaranteed by the substance abuse model governed in equation (1) is well posed in a feasible region Ω

Lemma 1.2 Suppose the initial conditions be $\{S(0) > 0, L(0) > 0, H(0) > 0, P(0) > 0, R(0) > 0, C(0) > 0\} \in \Omega$
Then the solution set $\{S, L, H, P, R, C\}(t)$ of the model system is positive $\forall t > 0$, employing similar methods in (Gao and Hethcote, 2006)
Proof: The first equation of the model (1) gives

$$\frac{dS}{dt} = \Lambda + \epsilon R - [\mu + \beta(1 + \alpha L)]S$$
$$\geq -[\mu + \beta(1 + \alpha L)]S$$

$$\frac{dS}{dt} \geq -[\mu + \beta(1 + \alpha L)]S$$

Applying the variable separable approach

$$\frac{dS}{S} \geq -[\mu + \beta(1 + \alpha L)]dt$$

Applying integrals to both sides

$$\int \frac{dS}{S} \geq -\int [\mu + \beta(1 + \alpha L)]dt$$

$$\ln S(t) \geq -[\mu + \beta(1 + \alpha L)]t + C$$

$$S(t) \geq K\exp(-[\mu + \beta(1 + \alpha L)]t)$$

At time $t = 0$ and applying the initial conditions yield

$$S(0) \geq 0$$

In a likewise manner

$$L(t), \quad H(t), \quad J(t), \quad R(t) \ and \ C(t) > 0 \ \ \forall t \geq 0$$

Basic Reproduction Number of Substance Abuse R_0

This is the average number of secondary cases per infectious case in a

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population made up of both susceptible and non-susceptible hosts. Here, we term it expected number of new substance abusers that arose by initial abuser in a susceptible population. Thus, R_0 is seen as a Threshold term to determine the condition for abusers outbreak

Substance Abuse Basic Reproduction Number

Here, we will use the concepts of Next Generation Matrix (Shaibu etal, 2018) to establish the linear stability of the disease-free equilibrium. The basic reproduction number is the number of secondary infections produced by one infected person in a completely susceptible population. The reproduction number combines the biology of infections with the social and behavioral factors causing contact rates (Van den Driessche P., 2017). The basic reproduction number R_0 is defined as the spectral radius of the next generation matrix.

Definition: The spectral radius of a matrix X is defined as the maximum of the absolute values of the matrix

$$i.e.\quad \rho(X) = \sup\{|\lambda| : \lambda \in \rho(X)\}$$

with $\rho(X)$ representing the set of eigenvalues of the matrix X.

Substance Abuse Basic Reproduction Number by Next Generation Matrix

Theorem 1.2 Li (2002) stated that suppose F a non zero non negative matrix and V a non-singular M-matrix such that $F - V$ is irreducible. Then

$$R_0 = \rho(FV^{-1})$$

and R_0 is the reciprocal of the smallest positive root x of the polynomial equation **Proof:** From the assumptions,

$$FV^{-1}$$

is a non-zero non-negative matrix, and by Li etal.(2002) and Perron-Frobenius theory the principal submatrix corresponding to the non-zero rows of F is irreducible.

Thus

$$\lambda = \rho(FV^{-1}) > 0$$

is the largest positive root of the polynomial equation

$$\det(\lambda I - FV^{-1}) = 0$$

Since V is non-singular and $\lambda > 0$, the polynomial equation is equivalent to

$$\det(F\lambda^{-1} - V) = 0$$

(where $V - F\lambda^{-1}$ is a singular M-matrix) and λ^{-1} is the smallest positive root. The proof suffices from the definition of

$$R_0 = \rho(FV^{-1})$$

Next Generation Matrix

This is the method used to derive the basic reproduction number, for a compartmental model of the spread of substance abuse.

Deriving R_0 with the next generation matrix (Van den Driessche, 2017) is sometimes difficult whenever the dimension of the model is increasing. An option of solution to this problem is employing the graph-theoretic method (de-Camino-Beck etal, 2009) to derive the reproduction number R_0 in a simpler form.

Computing R_0, we first assume $f_i(x)$ the rate of appearance of new abusers in compartment i, and $v_i(x)$ the net reducing rate of substance abusers in compartment i due to substance abusers flow inside system of abusers compartments.

Supposing

$$F = (f_1, \cdots, f_{2n}) \quad and$$
$$V = (v_1, \cdots, v_{2n}),$$

Let partition the derivative

$$TF(E_0)$$
$$and \quad TV(E_0) \quad as$$

$$TF(E_0) = \begin{pmatrix} F & 0 \\ 0 & 0 \end{pmatrix},\; TV(E_0) = \begin{pmatrix} V & 0 \\ J_3 & J_4 \end{pmatrix}$$

with

$$F = \begin{pmatrix} \frac{\partial F_1}{\partial L} & \frac{\partial F_1}{\partial H} & \frac{\partial F_1}{\partial J} \\ \frac{\partial F_2}{\partial L} & \frac{\partial F_2}{\partial H} & \frac{\partial F_2}{\partial J} \\ \frac{\partial F_3}{\partial L} & \frac{\partial F_3}{\partial H} & \frac{\partial F_3}{\partial J} \end{pmatrix}(x_0)$$

$$V = \begin{pmatrix} \frac{\partial V_1}{\partial L} & \frac{\partial V_1}{\partial H} & \frac{\partial V_1}{\partial J} \\ \frac{\partial V_2}{\partial L} & \frac{\partial V_2}{\partial H} & \frac{\partial V_2}{\partial J} \\ \frac{\partial V_3}{\partial L} & \frac{\partial V_3}{\partial H} & \frac{\partial V_3}{\partial J} \end{pmatrix}(x_0)$$

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Suppose $X = (S,\ L,\ H,\ P,\ R,\ C,)^T$, thus F and V can be derived from the model equation; also with its Jacobian matrices at substance-free equilibrium.

Thus $R_0 = \Gamma(FV^{-1})$
with Γ, the spectral radius

Using the Next Generation Matrix, we can derive the threshold value for substance abuse.

We recall that set of D.Es for Model 1 is;

$$\frac{dS(t)}{dt} = \Lambda + \epsilon R - \beta S(1+\alpha L) - \mu S$$
$$\frac{dL(t)}{dt} = \beta S(1+\alpha L) - (\sigma+\gamma+\mu+\rho)L$$
$$\frac{dH(t)}{dt} = \sigma L - (\psi+\theta+\mu)H$$
$$\frac{dP(t)}{dt} = \psi H - \mu P + (\phi - \tau)P$$
$$\frac{dR(t)}{dt} = \rho L + \phi P - (\epsilon+\mu)R$$
$$\frac{dC(t)}{dt} = \tau P - \mu C$$

Setting the Jacobian matrix of the system of differential equations, we have

$$F(t) = \begin{bmatrix} \beta S(1+\alpha) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$V(t) = \begin{bmatrix} (\sigma+\gamma+\mu+\rho) & 0 & 0 \\ -\sigma & (\psi+\theta+\mu) & 0 \\ 0 & -\psi & (\tau-\phi) \end{bmatrix}$$

$$F(t) = \begin{bmatrix} \beta\ S(1+\alpha) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

suppose $Z = \frac{1}{V}$ i.e V^{-1}, thus

$$Z = \begin{bmatrix} (\sigma+\gamma+\mu+\rho)^{-1} & 0 & 0 \\ \frac{\sigma}{(\psi+\theta+\mu)(\sigma+\gamma+\mu+\rho)} & (\psi+\theta+\mu)^{-1} & 0 \\ \frac{\sigma\ \psi}{(\sigma+\gamma+\mu+\rho)(\psi+\theta+\mu)(-\tau+\phi)} & \frac{\psi}{(\psi+\theta+\mu)(-\tau+\phi)} & (\tau-\phi)^{-1} \end{bmatrix} \quad (3.1)$$

$$F \cdot V^{-1} = \begin{bmatrix} \frac{\beta\ S(1+\alpha)}{\sigma+\gamma+\mu+\rho} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_0 = \Gamma(F \cdot V^{-1})$$

$$R_0 = \frac{\beta\ S(1+\alpha)}{\sigma+\gamma+\mu+\rho} \quad (3.2)$$

Thus;

$$\begin{aligned} R_0 &= \Gamma(F \cdot V^{-1}) \\ &= \frac{\beta S(1+\alpha)}{\sigma+\gamma+\mu+\rho} \end{aligned}$$

We deduce that; the substance-free equilibrium $E_0\left(\frac{\Lambda}{\mu},\ 0,\ 0,\ 0,\ 0,\ 0\right)$ associated with the mode set of D.Es is locally asymptotically stable if

$R_0 < 1$

i.e. substance abusers get apprehended to prosecution or rehabilitation.

E_0 becomes unstable iff

$R_0 > 1$

i.e. the act of abuse of substance is prevalent.

GRAPHICAL ILLUSTRATION AND NUMERICAL SOLUTION

We present the numerical Simulation of the model using DTM that is done by using some set of parameters from the literature as well as assumed values.

The following initial conditions are considered $S(0) = 0.82, L(0) = 0.08, H(0) = 0.06, J(0) = 0.02, R(0) = 0.6$ and C(0)=0.2.

The DTM is demonstrated against the Runge-Kutta o 4th order for the solution. Figure 1-6 shows the combined plots of the two solutions by DTM and Rk4.

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Parameter	Description
Λ	rate of recruitiment into susceptive class
α	rate at which individuals initiate colleagues who uses substance
β	rate at which individuals interact in the population
μ	natural death rate
σ	rate at which light substance users abuse them (progressive rate)
ρ	rate of natural recovery of light users
γ	mortal rate from light substance usage
θ	moral rate from heavy substance usage
ψ	rate at which heavy substance users get prosecuted
\phi	rates at which heavy users recover after prosecute
τ	rate at which heavy users get punishment after prosecution
ϵ	rate of re-absorption after rehab

Table 1: Parameter Values

Parameter	Value	Unit	Source
Λ	1500	$\frac{people}{year}$	Assumed
α	0.7	$\frac{1}{year}$	J.Li & M.Ma (2018)
β	0.8	$\frac{1}{year}$	J.Li & M.Ma (2018)
μ	0.02	$\frac{1}{year}$	J.Li & M.Ma (2018)
σ	0.03	$\frac{1}{year}$	J.Li & M.Ma (2018)
ρ	0.005	$\frac{1}{year}$	Analysis (Assumed)
γ	0.2	$\frac{1}{year}$	J.Li & M.Ma (2018)
θ	0.4	$\frac{1}{year}$	Assured
ψ	0.421	$\frac{1}{year}$	J.Li & M.Ma (2018)
ϕ	0.25	$\frac{1}{year}$	J.Li & M.Ma (2018), i.e $1-\phi=\tau$
τ	0.75	$\frac{1}{year}$	Analysis (Assumed)
ϵ	0.2	$\frac{1}{year}$	Analysis (Assumed)

Table 2: Initial Conditions for each point for variables and source

Variables	Value	Source
$S(t)$	500	UNODC (2019)
$L(t)$	250	,,
$H(t)$	150	,,
$P(t)$	100	,,
$R(t)$	120	,,
$C(t)$	20	,,

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Differential Transform Method (DTM)

According to Adesoye $et\ al$ (2018), Peter $et\ al$ (2018), , Peter and Ibrahim (2017), the DTM can be derived from Transform Series semi-method of solving both linear and non-linear of ordinary differential equation (ODE) to obtain approximate series of solution. The differential transform of the kth derivative of a function $f(\alpha)$ is given by

$$f(\alpha) = \frac{1}{\alpha!}\left(\frac{d^k f(t)}{dt^k}\right)_{t=0}$$

(5.0) Where $f(t)$ is the original function and $f(k)$ the transformed function. The inverse transformation is defined as follows

$$f(t) = \sum_{k=0}^{\infty} t^k f(\alpha) \quad (5.1)$$

Setting (5.0) in (5.1) gives

$$f(t) = \sum_{k=0}^{\infty}\left[\frac{1}{\alpha}\left(\frac{d^\alpha f(t)}{dt^k}\right)\right] \qquad (5.2)$$

The DTM is derived from Taylor's series expansion and does not find numerical value for derivatives.

Basic Rules of DTM

Suppose we set $(t), g(t)$ and $h(t)$ arbitrary functions , then the rules for manipulating DTM are here underlisted.

- : If $f(t) = g(t) \pm h(t)$, then $f(k) = G(k) \pm H(k)$
- : If $f(t)$ =a g(t), $then$f(k)=aG(k) for constant a
- : If $f(t) = \frac{d}{dx} g(t)$, then $f(k) = (k+1)G(k+1)$
- ; If $f(t) = \frac{d^2}{dx} g(t)$, then $f(k) = (k+1)(k+2)G(k+1)$
- ; If $f(t) = \frac{d^n}{dt^n} g(t)$, then $f(k) = \frac{k!}{k+1}$
- ; If $f(t) = g(t)h(t)$, then $f(\alpha) = \sum_{i=0}^{\alpha} G(i)H(\alpha-1)$
- ; If $f(t) = \frac{g(t)}{h(t)}$, then $f(k) = \sum_{i=0}^{\alpha} \frac{G(i)}{H(\alpha-1)}$
- If $f(t) = t^a$, then $f(k) = \delta(k-a)$ with $\delta(k-a) = \begin{cases} 1, & k = a \\ 0, & k \neq a \end{cases}$
- : If e^{at}, then $f(k) = \frac{a^k}{k!}$
- : If $f(t) = (1+t)^a$, then $f(k) = \frac{a(a-1)(a-k+1)}{k-1}$
- : If $f(t) = \sin(at-b)$, then $f(k) = \frac{a^k}{k!}\sin\left(\frac{\pi}{2}\alpha + b\right)$
- : If $f(t) = \cos(at-b)$, then $f(k) = \frac{a^k}{k!}\cos\left(\frac{\pi}{2}\alpha + b\right)$
- : If $f(t) = \sinh(at)$, then $f(k) = \frac{a^k}{2k!}(a^k - (-a)^k)$
- : If $f(t) = \cosh(at)$, then $f(k) = \frac{a^k}{2k!}(a^k + (-a)^k)$

Application of DTM

Here, DTM shall be used to solve the model equation using value of parameters from existing literature, Li & Ma (2018). UNODC (2019-2021). In this section we solve the model equation by applying DTM. Recall from

$$\frac{dS}{dt} = \lambda + \epsilon R - \beta S(1+\alpha L) - \mu S$$
$$\frac{dL}{dt} = \beta S(1+\alpha L) - (\sigma + \gamma + \mu + \rho)L$$
$$\frac{dH}{dt} = \sigma L - (\psi + \theta + \mu)H$$
$$\frac{dP}{dt} = \psi H - \mu P + (\phi - \tau)P$$
$$\frac{dR}{dt} = \rho L + \psi P - (\epsilon + \mu)R$$
$$\frac{dC}{dt} = \tau P - \mu c$$

Applying the DTM properties in the table 1 to (2) . From the first equation, we have

$$\frac{ds}{dt} = \lambda + \epsilon R - \beta S(1+\alpha L)\mu S$$

Thus transformed to

$$(k+1)S(k+1) = [\lambda G(k,0) - \beta S(k)S(L)\sum_{l=0}^{k} \alpha\beta(L-l) - \mu S(k)]$$

Dividing through by $(k+1)$

$$S(k+1) = \frac{1}{k+1}\left[\lambda a(k,0) - BS(k) - S(l)\sum_{l=0}^{k} \alpha\beta(L-L)\mu S(0)\right] \qquad (5.3)$$

Next, we apply DTM to second equation of ()

$$(k+1)L(k+1) = S(\ell)\sum_{l=0}^{k} \alpha\beta(L-\ell) - P_1 L(k)$$

Where $P_1 = (\sigma + \gamma + \mu + e)$

Dividing through by $k+1$

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$L(k+1) = \frac{1}{k+1}\left(S(\ell)\sum_{l=0}^{k} \alpha\beta(L-\ell) - P_1 L(k)\right)$

following the same procedure, the third equation of (2) is transformed to

$H(k+1) = \frac{1}{k+1}[\sigma L(k) - P_2 H(k)]$

Where $P_2 = (\psi + \theta + \mu)$

The 4 th equations of (2) is transformed into

$P(k+1) = \frac{1}{k+1}[\psi H(k) - P_3]$

where $P_3 = (\phi + \epsilon + \mu)$ Subsequently, the 5th equation of () transformed into

$R(k+1) = \frac{1}{k+1}[PL(k) + \phi J(k) - P_4 R(k)]$

Where $P_4 = (\epsilon + \mu)$ and

$C(k+1) = \frac{1}{k+1}[\tau J(k) - \mu(k)]$

Hence, solving the model upto 5th order, we obtain the solution for each of the variables

$S(t) = \sum_{l=0}^{k} S(k)t^k = 0.82 + 1.12462t - 0.018293t^2 + 0.00194t^3 + 2.62432t^4$
$-3.2t2428t^5 + \cdots$

$L(t) = \sum_{l=0}^{k} L(k)t^k = 0.08 - 0.01372t + 0.001342t^2 + 0.032644t^3$
$+0.66848436t^4 + 0.00563t^5 + \cdots$

$H(t) = \sum_{l=0}^{k} H(k)t^k = 0.06 - 0.00460927t - 0.0004392t^2 + 0.00023476t^3$
$+2.467286t^4 - 4.762632t^5 + \cdots$

$P(t) = \sum_{l=0} \sum_{l=0}^{k} t^k = 0.02 - 0.001723t - 0.000437t^2 - 0.332642t^3$
$+4.2736t^4 + 6.7326134t^5$

$R(t) = \sum_{l=0}^{k} R(k)t^k = 0.6 + 0.59626t - 0.043723t^2 = 0.00032648t^3$
$-0.0004672t^4 + 0.0002467328t^5$

$C(t) = \sum_{l=0}^{k} C(k)t^k = 0.2 - 0.042769t + 0.0061624t^3 - 0.00005932132t^3$
$+0.0000344700t^4 + 0.00032267328t^3 + \cdots$

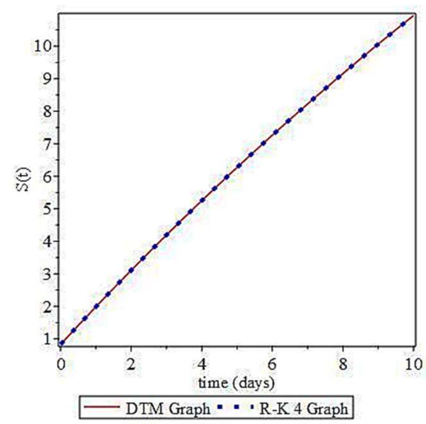

Figure 2: Graph outlining the dynamics of the Susceptible class $S(t)$ against time t for both DTM and R-K order 4

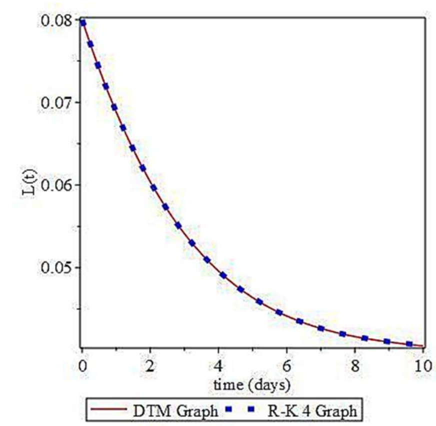

Figure 3: Graph outlining the dynamics of the Light user class $L(t)$ against time t for both DTM and R-K order 4

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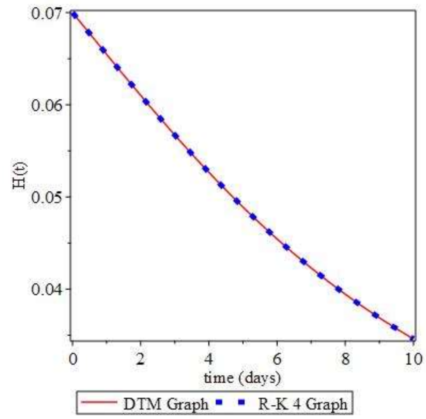

Figure 4: Graph outlining the dynamics of the Heavy user class $H(t)$ against time t for both DTM and R-K order 4

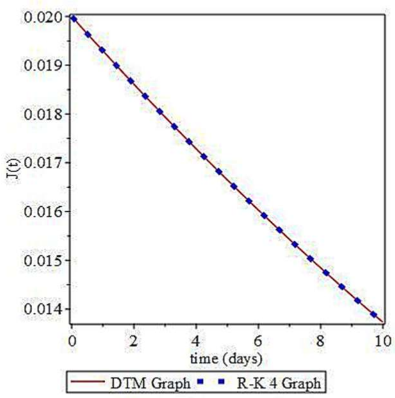

Figure 5: Graph outlining the dynamics of the Prosecuted class $P(t)$ against time t for both DTM and R-K order 4

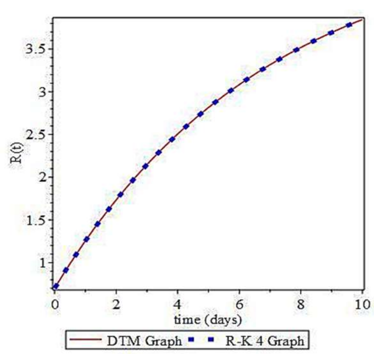

Figure 6: Graph outlining the dynamics of the Rehabilitated class $R(t)$ against time t for both DTM and R-K order 4

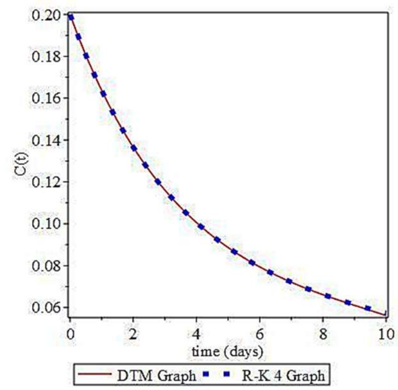

Figure 7: Graph outlining the dynamics of the class of Punishment $C(t)$ against time t for both DTM and R-K order 4

RESULTS AND DISCUSSION

This research centers on a unique method of solution - the Differential Transform Method(DTM) which reveals on analysis that; the Prosecution class $P(t)$ and also the Punishment $C(t)$ class significantly diminishes the abuse of substance, over intervals of time(see Fig.above) We also considered the approach of deriving the basic reproduction number R_0, known as the Next Generation Matrix method for modelling substance abuse. This approach is equivalent to Gaussian elimination digraph reduction method, and since the rules of digraph reduction method does not take the path of direct calculation of matrix inverse or determinants, this approach can be practical for larger systems.

CONCLUSION

In this research, the next generation matrix method was employed to compute the threshold number for the use and abuse of substance, R_0. The use and abuse of substances would continue to spread in the population if the substance abuse number is greater than one and the abuse of substances would die out of the system if the abuse number is less than one (1). The Differential Transform Method was employed as numerical solution of the system hence plot-

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visibility of each compartment behaviours as seen above. In a more developing studies, an optimal control approach to the spread of substance abusers population is examined using apprehension, prosecution and/or jailing facilities as a strategy for control.

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