Determination of Discharge Equation for Semi-Circular Broad Crested Weir Models

By

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ABSTRACT

The discharge characteristics of semi-circular crested weir models were investigated. Two categories of weir were considered; normal weirs and oblique weir models. Both group of models had constant weir height of 20cm and had the crest radius varied three times; 5cm, 7.5cm and 10cm.Twelve models were fabricated and tested. Nine models with angles 90°, 60° and 15° as well as having radius of 5cm, 7.5cm and 10cm respectively were used in generating the discharge equation while three weirs models of radius 5cm, 7.5cm and 10cm at constant oblique angle of 30° were used in validating the equation. Multiple nonlinear regression analysis was used in determining the discharge equation while the validation was done using statistical methods (total error, coefficient of determination, coefficient of correlation, and reliability) with attention to accuracy, validity and goodness of fit of the generated equation. The investigation showed that for the developed discharge equation, the overall total error is 0.195. Coefficient of determination value is 0.994. The value of coefficient of correlation is 0.997 and the reliability of 1.911% in term of relative difference. It can be concluded that the generated equation is a very good option for calculating discharges for semi-circular broad crested weir, because it gave high coefficient of determination, very low total error, and a good reliability value.

Key words: Oblique Weir Models, Weir Height, Crest Radius and Oblique Angle

INTRODUCTION

Weirs are stable over flow structures built across channels or rivers to spill or divert water. Weirs are obviously the most primitive, easiest and most accurate flow structures that have various uses (Nawzad, 2004). Weirs are often used for diversions and regulations of flow as well as depth control and discharge analysis in various engineering fields involving applications of canals, reservoirs and rivers. Weirs are mostly used for free-flow conditions. However, linear weir is commonly used in canal applications as a diversion structure and is positioned normal to the direction of flow (Tingey, 2011a, 2011b).

Forms of weirs ranges from sharp-crested weirs, to broad crested, ogee crest and circular weirs respectively. The curvature for circular weirs gives it some outstanding characteristics over other forms of weir such as stable overflow pattern, ability to pass floating debris easily, lower construction cost and ease of design when compared to ogee weir (Chanson & Montes, 1998).

A linear weir may not be able to convey the required discharge due to insufficient freeboard (difference between water height and channel depth). However, discharges can be increased without a corresponding increment in upstream head by just increasing the weir length. Increasing weir length is difficult since increasing the channel width in most engineering applications is impractical. Therefore, a reasonable solution is achieved through the use of oblique weir (Tingey, 2011b).

Crest configuration may influence weir discharge. A higher discharge may be obtained with a quarter round broad crested weirs for a given head than a sharp crested weir; a better discharge may be achieved with a half round broad crested weir than the quarter round broad crested weir or sharp crested weir (Tingey, 2011a).

(Al-Tabatabaie *et al.*, 1986) and (Abid-Ali, 1986) studied the influence of round crest for broad crested weir flow characteristics. However, performance of oblique sharp-crested weir where studied by (Aichel, 1953), (Ibraheem, 1983) and (Darvas, 1971).

Various studies were conducted for weirs under submerged and free flow conditions were; (Matthew, 1962) explained the influence of viscosity and surface tension on the discharge over round-crested weirs using a theoretical concept. He proposed an equation for C_d determination which includes Scale effects which are often a problem in hydraulic modelling of overflow structures (Nawzad, 2004).

(Fritz & Hager, 1998) studied hydraulics of embankment weirs with side slopes (1V:2H) for both upstream and downstream under free and submerged flow (Nawzad, 2004).

The influences of downstream and upstream weir slopes on the discharge coefficient values for circular weir were studied by (Ramamurthy & Vo, 1993). (Chilmeran, 1996) investigated experimentally the influence of semi-circular crests on discharge over normal and oblique weirs.

The behavior of cylindrical weirs due to the nature of flow was investigated by (Chanson & Montes, 1998). (Al-Tikrity, 2000) studied the nature of flow over the rectangular half round crested weirs and compared with the result obtained under free flow for sharp-crested weirs. (Fritz & Hager, 1998) studied hydraulics of embankment weirs with side slopes (1V:2H) for both upstream and downstream under free and submerged flow (Nawzad, 2004).

However, results from equations generated by (Noori & Chilmeran, 2005) could not be achieved (Tingey, 2011b).

In the light of (Tingey, 2011a) observations, this study evaluates the discharge of semi-circular crested weirs under laboratory pump flow with the aim of generating the weir equations. This study will focus on determining the discharge equation of semi-circular broad crested weir experimentally. The experiments were conducted in hydraulics laboratory, Department of Water Resources and Environmental Engineering, Ahmadu Bello University Zaria.

MATERIALS AND METHODS

Materials

The following materials were used in this work.

The physical model was fabricated from wood with a configuration of a semi-circular crest as shown in figure 1, the radius was determined according to the design criteria suggested by (Abid-Ali, 1986), in which the radius of the weir (R) should at least equal to the weir height (P) divided by 12.

$$R = \frac{P}{12} \qquad (1)$$

Experimental Models

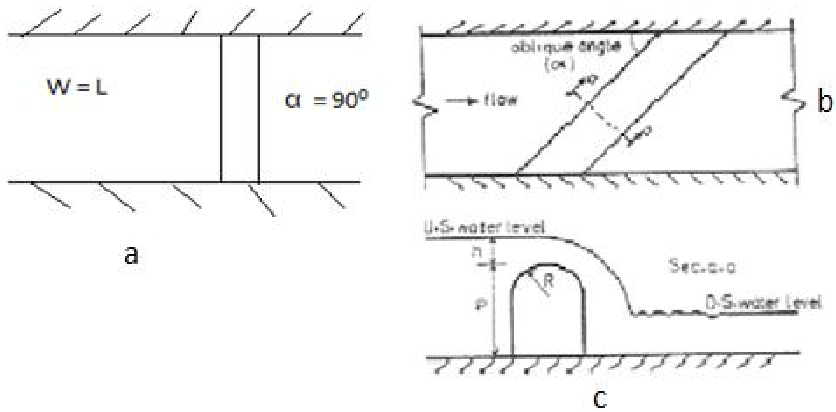

Figure1: Schematic Diagram of the Semi-Circular Crested Weir where: a) is the plan view of a semi-circular crested weir model; b) is the plan view of an oblique semi-circular crested weir, and c) is the side view of a normal semi-circular crested weir.

A Plan view of a semi-circular crested weir

B Plan view of an oblique semi-circular crested weir

C Side view of a semi-circular crested weir.

The experimental works were conducted on twelve (12) different fabricated models with angles of inclination to the direction of flow α = 15°, 30°, 60° and 90° and for different discharges. A summary of the weir geometries tested is given in Table 1

Table 1: Weir Geometry

Model No.	Group No.	Oblique Angle(α)	Series No	Crest Radius R(m)	Effective Crest Length L(cm)	Crest Height P(cm)
1 2 3	1	90° (Normal weir)	A B C	5.0 7.5 10.0	30.0 30.0 30.0	20

Model No.	Group No.	Oblique Angle(α)	Series No	Crest Radius R(m)	Effective Crest Length L(cm)	Crest Height P(cm)
4 5 6	2	60^o	A B C	5.0 7.5 10.0	34.6 34.6 34.6	20
7 8 9	3	30^o	A B C	5.0 7.5 10.0	60.0 60.0 60.0	20
10 11 12	4	15^o	A B C	5.0 7.5 10.0	115.9 115.9 115.9	20

Depth of water as well as the height above the crest was measured by the use of an accurate point gauge during the experiments. P the height of each weir was made constant by controlling the thickness of carpenter's putty used and variations made while installation. The measurement of height above the crest was taken with the point gauge located in accordance to (Subramanya, 1986) and (Finnemore & Franzini, 2002), the effective measurement point is considered as 3-4h away from the weir location in order to get rid of draw down effects. Where h is the head of above weir crest.

METHODS

Experimental Setup

The downstream and the upstream water surface profile as well as the crest height of the models were measured using a moveable point gauge. Water was circulated through the flume via a centrifugal pump. Twelve models were produced and tested. The models were categorized into four groups depending on the oblique angle (α). A constant weir height (P) was used for all models in each group with the crest radius (R) varied from 5cm, 7.5cm to 10cm. The groups consisted of the normal weir (90^o), the 60^o and 15^o oblique weirs respectively which was used for generating the model equation. While the 30^o weir was used in validating the model equation generated. Though all models were polished and varnished to give smooth surfaces, they were produced from wood. Each model was placed 3.5m from the inlet of the flume and various discharge were passed over the weir model. Six different discharges were passed over each model and the heights of water over specific grid point were measured in order to determine the upstream and downstream surface profiles.

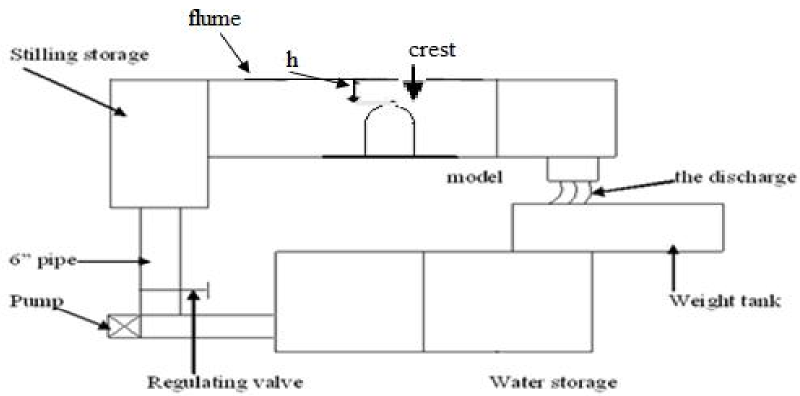

Figure 2: Schematic Representation of the Experimental Setup

Discharge Equation

Regression analysis was performed between the measured discharges 'Q', the head of water above crest 'h', the crest radius R and the sine of angle of weir inclination 'sinα' in order to arrive with a relationship in the following format of equation, as suggested by (Muhammad, 2012)

$$Q = \frac{CH^n R^x}{(\sin\alpha)^m} \quad (2)$$

Where c, x, n and m are parameters to be estimated by regression analysis.

The actual discharge (Q_{act}) was measured gravimetrically and then converted to m³/s using the equation below.

$$Q_{act} = \frac{M}{\rho t_{av}} \quad (3)$$

Surface Profile Study

In order to decide the point gauge location, water surface profile was made for different discharge values. Water depth was recorded for different discharges and a graph was used to illustrate water surface in the channel of the flume. The x-axis of the graph represented the distance from the weir while the y-axis represented the water height from the bottom of the channel.

Dimensional Analysis for the Semi-Circular Weir

A functional relationship was developed using dimensional analysis based on the idea of (Noori & Chilmeran, 2005), that the overflow characteristics of semicircular crested oblique weirs can be expressed as follows:

$$q = \phi(h, P, R, g, \alpha) = 0 \quad (4)$$

The general relationship among all variables is (Noori & Chilmeran, 2005):

$$\Phi(q, h, P, R, g, \alpha) = 0 \quad (5)$$

The variable P, and g were not considered since P was constant for all weir hence, the relationship reduced to

$$q = \phi_2(h, R, \sin\alpha) = 0 \quad (6)$$

Statistical Evaluation

The model was evaluated statistically in order to validate the equation generated. Statistical tools like the total error, coefficient of determination, correlation coefficient and the reliabity was tested.

RESULTS

For model equation generation, fifty-four experiments were conducted in the hydraulics laboratory to determine flow parameter namely water surface profile, head of water above the crest 'h' and actual discharge Q_{act}, The Tables show the various values of h and Q_{act} based on various crest radius R= 5cm, 7.5cm, and 10cm respectively. Each of these crest radii consist of three different angles of inclination to the direction of flow (that is α = 15°, 60° and 90°). Eighteen experiments were performed to test the validity of the generated equation at a constant oblique angle of 30° with various crest radius R= 5cm, 7.5cm, and 10cm respectively.

Water surface Profile

The graph in Figure illustrates the water surface profile upstream of the weir using the measured water depth for seven various discharges Q_1 to Q_7 (2.143 to11.765 (l/s)). The water surface appears virtually stationary at 0.5m from the weir. At this point, it shows that the depth becomes linearly dependent on the discharge. Hence, the respective head above the crest for different discharges could be measured at this section using the point gauge. However, the point gauge was placed at 0.70m in accordance to (Subramanya, 1986) and (Finnemore & Franzini, 2002).

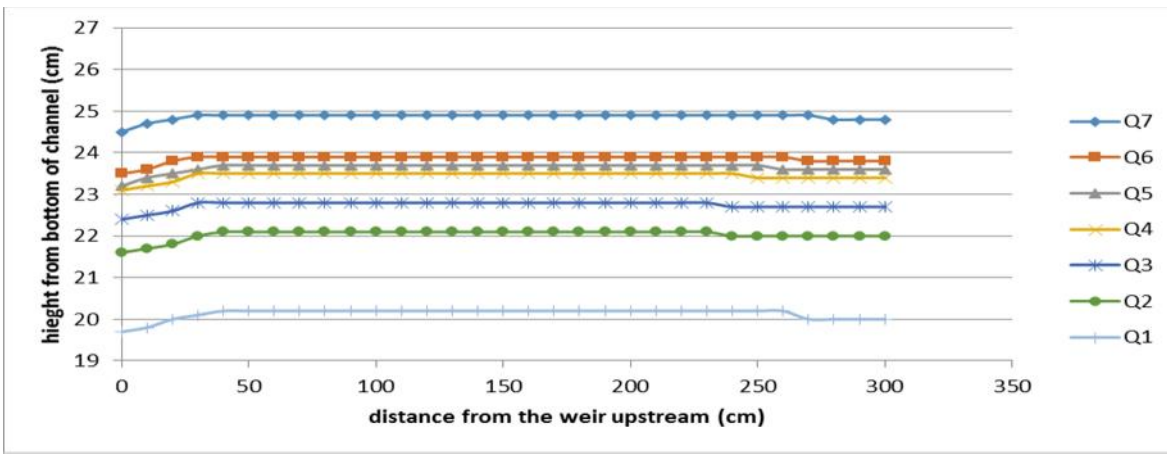

Figure 3: Water Surface Profile for Different Discharges

Measured Flow Parameter on the Semi-Circular Broad Crested Weir

The values in tables were analyzed using multiple regression analysis model to develop the equations (2) for head-discharge relationship.

Table 2: Measured flow parameters for weir models

Angle of inclination α (degree)	Effective head (H_{eff})	CRR	Sine α	Actual discharge Qact	Angle of inclination α (degree)	Effective head (H_{eff})	Crest Radius R	Sine α	Actual discharge Qact
90	4.582	10	1	6.71	60	4.246	7.5	0.867	6.9
	5.057	10	1	7.93		4.589	7.5	0.867	7.9
	5.367	10	1	8.72		4.994	7.5	0.867	9.01
	5.854	10	1	9.92		5.255	7.5	0.867	9.77
	5.989	10	1	10.29		5.423	7.5	0.867	10.25
	6.2	10	1	10.95		5.561	7.5	0.867	10.74
60	4.391	10	0.867	7.32	15	2.02	7.5	0.259	7.7
	4.662	10	0.867	8.12		2.189	7.5	0.259	8.78
	5.036	10	0.867	9.15		2.371	7.5	0.259	9.86
	5.371	10	0.867	10.11		2.418	7.5	0.259	10.17
	5.58	10	0.867	10.71		2.465	7.5	0.259	10.51
	5.761	10	0.867	11.35		2.571	7.5	0.259	11.58
15	2.065	10	0.259	8.49	90	4.387	5	1	6.25
	2.2	10	0.259	9.3		4.809	5	1	7.26
	2.293	10	0.259	9.9		5.232	5	1	8.31
	2.367	10	0.259	10.55		5.636	5	1	9.36
	2.445	10	0.259	11.28		5.781	5	1	9.74
	2.497	10	0.259	11.76		6.072	5	1	10.49
90	4.449	7.5	1	6.4	60	4.164	5	0.867	6.69
	4.871	7.5	1	7.48		4.598	5	0.867	7.84
	5.304	7.5	1	8.51		5.117	5	0.867	9.25
	5.771	7.5	1	9.71		5.191	5	0.867	9.52
	5.948	7.5	1	10.18		5.348	5	0.867	9.98
	6.198	7.5	1	10.84		5.535	5	0.867	10.4

15	1.996	5	0.259	7.46
	2.153	5	0.259	8.47
	2.268	5	0.259	9.22
	2.371	5	0.259	9.87
	2.476	5	0.259	10.55
	2.595	5	0.259	11.38

Table 3: Values of parameters as a result of regression analysis

Regression Statistics	
Multiple R	0.995
R Square	0.990
Adjusted R Square	0.989
Standard Error	0.00747
Observations	54

Table 4: Values of model equation parameters as a result of regression analysis

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.00261	0.0312	0.0836	0.934
Log H_{eff} (n)	1.590	0.0242	65.794	3.112E-50
Log sinα (m)	-1.0782	0.0157	-68.509	4.211E-51
Log R (x)	0.0410	0.00826	4.966	8.370E-06

The following expressions were developed using the experimental results in tables 2.

$$Q = \frac{CH^n R^x}{\sin\alpha^m} \quad (7)$$

Developed Model Equations

The parameters of Equations have been obtained by regression analysis as indicated in Table 4 above Equation has coefficient of determination 0.9897 that is, 98.97 percent of the variance in the observed values of the dependent variable is explained by the model and 1.03 percent of this variable remains unexplained in the error (residual) term. The high adjusted coefficient of determination (0.9891) computed shows the model has regression with the ideal combination of having the best fit without excess/unnecessary terms. Furthermore, the model equation is reliable in predicting discharge due to the high coefficients of determination as well as high adjusted coefficients of determination (R^2 and R^2_{adj}) and the respective small errors of standard estimate. The extremely low p-value of the independent variables shows their high significance in predicting the dependent variable.

$$Q = \frac{1.00602 H^{1.58953} R^{0.04103}}{\sin\alpha^{1.078}} \quad (8)$$

The equation above is the discharge relationship with the head, crest radius and angle of inclination which can be adopted in calculating flow rate over any of the semi-circular crested weir models.

Model Validation

The model equation was validated experimentally through the use of models with oblique angle of 30° but radii of 5cm, 7.5cm and 10cm respectively. The table below show the results obtained.

Table 5: Actual Discharge and Calculated discharges From Weir Equation Developed

head (H_{eff})	sinα	R	Q_{act}	Q_{est}	Total error2	Q_{act}-Q_{mean}	$\ln Q_{est}$-$\ln Q_{act}$
3.111	0.5	10	7.58	7.772389	0.037014	3.669353086	2.506438245
3.275	0.5	10	8.47	8.433718	0.001316	1.051764198	-0.429277729
3.543	0.5	10	9.52	9.556906	0.001362	0.000597531	0.386917574
3.717	0.5	10	10.25	10.31368	0.004055	0.56918642	0.619343661
3.818	0.5	10	10.82	10.7627	0.003284	1.754153086	-0.53099969
3.913	0.5	10	11.51	11.19149	0.101452	4.05798642	-2.806299453
3.069	0.5	7.5	7.42	7.516992	0.009408	4.307930864	1.2987049
3.444	0.5	7.5	9.01	9.028718	0.00035	0.235764198	0.207535676
3.594	0.5	7.5	9.58	9.661759	0.006685	0.007130864	0.849815739
3.67	0.5	7.5	9.90	9.988537	0.007839	0.163575309	0.890337346
3.801	0.5	7.5	10.47	10.5612	0.008318	0.949541975	0.867307044
3.889	0.5	7.5	10.91	10.9525	0.001807	2.000653086	0.388832822
2.952	0.5	5	6.96	6.950012	9.98E-05	6.429041975	-0.143612899
3.217	0.5	5	7.97	7.967651	5.52E-06	2.327319753	-0.029475148
3.593	0.5	5	9.54	9.498121	0.001754	0.001975309	-0.439945614
3.67	0.5	5	9.89	9.823709	0.004395	0.15558642	-0.672539929
3.746	0.5	5	10.2	10.14904	0.002597	0.496241975	-0.500839419
3.909	0.5	5	10.92	10.85996	0.003605	2.029041975	-0.551366316
		sum	170.92		0.195344	30.20684444	1.910876811

Statistical Evaluation of the Model Equation

Total Error

Total error, TE, is the entire or overall error that may occur in a model result due to either inaccuracy (systematic error) or imprecision (random error) or even both of the measurement procedure (Oke, 2007). Total error is the sum of the squares of the errors between the obtained values of discharges observed and the predicted values of discharges from the model equation, thus can be interpreted as a measure of variation in the values predicted unexplained by the values obtained by the model equation (Oke & Akindahunsi, 2005). Accuracy, validity as well as the goodness of fit are achieved when the value of total error is low (Montgomery *et al.*, 2010; Oke, 2007). Total error $\left(Err^2\right)$ was obtained using equation below.

$$Err^2 = \sum_{i=1}^{n}\left(Y_{obsi} - Y_{cali}\right) \qquad (9)$$

The observed values of discharge were the discharges measured from the experiment while the calculated values of discharges were obtained from the model equation. The error 0.195344 was obtained. The low total error obtained indicates that the model equation generated have good validity, accuracy and goodness of fit.

Coefficient of Determination and Correlation Coefficient

The coefficient of determination, r^2, is highly significant because it explains the percentage of the variance (alternation) of one variable that is estimated from the other variable. It is a measure that enable us ascertain how certain we are in making predictions from a model/graph. It represents the percent of the data that is the closest to the line of best fit. Coefficient of determination is such that $0 < r^2 < 1$, and show the strength of the linear association between two variables. It is expressed mathematically as the square of the correlation coefficient (Montgomery *et al.*, 2010):

$$r = \sqrt{r^2} = \sqrt{\frac{\sum_{i=1}^{n}\left(Y_{obsi} - \bar{Y}_{cali}\right)^2 - \sum_{i=1}^{n}\left(Y_{obsi} - Y_{cali}\right)^2}{\sum_{i=1}^{n}\left(Y_{obsi} - \bar{Y}_{cali}\right)^2}} \qquad (10)$$

A correlation greater than 0.8 is considered as strong, whereas a correlation less than 0.5 is generally described as weak. The variation of the value of r depends on the "type" of data under examination. A study utilizing scientific data may require a stronger correlation than a study using social science data (Montgomery *et al.*, 2010).

r and r^2 values obtained from the validation experiment were 0.993533137 and 0.996761324 respectively. The high value of r^2 and r obtained indicates that the model equation generated have high accuracy, validity and goodness of fit.

Reliability

Reliability of an equation depends on its validity and accuracy. The statistical methodology normally adopted for reliability of any method is using the null hypothesis that there is no difference between the expected results and that of the equation Reliability of an equation depends on its validity and accuracy. The statistical methodology normally adopted for reliability of any method is using the null hypothesis that there is no difference between the expected results and that of the equation (Oke, 2007; Sartory, 2005) describes statistical relative difference between measured discharges with that obtained from the equation as follows:

$$RD = \left(\ln A_q - \ln B_q\right) x100 \qquad (11)$$

A lower RD value implies that the equation is reliable. An RD of 1.910876811% was obtained during validation. The low RD value obtained indicates that the model equation is reliable.

CONCLUSION

The parameter of the generated equations was obtained by regression analysis. The equation had coefficient of determination r^2 of 0.989736253 and adjusted r^2 0.989120429 that is (98.91%) ninety-nine percent of the variance in the observed values of the dependent variable is explained by the model and (1.09%) one percent of this variable remains unexplained in the error (residual) term. Model equation generated was tested for total error, reliability, coefficients of determination and correlation. It had total error of 0.195344, reliability 1.910876811% in terms of relative difference between measured discharges with that obtained from the equation developed, coefficients of determination of 0.996761324 and correlation coefficient 0.993533137. The model equation generated in this study was found to be statistically accurate, valid, reliable and have good fitness and therefore can be used to predict discharge over semi-circular broad crested weirs of all angle of inclinations.

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