

A Study on Inverted Exponentiated Generalized Skew Student T Distribution: Definition, Properties and Estimation with Application to Financial Data

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ABSTRACT

This article aims to derive a new generalization of the exponentiated generalized family of distribution and with it a sub model called the Inverted Exponentiated Generalized Skew Student t distribution (IEGSS t D) for modeling financial data. Some statistical properties of the IEGSS t D are derived, such as quantile function, raw moments, entropy, reliability function, and hazard function. Parameter estimates using maximum likelihood estimation were consistent and the behavior of the estimates with increase in sample size was studied using Montecarlo simulation. In a practical application to the simple returns of two financial idiocies, the Guaranty Trust Holding Company PLC (GTCO) and Nigeria Breweries (NB). The flexibility of the Inverted Exponentiated Generalized Skew Student t distribution demonstrated by application to the simple returns revealed that the Inverted Exponentiated Generalized Skew Student t model yielded a better fit.

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INTRODUCTION

Parametric statistical analysis intricately relies on empirical probability models also known as probability distributions. However, a prevailing reality across diverse fields such as finance, environmental science, biomedical sciences, engineering and others indicates that classical distributions (like the skew student t) are more of an anomaly when describing several data and data generation processes. This recognition underscores a compelling necessity for the hybridization of classical probability models and in addressing this intricate challenge (Mohammed et al 2020, Mathew et al., 2024). Significant scholarly efforts have been dedicated to formulating hybrid classes of probability distributions and enhancing statistical methodologies. Among such methodologies is the addition of one or more parameters to classical probability distributions in an attempt to add flexibility (David et al., 2024).

Consider Cordeiro et al. (2013) derived the Exponentiated Generalized-G family of distributions which have been utilized by several scholars to modify classical models. Lemonte et al., (2013) proposed a new member of the Exponentiated distribution called the Exponentiated Kumaraswamy distribution, Hamdy (2014), proposed the Exponentiated Lomax distribution, Ashour and Eltehiwy (2015), the authors presented a new distribution called the Exponentiated power Lindley distribution, Fattah et al., (2017) proposed a new member of the Exponentiated family of distributions called the Exponentiated transmuted Weibull distribution, Fatima and Ahmad (2017) derived a positively skewed distribution called the Exponentiated Inverted Exponential Distribution, Dikko and Agboola (2017a) derived a two-parameter distribution called the Exponentiated skewed Student- t distribution, Dikko, and Agboola (2017b) studied the three-parameter Exponentiated Generalized Skewed Student- t

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Distribution, Hassan and Abd-Allah (2018) proposed a new class of distributions called the Exponentiated Weibull-Lomax distribution, Rather and Subramanian (2018) discussed on the Exponentiated Mukherjee-Islam distribution and derived some of the properties, Muhammad and Liu (2019) proposed a new class of distributions called the Exponentiated generalized half-logistic distribution, Al-Omari et al., (2019) proposed a new continuous distribution called the exponentiated new Weibull-Pareto (ENWP) distribution, Sunday and Oluwafunmilola (2021) study Standardized Exponentiated Gumbel, Rather et al., (2022) worked on the Exponentiated Ailamujia distribution amongst others.

In this article we use Exponentiated Generalized-G family approach suggested by Cordeiro et al. (2013) to define a new family of generalized distribution called the Inverted Exponentiated Generalized-G family of model and derive a model called the Inverted Exponentiated Generalized Skew Student t distribution and study its performance in modelling financial data.

Inverted Exponentiated Generalized Family of Distribution

Cordeiro *et al.* (2013) defined the CDF generator of the Exponentiated Generalized Distribution as;

$$F(y) = \left[1 - (1 - G(y))^u \right]^v \quad (1)$$

Where $G(y)$ is the CDF of an arbitrary baseline distribution. If we let $F(y) = Y$ and $G(y) = X$ and substitute into (1), we will have that;

$$Y = \left(1 - (1 - X)^u \right)^v \quad (2)$$

Thus, inverting the generator implies finding the inverse of the function (i.e., making X subject of the equation (2))

$$\left(1 - Y^{\frac{1}{v}} \right)^{\frac{1}{u}} = 1 - X$$

Undoing the substitution for $F(y) = Y$ and $G(y) = X$, we have that

$$G(y) = 1 - \left(1 - F(y)^{\frac{1}{v}} \right)^{\frac{1}{u}} \quad (3)$$

Which is the therefore, the CDF generator of the Inverted Exponentiated Generalized family of Distribution is presented in equation(3).

Inverted Exponentiated Generalized Skewed Student-t Distribution (IESS t D)

The CDF of the Jones and Faddy, (2007) Skewed Student-t Distribution is given as;

$$F(y) = \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]; -\infty < y < \infty \quad (4)$$

Substituting equation (4) into (3), the CDF of the Inverted Exponentiated Generalized Skewed Student-t Distribution (IEGS t D) is obtained as;

$$G(y) = 1 - \left(1 - \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}} \right)^{\frac{1}{u}} \quad (5)$$

The PDF can be obtained from the CDF by differentiation. Thus, the PDF of IEGSStD can be expressed as;

$$PDF = g(y) = \frac{dG(y)}{dy} = \frac{d}{dy} \left\{ 1 - \left(1 - \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}} \right)^{\frac{1}{u}} \right\}$$

Therefore, $y \sim IEGSStD(u, v, \lambda)$ if its PDF is given by;

$$g(y) = \frac{\lambda}{2uv} \left(1 - \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}} \right)^{\frac{1-u}{u}} \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1-v}{v}} \frac{1}{(\lambda + y^2)^{\frac{3}{2}}}; -\infty < y < \infty \quad (6)$$

Inverted Exponentiated Generalized Skewed Student-t Distribution (IEGSStD)

Let y be a random variable from an arbitrary baseline distribution. Hence, y follows the proposed Inverted Exponentiated Generalized Skewed student-t distribution (IEGSStD) if its CDF and PDF are given by (5) and (6).

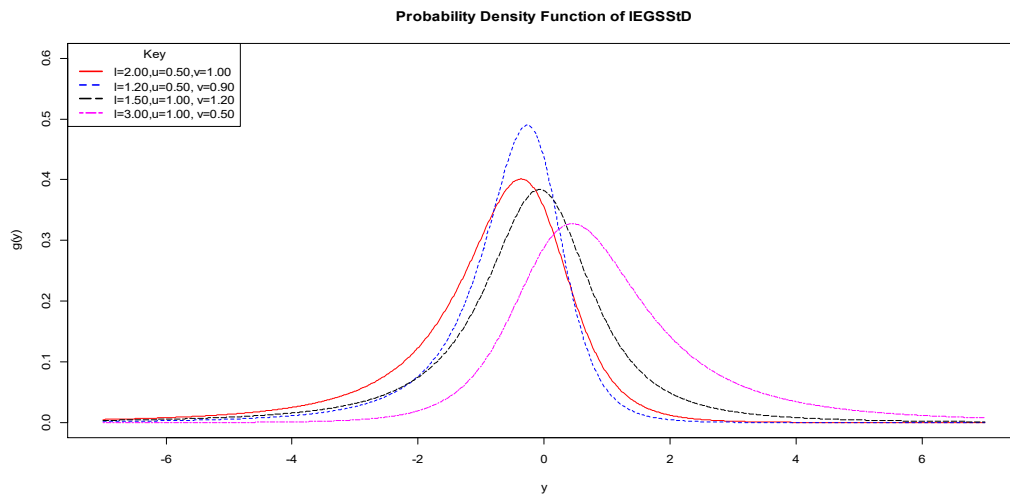


Figure 1: Probability Density Function plot of IEGSStD at different parameter values where $l = \lambda$

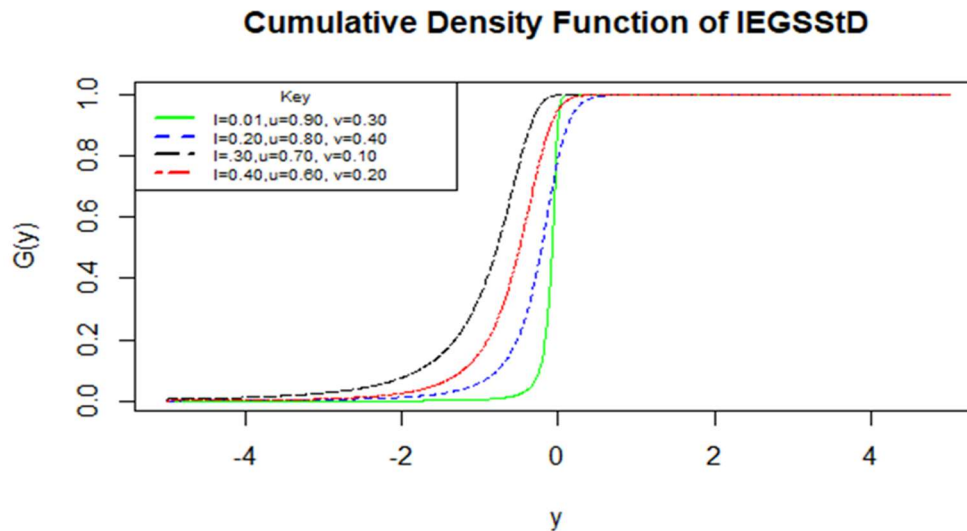


Figure 2: Cumulative Density Function plot of IEGSStD at different parameter values where $l = \lambda$

The IEGSStD's PDF plot, shown in Figure 1, provided important information about the distribution's form and probability of various values. With a longer tail on one side and a shorter, steeper tail on the other, there is clear asymmetry. The curve's apex represents the mean, or the most likely value, but the IEGSStD's heavier tails point to a higher likelihood of extreme values. The CDF plot of the IEGSStD is shown in Figure 2. The gradual increase from 0 to 1 (on the $G(y)$ axis) indicates the increasing likelihood of higher values, with any changes in slope corresponding to critical points. The central tendency of the distribution is shown by the

distribution's median, which is the point at which the CDF approaches 0.5.

Statistical Asymptotic Behaviour

These sections give the asymptotic behaviour of our proposed distributions. In order to investigate the behaviour of the proposed distributions, we proceed thus,

Asymptotic behavior of IEGSStD

The limit of Inverted Exponentiated Generalized Skewed student-t density function as $y \rightarrow \infty$ is 0 and the limit as $y \rightarrow -\infty, \infty$ is 0.

$$\lim_{y \rightarrow \infty} \left[\frac{\lambda}{2uv} \left(1 - \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}} \right)^{\frac{1}{u}-1} \times \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}-1} \frac{1}{(\lambda + y^2)^{\frac{3}{2}}} \right] \quad (7)$$

Since the $\lim_{y \rightarrow \infty} \left[\frac{1}{(\lambda + y^2)^{\frac{3}{2}}} \right] = 0$, then it implies that

$$\lim_{y \rightarrow \infty} \left[\frac{\lambda}{2uv} \right] \times \lim_{y \rightarrow \infty} \left[\left(1 - \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}} \right)^{\frac{1}{u}-1} \right] \times \lim_{y \rightarrow \infty} \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}-1} \times 0 = 0 \quad (8)$$

Also, as $y \rightarrow (-\infty)$

Since the $\lim_{y \rightarrow -\infty} \left[\frac{1}{(\lambda + y^2)^{\frac{3}{2}}} \right] = 0$, then it implies that

$$\lim_{y \rightarrow -\infty} \left[\frac{\lambda}{2uv} \right] \times \lim_{y \rightarrow -\infty} \left[\left(1 - \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}} \right)^{\frac{1}{u}-1} \right] \times \lim_{y \rightarrow -\infty} \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}-1} \times 0 = 0 \quad (9)$$

As observed in both cases, as $y \rightarrow (-\infty, \infty) = \lim_{y \rightarrow [-\infty, \infty]} [g(y)] = 0..$

Quantile Function for Proposed Distributions

These sections present the Quantile function of the proposed distribution. By definition, the Quantile function is given by;

$$\Phi(\mu) = G^{-1}(\mu) \quad (10)$$

Quantile function of IEGSStD

The proposed Inverted Exponentiated Generalized Skewed student-t distribution (IEGSStD) CDF is given in

(5). Given that $\Phi(\mu) = G^{-1}(\mu)$; Let $\mu = G(y)$, then

$$\mu = 1 - \left(1 - \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}} \right)^{\frac{1}{u}} \quad (11)$$

$$y^2 = 4\lambda \left(\left(1 - (1 - \mu)^u \right)^v - \frac{1}{2} \right)^2 + 4y^2 \left(\left(1 - (1 - \mu)^u \right)^v - \frac{1}{2} \right)^2$$

$$y = \pm \sqrt{\frac{4\lambda \left(\left(1 - (1 - \mu)^u \right)^v - \frac{1}{2} \right)^2}{\left(1 - \left(\left(1 - (1 - \mu)^u \right)^v - \frac{1}{2} \right)^2 \right)}} = y \quad (12)$$

Binomial Expansion of the Proposed Probability Distributions

In this subsection, we expand the PDF and CDF of the proposed distributions for error innovation using the binomial series expansion in order to easily compute the Moment of the distributions. Thus, if “a” is a positive real non-integer and $|y| \leq 1$, we can consider the power series expansion given as;

$$(1-y)^{b-1} = \sum_{i=0}^{\infty} b_i y^i \quad (13)$$

$$\text{where } b_i = (-1)^i \binom{b-1}{i}$$

$$\text{and the sum of the binomial series given by } (a+b)^n = \sum_{i=0}^{\infty} \binom{n}{i} (a^{n-i} b^i) \quad (14)$$

Binomial expansion of PDF of IEGSSStD

The PDF IEGSSStD in equation (6) can be represented as a sum of binomial series as follows.

$$g(y) = \frac{\lambda}{2uv} \left(1 - \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda+y^2}} \right]^{\frac{1}{v}} \right)^{\frac{1}{u}-1} \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda+y^2}} \right]^{\frac{1}{v}-1} \frac{1}{(\lambda+y^2)^{\frac{3}{2}}} : -\infty < y < \infty \quad (15)$$

$$g(y) = \frac{\lambda}{2uv} \left(1 - \left(\frac{1}{2} \right)^{\frac{1}{v}} \left[1 + \frac{y}{\sqrt{\lambda+y^2}} \right]^{\frac{1}{v}} \right)^{\frac{1}{u}-1} \left(\frac{1}{2} \right)^{\frac{1}{v}-1} \left[1 + \frac{y}{\sqrt{\lambda+y^2}} \right]^{\frac{1}{v}-1} \frac{1}{(\lambda+y^2)^{\frac{3}{2}}}$$

$$\left(1 - \left(\frac{1}{2} \right)^{\frac{1}{v}} \left[1 + \frac{y}{\sqrt{\lambda+y^2}} \right]^{\frac{1}{v}} \right)^{\frac{1}{u}-1} = \sum_{i=0}^{\infty} \binom{\frac{1}{u}-1}{i} (-1)^i \left(\frac{1}{2} \right)^{\frac{i}{v}} \left[1 + \frac{y}{\sqrt{\lambda+y^2}} \right]^{\frac{i}{v}} \quad (16)$$

$$g(y) = \frac{\lambda}{2uv} \left(\frac{1}{2} \right)^{\frac{1-v}{v} + \frac{i}{v}} \sum_{i=0}^{\infty} \binom{\frac{1}{u}-1}{i} (-1)^i \left[1 + \frac{y}{\sqrt{\lambda+y^2}} \right]^{\frac{i+1-v}{v}} \frac{1}{(\lambda+y^2)^{\frac{3}{2}}}$$

$$g(y) = \frac{\lambda}{2uv} \left(\frac{1}{2} \right)^{\frac{1-v}{v} + \frac{i}{v}} \sum_{i=0}^{\infty} \binom{\frac{1}{u}-1}{i} (-1)^i \left[1 + \frac{y}{\sqrt{\lambda+y^2}} \right]^{\frac{i+1-v}{v}} \frac{1}{(\lambda+y^2)^{\frac{3}{2}}}$$

$$\left[1 + \frac{y}{\sqrt{\lambda + y^2}}\right]^{\frac{i+1-v}{v}} = \sum_{j=0}^{\frac{i+1-v}{v}} \binom{\frac{i+1-v}{v}}{j} (-1)^{\frac{i+1-v}{v}-j} \left[\frac{y}{\sqrt{\lambda + y^2}}\right]^j \quad (17)$$

$$g(y) = \frac{\lambda}{2uv} h_{i,j} y^j \frac{1}{(\lambda + y^2)^{\frac{j+3}{2}}} = \frac{\lambda}{2uv} h_{i,j} y^j \frac{1}{(\lambda + y^2)^{\frac{j+3}{2}}}$$

$$g(y) = h_{i,j} y^j \frac{1}{(\lambda + y^2)^{\frac{j+3}{2}}} \quad (18)$$

$$\text{where } h_{i,j} = \frac{\lambda}{2uv} \left(\frac{1}{2}\right)^{\frac{1-v+i}{v}} \sum_{i=0}^{\infty} \binom{\frac{1-u}{u}}{i} (-1)^i \sum_{j=0}^{\frac{i+1-v}{v}} \binom{\frac{i+1-v}{v}}{j} (-1)^{\frac{i+1-v}{v}-j}$$

Binomial expansion of CDF of IEGSStD

Using (13) and (14), the CDF of IEGSStD in equation (5) can be represented as a sum of series as follows.

$$\left(1 - \left(\frac{1}{2}\right)^{\frac{1}{v}} \left[1 + \frac{y}{\sqrt{\lambda + y^2}}\right]^{\frac{1}{v}}\right)^{\frac{1}{u}} = \sum_{i=0}^{\infty} \binom{\frac{1}{u}}{i} (-1)^i \left(\frac{1}{2}\right)^{\frac{i}{v}} \left[1 + \frac{y}{\sqrt{\lambda + y^2}}\right]^{\frac{i}{v}} \quad (19)$$

$$\left[1 + \frac{y}{\sqrt{\lambda + y^2}}\right]^{\frac{i}{v}} = \sum_{l=0}^{\frac{i}{v}} \binom{\frac{i}{v}}{l} (-1)^{\frac{i}{v}-l} \left(\frac{y}{\sqrt{\lambda + y^2}}\right)^l$$

$$G(y) = 1 - H \left(\frac{y}{\sqrt{\lambda + y^2}}\right)^l = 1 - Hy^l (\lambda + y^2)^{-\frac{l}{2}} \quad (20)$$

$$\text{where } H = \sum_{i=0}^{\infty} \binom{\frac{1}{u}}{i} (-1)^i \left(\frac{1}{2}\right)^{\frac{i}{v}} \sum_{l=0}^{\frac{i}{v}} \binom{\frac{i}{v}}{l} (-1)^l \left(\frac{1}{2}\right)^{\frac{l}{v}}$$

Moment Function

In this section, the moment of the IEGSStD will be derived using the sum of the binomial series for the respective PDF in (18). By definition, the r^{th} moment of a continuous random variable y from a valid probability distribution is given by;

$$u_r = E[y^r] = \int_{-\infty}^{\infty} y^r f(y) dy \quad (21)$$

On substituting (18) into (21), we have that

$$u_r = \int_{-\infty}^{\infty} y^r h_{i,j} y^j \frac{1}{(\lambda + y^2)^{\frac{j+3}{2}}} dy \quad (22)$$

$$u_r = 2h_{i,j} \lambda^{-\left(\frac{3+i}{2}\right)} \int_0^{\infty} y^{j+r} \left[1 + \frac{y^2}{\lambda}\right]^{-\left(\frac{3+i}{2}\right)} dy \quad (23)$$

By transformation, let $q = \frac{y^2}{\lambda}; q\lambda = y^2; y = (q\lambda)^{\frac{1}{2}}; \frac{dq}{dy} = \frac{2}{\lambda} y; dy = \frac{\lambda}{2y} dq$ (24)

Substitute (24) into (23). Thus

$$u_r = h_{i,j} \lambda^{-\left(\frac{3+i}{2}\right)} \int_{-\infty}^{\infty} y^{j+r} [1+q]^{-\left(\frac{3+i}{2}\right)} \frac{\lambda}{2(q\lambda)^{\frac{1}{2}}} dq \quad (25)$$

$$u_r = \frac{2}{2} h_{i,j} \lambda^{-\left(\frac{3+i}{2}\right)} \int_0^{\infty} (q\lambda)^{\frac{1}{2}(j+r)} [1+q]^{-\left(\frac{3+i}{2}\right)} \frac{\lambda^{1-\frac{1}{2}}}{q^{\frac{1}{2}}} dq$$

$$u_r = h_{i,j} \lambda^{-\left(\frac{3+i}{2}\right) + \frac{1}{2} + \frac{1}{2}(j+r)} \int_0^{\infty} q^{\frac{1}{2}(j+r) - \frac{1}{2}} [1+q]^{-\left(\frac{3+i}{2}\right)} dq$$

$$u_r = h_{i,j} \lambda^{\frac{-2-i+j+r}{2}} \int_0^{\infty} \frac{q^{\frac{j+r+1}{2}-1}}{[1+q]^{\frac{3+i-2}{2}+1}} dq \quad (26)$$

Recall that; $B[p, q] = \int_0^{\infty} \frac{u^{p-1}}{(1+u)^{p+q}} du$ (27)

Therefore, on transforming (26) into the form in (27), the moment of the IEGSStD is given as;

$$u_r = h_{i,j} \lambda^{\frac{-2-i+j+r}{2}} B\left[\frac{j+r+1}{2}, \frac{1+r}{2}\right] \quad (28)$$

Characteristic Functions (CF)

In this section, the characteristic function of the distribution will be derived by using the sum of the binomial series of the PDF of the distribution presented in (18). The characteristic function for a random variable y with PDF $g(y)$ is defined for $y \in R$ as;

$$\varphi_y(t) = E[e^{itz_t}] = \int_{-\infty}^{\infty} e^{ity} g(y) dy \quad (29)$$

Characteristic function of IEGSStD

To obtain the characteristic function for IEGSStD, substitute equation (18) into (29). on doing that, the following is obtained:

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$$\varphi_y(t) = E[e^{ity}] = \int_{-\infty}^{\infty} e^{ity} h_{i,j} y^j (\lambda + y^2)^{-\left(\frac{j+3}{2}\right)} dy$$

$$\varphi_y(t) = \left(\sum_0^{\infty} \frac{(it)^r}{r!} \right) \int_{-\infty}^{\infty} y^r h_{i,j} y^j (\lambda + y^2)^{-\left(\frac{j+3}{2}\right)} dy \quad (30)$$

$$= \int_{-\infty}^{\infty} \left(\sum_0^{\infty} \frac{(it)^r}{r!} y^r \right) h_{i,j} y^j (\lambda + y^2)^{-\left(\frac{j+3}{2}\right)} dy \quad (31)$$

$$\varphi_y(t) = \left(\sum_0^{\infty} \frac{(it)^r}{r!} \right) u_{rg} \quad (32)$$

$$\text{Where; } u_r = h_{i,j} \lambda^{\frac{-2-i+j+r}{2}} B\left[\frac{j+r+1}{2}, \frac{1+r}{2}\right] \text{ from equation (28).}$$

Order Statistic

The c^{th} order statistic of a random variable is given as;

$$f_{(c,n)}(y) = \frac{n!}{(c-1)!(n-c)!} g(y) [G(y)]^{c-1} [1-G(y)]^{n-c} \quad (33)$$

Expanding $[1-G(y)]^{n-c}$ using the Binomial theorem, we have

$$[1-G(y)]^{n-c} = \sum_{p=0}^{\infty} \binom{n-c}{p} (-1)^p [G(y)]^p \quad (34)$$

Therefore, equation (33) becomes;

$$f_{(c,n)}(y) = \sum_{p=0}^{\infty} \binom{n-c}{p} \frac{n!}{(c-1)!(n-c)!} (-1)^p g(y) [G(y)]^{p+c-1} \quad (35)$$

To obtain the c^{th} order statistic of the IEGSSStD, we make the substitution (18) for $g(y)$ and (20) for $G(y)$ in equation (35). Therefore,

$$f_{(c,n)IEG}(y) = \sum_{p=0}^{\infty} \binom{n-c}{p} \frac{n!}{(c-1)!(n-c)!} (-1)^p h_{i,j} y^j (\lambda + y^2)^{-\frac{j+3}{2}} \left[1 - Hy^l (\lambda + y^2)^{-\frac{l}{2}} \right]^{p+c-1} \quad (36)$$

$$\left[1 - Hy^l (\lambda + y^2)^{-\frac{l}{2}} \right]^{p+c-1} = \sum_{m=0}^{\infty} \binom{p+c-1}{m} (-1)^m \left[Hy^l (\lambda + y^2)^{-\frac{l}{2}} \right]^m = \psi H^m y^{lm} (\lambda + y^2)^{-\frac{lm}{2}}$$

$$\text{where; } \psi = \sum_{m=0}^{\infty} \binom{p+c-1}{m} (-1)^m$$

Thus, the c^{th} order statistic for the IEGSSStD is given by,

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$$f_{(c,n)IEG}(y) = \sum_{p=0}^{\infty} \binom{n-c}{p} \frac{n!}{(c-1)!(n-c)!} (-1)^p h_{i,j} [\psi H^m]^{p+c-1} y^{j+lm(p+c-1)} (\lambda + y^2)^{-\frac{j+3}{2} - \frac{lm(p+c-1)}{2}}$$

$$f_{(c,n)IEG}(y) = h_{(c,n)IEG} y^{j+lm(p+c-1)} (\lambda + y^2)^{-\frac{j+3}{2} - \frac{lm(p+c-1)}{2}} \quad (37)$$

Where; $h_{(c,n)IEG} = \sum_{p=0}^{\infty} \binom{n-c}{p} \frac{n!}{(c-1)!(n-c)!} (-1)^p h_{i,j} [\psi H^m]^{p+c-1}$

The smallest and largest order statistic for the IEGSStD will be obtained by making the substitution $c = 1$ and $c = n$ in equation (37).

Smallest order statistic

$$f_{(1,n)IEG}(y) = h_{(1,n)IEG} y^{j+lm(p+1-1)} (\lambda + y^2)^{-\frac{j+3}{2} - \frac{lm(p+1-1)}{2}}$$

$$f_{(1,n)IEG}(y) = h_{(1,n)IEG} y^{j+lm(p)} (\lambda + y^2)^{-\frac{j+3}{2} - \frac{lm(p)}{2}} \quad (38)$$

Largest order statistic

$$f_{(n,n)IEG}(y) = h_{(n,n)IEG} y^{j+lm(p+n-1)} (\lambda + y^2)^{-\frac{j+3}{2} - \frac{lm(p+n-1)}{2}} \quad (39)$$

Reliability Functions

Reliability analysis in distribution theory is basically concerned with the analysis of time to failure for a system under study and in this section, the reliability measures/functions for the proposed distributions will be derived.

Survival [S(y)] and cumulative hazard [H(y)] function

One of the functions usually estimated in reliability analysis is the survival function. The survival function is defined as the probability that the time to the event is greater than a specified time t . This is obtained by subtracting 1 from the CDF of a distribution. Mathematically, it can be expressed as;

$$S(y) = 1 - G(y) \quad (40)$$

The hazard function, represents the instantaneous rate of occurrence of an event at time t , given survival up to that time. The cumulative hazard function is given by;

$$H(y) = \int_0^y \log(y) du = \log(S(y)) \quad (41)$$

In the subsequent subsections, the $H(y)$ and $S(y)$ for the proposed probability distribution will be derived. Substituting (5) into (40), the survival function of IEGSStD is obtained and expressed as follows;

$$S(y) = 1 - \left\{ 1 - \left[1 - \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}} \right]^{\frac{1}{u}} \right\} = \left[1 - \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}} \right]^{\frac{1}{u}} \quad (42)$$

The cumulative hazard function of IEGSStD can be obtained by substituting equation (42) into equation (41).

$$H(y) = \log \left(\left[1 - \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}} \right]^{\frac{1}{u}} \right) = \frac{1}{u} \log \left(1 - \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}} \right) \quad (43)$$

Hazzard function

The hazard function is one important function in reliability analysis. This can be defined as a ratio of the probability density function to the survival function. Utilizing the densities of the proposed distributions, the hazard function would be derived.

$$H_z(y) = \frac{g(y)}{S(y)} \quad (44)$$

For a random variable with IEGSStD density, the hazard function can be obtained by substituting (6) for $g(y)$ and equation (42) for $S(y)$ into equation (44). This is illustrated as follows;

$$H_{z(IEG)}(y) = \frac{\frac{\lambda}{uv} \left(1 - \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}} \right)^{\frac{1}{u}-1} \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}-1} \frac{1}{(\lambda + y^2)^{\frac{3}{2}}}}{\left(1 - \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}} \right)^{\frac{1}{u}}} \quad (45)$$

Information Measures

Information measures are quantities used to quantify various aspects of information, uncertainty, and entropy in information theory. These measures have had different applications in various fields including communication theory, cryptography, machine learning, and statistics, where understanding and quantifying information and uncertainty are crucial. In this study the

Renyi entropy and its Arimoto measure will be derived in this subsection.

Renyi entropy

In information theory, the Rényi entropy family of entropy measures bear the name of the mathematician Alfréd Rényi. It generalizes the concept of Shannon entropy, providing a parameterized family of entropy measures. This can be defined as follows;

$$I_R(c) = \frac{1}{1-c} \log \int_0^{\infty} g^c(y) dy; \text{ for } c > 1, c \neq 1 \quad (46)$$

Suppose a random variable $y \sim IEGSSD(\lambda, u, v)$, then the degree of uncertainty can be obtained by substituting (6) for $g(y)$ in equation (46) as follows;

$$I_{R(IEG)}(c) = \frac{1}{1-c} \log \left[\left(\frac{\lambda}{uv} \right)^c \left(\frac{1}{2} \right)^{\frac{c(1-v)}{v}} \int_0^{\infty} \left[1 - \left(\frac{1}{2} \right)^{\frac{1}{v}} \left[1 + \frac{y}{2\sqrt{\lambda+y^2}} \right]^{\frac{1}{v}} \right]^{\frac{c(1-u)}{u}} \left[1 + \frac{y}{2\sqrt{\lambda+y^2}} \right]^{\frac{c(1-v)}{v}} (\lambda+y^2)^{-\frac{3c}{2}} dy \right]$$

$$\left[1 - \left(\frac{1}{2} \right)^{\frac{1}{v}} \left[1 + \frac{y}{2\sqrt{\lambda+y^2}} \right]^{\frac{1}{v}} \right]^{\frac{c(1-u)}{u}} = \pi \left[1 + \frac{y}{\sqrt{\lambda+y^2}} \right]^{\frac{r}{v}} \quad (47)$$

$$\text{Where } \pi = \sum_{r=0}^{\infty} \left(\frac{c(1-u)}{u} \right)^r \frac{(-1)^r}{r} \left(\frac{1}{2} \right)^{\frac{r}{v}}$$

$$I_{R(IEG)}(c) = \frac{1}{1-c} \log \left[\pi \left(\frac{\lambda}{uv} \right)^c \left(\frac{1}{2} \right)^{\frac{c(1-v)}{v}} \int_0^{\infty} \left[1 + \frac{y}{\sqrt{\lambda+y^2}} \right]^{\frac{r}{v}} \left[1 + \frac{y}{2\sqrt{\lambda+y^2}} \right]^{\frac{c(1-v)}{v}} (\lambda+y^2)^{-\frac{3c}{2}} dy \right]$$

$$I_{R(IEG)}(c) = \frac{1}{1-c} \log \left[\pi \left(\frac{\lambda}{uv} \right)^c \left(\frac{1}{2} \right)^{\frac{c(1-v)}{v}} \int_0^{\infty} \left[1 + \frac{y}{\sqrt{\lambda+y^2}} \right]^{\frac{r+c(1-v)}{v}} (\lambda+y^2)^{-\frac{3c}{2}} dy \right]$$

$$\left[1 + \frac{y}{\sqrt{\lambda+y^2}} \right]^{\frac{r+c(1-v)}{v}} = \mu y^{\tau} (\lambda+y^2)^{-\frac{\tau}{2}} \quad (48)$$

$$\text{where } \mu = \sum_{\tau=0}^{\frac{r+c(1-v)}{v}} \left(\frac{r+c(1-v)}{v} \right)^{\tau} \frac{(1)^{\frac{r+c(1-v)}{v}-\tau}}{\tau}$$

$$I_{R(IEG)}(c) = \frac{1}{1-c} \log \left[\pi \mu \left(\frac{\lambda}{uv} \right)^c \left(\frac{1}{2} \right)^{\frac{c(1-v)}{v}} \int_0^{\infty} y^{\tau} (\lambda + y^2)^{\frac{\tau}{2}} (\lambda + y^2)^{\frac{3c}{2}} dy \right]$$

$$I_{R(IEG)}(c) = \frac{1}{1-c} \log \left[\alpha_{(IEG)} \left(\frac{\lambda}{uv} \right)^c \left(\frac{1}{2} \right)^{\frac{c(1-v)}{v}} \int_0^{\infty} y^{\tau} (\lambda + y^2)^{\frac{\tau-3c}{2}} dy \right] \quad (49)$$

Where $\alpha_{(IEG)} = \pi \mu$

$$I_{R(IEG)}(c) = \frac{1}{1-c} \log \left[\alpha_{(IEG)} \left(\frac{\lambda}{uv} \right)^c \left(\frac{1}{2} \right)^{\frac{c(1-v)}{v}} \int_0^{\infty} \frac{y^{\tau}}{(\lambda + y^2)^{\frac{\tau+3c}{2}}} dy \right]$$

$$I_{R(IEG)}(c) = \frac{1}{1-c} \log \left[\alpha_{(IEG)} \left(\frac{\lambda}{uv} \right)^c \left(\frac{1}{2} \right)^{\frac{c(1-v)}{v}} \beta \left[(\tau+1), \left(\frac{\tau+3c-2}{2} \right) \right] \right] \quad (50)$$

Arimoto measure of entropy

The Arimoto measure of entropy, also known as the "information radius entropy" or "information divergence entropy," provides a way to compare the uncertainty or disorder in probability distributions. This is defined by the following equation;

$$A_{\varepsilon}(c) = \frac{c}{1-c} \left[\left(E_c(\theta) \right)^{\frac{1}{c}} - 1 \right] = \frac{c}{1-c} \left(\int_0^{\infty} (g^c(y) dy)^{\frac{1}{c}} - 1 \right); \text{ for } c > 1, c \neq 1 \quad (51)$$

Suppose a random variable $y \sim IEGSSID(\lambda, u)$, then on substituting (18) into (51), the Arimoto measure of entropy is derived as follows;

$$A_{\varepsilon(IEG)}(c) = \frac{c}{1-c} \left(\left(\int_0^{\infty} (h_{i,j})^c y^{cj} [\lambda + y^2]^{-c \left(\frac{j+3}{2} \right)} dy \right)^{\frac{1}{c}} - 1 \right) \quad (52)$$

$$A_{\varepsilon(IEG)}(c) = \frac{c}{1-c} \left(\left((h_{i,j})^c \frac{1}{\lambda} \int_0^{\infty} \frac{y^{cj}}{\left[1 + \frac{y^2}{\lambda} \right]^{c \left(\frac{j+3}{2} \right)}} dy \right)^{\frac{1}{c}} - 1 \right)$$

By substituting the transformation in (24) into (52), we have that;

$$\begin{aligned}
 A_{\varepsilon(IEG)}(c) &= \frac{c}{1-c} \left(\left(h_{i,j} \right)^c \frac{1}{\lambda} \int_0^{\infty} \frac{y^{cj}}{\left[1 + \frac{y^2}{\lambda} \right]^{c \left(\frac{j+3}{2} \right)}} dy \right)^{\frac{1}{c}} - 1 \\
 A_{\varepsilon(IEG)}(c) &= \frac{c}{1-c} \left(\left(h_{i,j} \right)^c \frac{1}{\lambda} \int_0^{\infty} \frac{(q\lambda)^{\frac{1}{2}cj}}{\left[1+q \right]^{c \left(\frac{j+3}{2} \right)}} \frac{\lambda}{2(q\lambda)^{\frac{1}{2}}} dq \right)^{\frac{1}{c}} - 1 \\
 A_{\varepsilon(IEG)}(c) &= \frac{c}{1-c} \left(\left(h_{i,j} \right)^c \frac{\lambda^{\frac{cj-1}{2}-1+1}}{2} \int_0^{\infty} \frac{q^{\frac{cj-1}{2}}}{\left[1+q \right]^{c \left(\frac{j+3}{2} \right)}} dq \right)^{\frac{1}{c}} - 1
 \end{aligned} \tag{53}$$

Therefore, on transforming (53) into the form in (27), the Arimoto measure of entropy of the IEGSStD is given as;

$$\begin{aligned}
 A_{\varepsilon(IEG)}(c) &= \frac{c}{1-c} \left(\left(h_{i,j} \right)^c \frac{\lambda^{\frac{cj-1}{2}}}{2} \int_0^{\infty} \frac{q^{\frac{cj-1}{2}-1+1}}{\left[1+q \right]^{c \left(\frac{j+3}{2} \right)-1+1}} dq \right)^{\frac{1}{c}} - 1 \\
 A_{\varepsilon(IEG)}(c) &= \frac{c}{1-c} \left(\left(h_{i,j} \right)^c \frac{\lambda^{\frac{cj-1}{2}}}{2} B \left[\frac{cj+1}{2}, c \left(\frac{j+1}{2} \right) \right] \right)^{\frac{1}{c}} - 1
 \end{aligned} \tag{54}$$

Estimation of Parameters and Montecarlo Simulation

This section presents the MLE estimates and a stability analysis of the estimates of the proposed model. The method of maximum likelihood was used to estimate the values of the model's parameters. This will be illustrated in the following subsections.

The Mean Squared Error (MSE) and Root Mean Square Error (RMSE) serve as critical metrics in statistics and data analysis. These

metrics offer a comprehensive evaluation of the accuracy and precision of predictions generated by statistical models or estimators. For the proposed probability distributions, the stability of the Maximum Likelihood Estimators (MLE) was studied by observing the behaviors of the MSE and RMSE at different sample sizes via Monte Carlo simulations. Random numbers were generated using the quantile functions and the uniform distribution with fixed parameter values for the simulations.

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Maximum likelihood estimators (MLE)

The Maximum Likelihood Estimation (MLE) method is widely utilized in academic literature due to its favorable properties as an estimator. MLE-derived estimates exhibit key qualities such as consistency, asymptotic

efficiency, and invariance. These attributes collectively underscore the reliability and effectiveness of MLE in statistical estimation. Let Y_1, Y_2, \dots, Y_n be random samples IEGSSd with PDF (6), then the MLE can be obtained as follows:

$$L(\theta) = \prod_{i=1}^n \left[\frac{\lambda}{uv} \left(1 - \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}} \right)^{\frac{1}{u}-1} \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}-1} \frac{1}{(\lambda + y^2)^{\frac{3}{2}}} \right] \quad (55)$$

$$L(\theta) = \left(\frac{\lambda}{uv} \right)^n \sum_{i=1}^n \left(1 - \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}} \right)^{\frac{1}{u}-1} \sum_{i=1}^n \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}-1} \sum_{i=1}^n (\lambda + y^2)^{-\frac{3}{2}}$$

$$\log(L(\theta)) = n \log\left(\frac{\lambda}{uv}\right) + \left(\frac{1-u}{u}\right) \sum_{i=1}^n \log\left(1 - \left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}} \right]^{\frac{1}{v}}\right) + \left(\frac{1-v}{v}\right) \sum_{i=1}^n \log\left[\frac{1}{2} + \frac{y}{2\sqrt{\lambda + y^2}}\right] \quad (56)$$

$$- \frac{3}{2} \sum_{i=1}^n \log(\lambda + y^2)$$

Stability Analysis via Montecarlo Simulations for IEGSSd

Table 1: Stability measures for IEGSSd

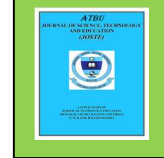
	$u = 0.01,$	$\lambda = 0.40,$	$v = 0.60$
Size	30		
Parameter Type	u	λ	v
MLE	0.00504	2.59909	0.54303
Absolute Bias	0.00496	2.19909	0.05697
MSE	0.00008	52.28891	0.04377
RMSE	0.00886	7.23111	0.20921
Size	100		
MLE	0.00759	0.60483	0.58861
Absolute Bias	0.00241	0.20483	0.01139
MSE	0.00004	1.74601	0.01299
RMSE	0.00616	1.32137	0.11396
Size	700		
MLE	0.00948	0.40558	0.60092

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Absolute Bias	0.00052	0.00558	0.00092
MSE	0.00001	0.02639	0.00183
RMSE	0.00266	0.16246	0.04278
Size	1000		
MLE	0.00962	0.40314	0.60055
Absolute Bias	0.00038	0.00314	0.00055
MSE	0.00000	0.01757	0.00128
RMSE	0.00222	0.13257	0.03577
Size	2000		
MLE	0.00979	0.40210	0.59982
Absolute Bias	0.00021	0.00210	0.00018
MSE	0.00000	0.00867	0.00064
RMSE	0.00152	0.09311	0.02533
Size	3000		
MLE	0.00987	0.40207	0.59948
Absolute Bias	0.00013	0.00207	0.00052
MSE	0.00000	0.00550	0.00040
RMSE	0.00123	0.07415	0.02000
Size	5000		
MLE	0.00992	0.40208	0.59946
Absolute Bias	0.00008	0.00208	0.00054
MSE	0.00000	0.00326	0.00025
RMSE	0.00093	0.05707	0.01592

The results from the simulations, as presented in Table 1 with the Mean Squared Error (MSE) and Root Mean Square Error (RMSE), the parameters of the IEGSSStD, affirmed the expected trend. It becomes increasingly apparent that, as the sample size systematically increase, there is a consistent decline in both MSE and RMSE values. This conspicuous pattern suggests that larger sample sizes significantly enhance the stability of MLE for the IEGSSStD parameters. The diminishing trend in both MSE and RMSE values underscores the heightened stability of the model estimators.

$$Pr_t = \frac{k_t}{k_{t-1}} - 1$$

Application

To determine the best-fitted model, the analysis involves computing the AIC (Akaike Information Criterion) for the proposed model as well as the models chosen for comparison. The model exhibiting the minimum values for AIC is deemed the best fit for the provided datasets.

The first data was on Guaranty Trust Holding Company PLC (GTCO) and the second data was for the Nigeria Breweries (NB) from 3/4/2023 to 27/12/2023 a total of 180 trading days. From the prices of the indices, the simple return series was calculated by the expression;

(57)

where k_t and k_{t-1} be the recent and immediate past price observed and (Pr_t) is the simple return series.

Tables 2 provide a comprehensive summary of key descriptive, including the mean, standard error, standard deviation, kurtosis, skewness, minimum, etc. for the two datasets.

Guaranty Trust Holding Company PLC (GTCO) Simple Return Series

0, 0.0012, 0, 0.0076, 0.0025, -0.0075, 0, 0, 0.0101, 0.0064, 0.0038, 0.0026, -0.0176, 0.0051, 0.0077, 0.0026, 0.0026, 0.0143, -0.0013, -0.009, -0.0038, 0.0013, 0.0196, 0.0241, 0.0054, 0.0027, 0, 0, 0.0193, 0.0083, 0.0084, -0.0056, 0.0056, 0.0028, 0.0085, 0.0043, 0.0043, -0.0113, 0.0172, -0.0156, 0, 0.01, -0.0155, -0.0125, 0.0227, -0.0085, -0.0056, 0.0071, 0.0115, 0.0029, 0, -0.0086, -0.0182, -0.007, -0.0042, 0.0014, 0, 0.0227, 0.0218, -0.0014, -0.0072, -0.0293, -0.0042, -0.0014, 0.0112, 0.0056, 0.0043, 0.0014, -0.0222, -0.0296, 0.06, -0.0141, -0.0862, -0.0038, 0.0052, -0.0177, -0.0247, 0.0658, 0.0354, 0.011, -0.0095, 0.0181, -0.0028, 0.0028, 0.0056, -0.0178, -0.0014, 0.0014, -0.0175, -0.0027, -0.0053, -0.0053, -0.0079, -0.0026, 0.0133, 0, 0.0135, 0.0165, 0.0028, 0.0225, 0, -0.0314, -0.0108, -0.012, -0.031, 0, 0.0052, 0, 0.1, 0.0057, 0.0087, 0.0147, 0.0089, -0.0088, -0.0556, 0.0028, -0.0028, -0.0191, 0.0209, -0.0014, 0.0256, -0.0384, 0.0429, 0.0174, 0.052, 0.0397, -0.0016, -0.0156, -0.0093, 0.0078, 0.0474, -0.0392, -0.0549, 0.0942, 0.1, 0.0036, 0.0036, -0.0018, -0.0089, -0.0088, -0.0224, 0.0105, -0.0238, 0.0691, 0.0223, -0.0147, -0.0144, 0.0203, 0.0207, 0.0153, 0.0019, 0.0418, 0.0415, -0.0062, 0.0104, -0.0244, 0, 0.0041, 0.0187, 0.0021, 0.0412, 0.0132, -0.0845, -0.002, -0.004, 0, 0, -0.006, 0, 0.006, 0.0246, 0.0021, 0.0021, 0, -0.0041, -0.0121, 0.0041, -0.0315

Nigeria Breweries (NB) Simple Return Series

0.027777778, 0, 0, 0, -0.050131926, 0, 0.041208791, -0.01754386, 0, 0, 0, -0.001347709, 0.001349528, 0, 0, 0, 0, -0.025, -0.002624672, 0.002631579, -0.025641026, 0, 0, 0, -0.025, 0, 0.052631579, 0, 0, 0, 0, 0.006622517, 0, 0.006666667, 0.006711409, 0.00269179, 0, 0, -0.021080369, 0.052704577, -0.051315789, 0, -0.025641026, 0, 0.026315789, 0, -0.023136247, 0, -0.073809524, 0, 0, 0, 0, 0, 0.090909091, -0.012820513, 0, 0, -0.045287638, 0, 0, 0.034177215, 0, 0, -0.035409035, 0, 0, 0, 0.056774194, 0, 0, 0, 0.017060367, 0, -0.003921569, -0.006493506, -0.012820513, 0, 0, 0, 0, 0.051212938, -0.021108179, -0.002631579, 0.018766756, -0.024836601, -0.1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.016746411, 0.1, 0.027027027, 0.01369863, 0, -0.0875, -0.047619048, 0, -0.037800687, 0, 0.099496222, 0, 0.044736842, 0.027027027, -0.051282051, -0.012658228, -0.070588235, 0, -0.002347418, -0.025171625, -0.006818182, -0.022222222, 0.005586592, -0.005555556, 0, 0.023890785, 0.022093023, -0.016018307, -0.002283105, 0, 0.00228833, -0.006818182, 0.002277904, 0.032941176, -0.059734513, 0.057309942, 0.042682927, 0, -0.017964072, 0, 0.018292683, 0, 0.001221001, -0.020334928, -0.012987013, 0.1, 0.015831135, 0, 0, -0.002631579, 0, -0.018087855, 0, 0.09631728, 0.099688474, 0, 0, 0, 0, -0.082857143, 0.038575668, 0.053125, -0.02587519, -0.061428571, 0.09375, -0.02587519, -0.1, 0.009681881, 0, -0.001381215, -0.014965986, 0.011004127, 0, 0, -0.004109589, 0, -0.013513514, 0, 0, 0

Table 3: Descriptive statistics of the datasets

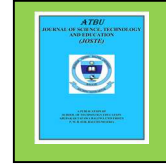
	Data 1		Data 2	
	Price	Simple Return	Price	Simple Return
Mean	33.8008	0.0028	39.0378	0.0005
Standard Error	0.3746	0.0018	0.2205	0.0024
Standard Deviation	5.0255	0.0240	2.9585	0.0314
Sample Variance	25.2559	0.0006	8.7529	0.0010
Kurtosis	-0.6064	5.3602	-0.1622	3.5936
Skewness	-0.8599	0.6857	-0.0733	0.4182
Minimum	22.7500	-0.0862	32.0000	-0.1000

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Maximum	40.5000	0.1000	45.2000	0.1000
Count	180	179	180	179

Table 3: MLE Goodness of Fit of the Datasets

	Data 1			Data 2		
	MLE	AIC	log L	MLE	AIC	log L
ESStD	0.1039995 0.0000944	-402.7407	203.3704	0.8206216 0.0001083	-849.7382	426.8691
Gumbel D	0.0000005 0.0334746	-752.7335	378.3668	0.0000004 0.0515047	-626.3400	315.1700
Logistic D	0.0017764 0.0111462	-867.9498	435.9749	0.0000000 0.0144232	-768.4066	386.2033
IEGSStD	0.0007815 0.4983699 0.4985950	-887.6275	446.8138	0.0000005 2.0015097 2.3045528	-962.5599	484.2800
EGSStD	0.9855961 0.0001550 0.5634654	-843.6393	424.8196	0.4611142 0.0000002 0.5107727	-943.9397	474.9698

Table 3 presents the results of fitting the Guaranty Trust Holding Company PLC (GTCO), Nigeria Breweries (NB) return series to five probability models (IEGSStD, EGSStD, ESStD, Gumbel D and Logistic D). The AIC, MLE, and log-likelihood values for each model are provided, revealing that the IEGSStD model had the smallest AIC and the largest log-likelihood values. A larger likelihood value indicates a better model fit, while a lower AIC suggests better fit. Therefore, based on the presented results, the IEGSStD model emerges as the most suitable among the other probability models for the simple return series.

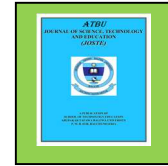
CONCLUSION

IEGSSt distribution was derived in this study by inverting the CDF generator of the exponentiated generalized family of distribution. Probability density plot and the corresponding cumulative distribution function for the IEGSSt model were plotted for some arbitrary parameter values. Some statistical properties of the model such as the raw moments, the moment generating function, the quantile function, the c^{th} order statistic were derived. The model

parameters were estimated using the maximum likelihood estimation (MLE) method. Through Montecarlo simulation the MLE was found to be consistent and as expected, with increased sample size, the RMSE almost converges to zero. The model potential of the in financial data analysis was illustrated using two financial datasets. The results from the fitting revealed that the IEGSSt distribution fitted better to the simple return series among the other distributions in terms of having the smallest AIC and the largest loglikelihood values. For modeling financial data, the application of IEGSSt distribution should be highly considered.

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