



Mathematical Modelling of an Autonomous Photovoltaic Standalone System

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ABSTRACT

Autonomous stand-alone Photovoltaic power systems are an essential element in providing power in Nigeria looking forward. In order to effectively exploit this technology while avoiding the pitfalls of the past, it is essential to establish a well-structured and formal framework for the design, maintenance and operations. This paper presents the mathematical model for the study and possibly design of such systems by solving the dynamical equations that describe the system during sunshine and dark periods. Both periods can be represented by nonlinear differential equations which are solved numerically and used to develop scenarios. The results of the studies indicate that under circumstances similar to those encountered during the rainy season when insolation can be markedly reduced, create conditions whereby for specific systems, the loading may have to be markedly decreased. Tests with larger battery capacities indicate that the batteries may fail to charge fully and subsequently the system fails unless an external intervention is made to augment the PV power.

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INTRODUCTION

An autonomous solar-PV system consists of only PV panels as the source of generation (Glavin, 2012). Thus, it is required for the size of the PV modules to be large enough to provide the energy that can satisfy the load demand at all time. Since the solar irradiance is the only source of energy for this type of system and that the insolation is highly stochastic there is, therefore, the need to store the energy in devices such as a battery. A trade-off

between the sizes of the battery and the PV panels is considered to mitigate the vulnerability of solar PV-battery system regarding low and/or unavailable solar irradiance due to night periods, cloud cover, bird dropping, and dust (Bayod, 2013).

An autonomous solar PV system provides electricity independent of the national or local grid and is mostly installed in isolated areas not privy to the main grid. Also, other areas of application of solar PV standalone system may include

telecommunication stations, rural housing, residential homes, hospitals, schools, and water irrigation systems (Abubakar, et al., 2018). One of the factors to consider in the design of this system is to employ the use of low energy-efficient appliances that will ensure a little load demand and consequently reduce system cost (Glavin, 2012).

Globally, solar-PV installation has witnessed a magnificent increase as shown in Figure 1, where it became the leader in the RES family (Zervos, 2016). This became feasible due to panel price, policies by various governments, public awareness and adaptation, modularity and better batteries in market with increased lifetime and higher efficiency (Bapat, 2016; iea, 2019).

The projection of investment in the solar energy shows that Asia will perform better than any other continent, where it is expected that about 60% of all installations will be made in Asia. The tropical region (Nigeria inclusive) despite having abundance of solar energy continue to witness a slow installations of solar-PV. However, the sharp drop in the prices of panels, batteries and other associated components significantly improved the investment within the African region (Abubakar, et al., 2018; Timilsina, 2011; Kilisek, 2015).

There are several attempts in Nigeria recently to deploy the solar-PV system in electricity provision to communities via numerous memoranda of understanding between governments at all levels and investors (tribuneonline, 2017; Ozoegwu, 2017). However, there are so many obstacles that became stumbling blocks to the success of such deployment ranging from lack of awareness, theft, cost and stringent conditions (Ohunakin, 2014). There are numerous parameters considered in the design of autonomous renewable energy sources (RES) such as solar-PV/battery system. These parameters are used to have an appropriate models for the generation, battery charging and discharging. Several works concluded that off-grid autonomous hybrid system can be achievable to provide reliable electricity to the specific desired load (George, 2015).

In this paper the capability of autonomous solar-PV/battery based system in providing reliable electricity to a specified load demand is proven. The mathematical models of the solar-PV generator, battery charging and discharging are presented that consider semi-autonomous days (days with minimum sunshine hours). The sunshine hours were extracted from the solar radiation of the area and used in the design of a robust solar-PV autonomous system.

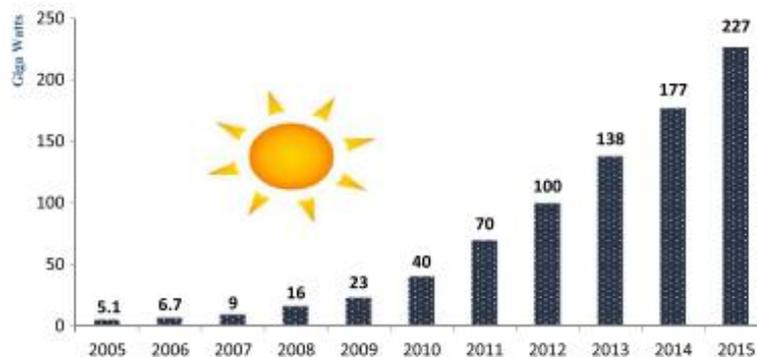


Figure 1: Total Global Installed PV Power System between 2005 and 2015 (Zervos, 2016)

COMPONENTS AND METHODS

In Figure 2 the proposed system is shown, consisting of inverter, PV array, battery (storage) bank, load and voltage controller. The solar-PV of the proposed system is expected to generate enough

energy for the voltage load and an excess energy that is stored in the battery bank, to provide electricity during the night-time or at low solar irradiance periods. The voltage controller keeps the battery safe by avoiding overcharging.

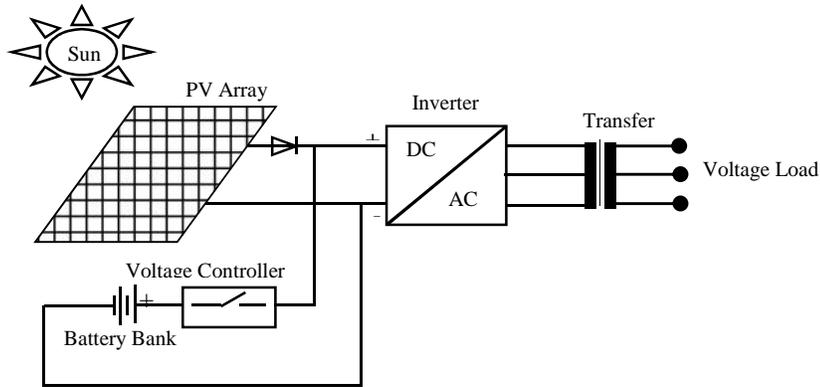


Figure 2. Proposed Solar-PV Microgrid

Daily Voltage Load

A residential load demand is used in this work and the total daily energy demand considers seasonal effects, where heavy load (air conditioners) are expected to work at temperatures above 31°C. The total load demand is given by:

$$E_D = \begin{cases} \sum_{i=1}^N E_i; & \theta_a < 32 \\ \sum_{i=1}^N (E_i + \hat{E}_i); & \theta_a \geq 32 \end{cases} \quad (1)$$

$$E_i = N_i \times P_i \times T_i \quad (2)$$

$$\hat{E}_i = \hat{N}_i \times \hat{P}_i \times \hat{T}_i \quad (3)$$

where E_D is the total daily energy demand (Wh), E_i is the electrical appliance required energy (Wh), N_i is the number of appliances, P_i is the power rating of the appliance (W), T_i is the time of usage of the appliance in hours, \hat{N}_i is the number of air conditioners, \hat{P}_i is the power rating of air conditioner unit (W), \hat{T}_i is the time of usage of air conditioner unit in hours, and θ_a is ambient temperature in °C.

Solar Photovoltaic Generation System

The solar-PV generator depends on the availability of sunshine and other

environmental variables such as dust and temperature. The mathematical model of the solar-PV conversion system is defined in (Abubakar, et al., 2018) by:

$$P_p = P_m \times D_m \times \frac{Y_b}{Y_g} \times \eta_s \quad (4)$$

where P_p is the panel output power in kW, P_m is the PV panel rating in W, Y_b is the solar irradiance in W/m^2 , Y_g is the solar irradiance at standard test condition (STC) in W/m^2 , D_m is the panel de-rating factor chosen as 97%, and η_s is the temperature de-rating factor defined as:

$$\eta_s = 1 + \gamma(\theta_p - \theta_s) \quad (5)$$

γ is temperature coefficient (ranging between 0.4% and 0.6% per °C) $\gamma=0.45\%$ is considered in this work. θ_p is PV panel temperature in °C and θ_s is panel temperature at standard test condition (STC), usually taken as 25°C.

The solar irradiance Y_b in W/m^2 is computed in (Robishaw, 2019) as:

$$Y_b = Y_c \times 11.6 \quad (6)$$

Where Y_c is solar radiation in $MJ/m^2/day$.



The corresponding solar energy of the PV array is defined in (letitgo, 2017) by:

$$E_p = P_p \times P \quad (7)$$

where E_p is the solar PV array energy output (Wh), and P_p is the peak sunshine hour (h) computed from

$$P_p = \frac{Y_B \times 2.9}{Y_g} \quad (8)$$

Battery Storage System

The stochastic nature of the solar irradiation makes the output of the solar-PV to be stochastic too, hence the danger of battery overcharging. Also, ageing factor and temperature affect the operation of the battery storage system. Conventionally, charge controllers are used to mitigate such problem yet they too suffer from substandard designs and calibrations. This posed a serious challenge to autonomous solar-PV installations.

Battery Capacity Constant

A new parameter, the battery capacity constant is introduced in this work to avert the battery over- and under-charging scenarios. This ensures that the battery operates within the minimum and maximum boundaries determined by the storage system (12V, 24V or 48V). This is defined by:

$$\xi_t(t) = \frac{\Delta V_B(t)}{\Delta \mathfrak{B}(t)} \quad (9)$$

Where

$$\Delta V_B = V_B^+ - V_B^- \quad (10)$$

$$\mathfrak{B} = Q_B \times \frac{D}{1} \quad (11)$$

Where V_B^+ the battery allowable upper is limit (12.9V for 12V battery design, 25.8V for 24V battery design etc.), V_B^- is the battery allowable lower limit (11.7V for 12V battery design, 23.4V for 24V battery design etc.), \mathfrak{B} is the rate of change of battery with respect to depth of discharge (DoD) in Ah, and Q_B is the battery rate of discharge in Ah.

Battery Charging Model

When sunshine is available the solar PV charges the battery and the general charging equation is given by:

$$\frac{dV_B}{dt} = \xi \left(I_p - \frac{P_d}{V_B} \right); T_{sh,u} \leq t \leq T_{sh,k} \quad (12)$$

$$V_B(t) = V_{B_0} \quad (13)$$

$$I_p = \frac{E_d}{V_B \times T_{sh,k}} \quad (14)$$

where I_p is the nominal current (A) at $T_{sh,k}$, P_d is load (Watt), V_B is instantaneous battery voltage (Volt), $T_{sh,k}$ is the reference sunshine hours (h) of the area (taken as 6 hours in this work), V_{B_0} is the voltage at the starting time of the day (where it is V_B^- in the first day) and E_d is the total daily energy (Wh).

The charging equation (12) is a nonlinear differential equation that can be solved by using the Picard's iterate expressed as:

$$\hat{V}_B(t) = V_{B_0} + \int_0^t f(t, V_{B_n}) dt \quad (15)$$

However, the Picard's iterate should satisfy the Lipschitz condition described by:

$$L \left\| \int_0^t [f(t, V_{B_{n+1}}) - f(t, V_{B_n})] dt \right\| \leq \|V_{B_{n+1}} - V_{B_n}\| \quad (16)$$

Where L is a Lipschitz constant

Battery Discharging Model

When sunshine is unavailable or could not meet up with the load then the battery will discharge to supply the load.

The discharge model is a linear differential equation, given as:

$$\frac{dV_B}{dt} = \frac{-\xi E_d}{V_B} \quad (17)$$

The linear differential equation (17) is solved as:

$$V_B(t) = \sqrt{[V_B^2(t) - 2\xi P_d(t - T_{sh,k})]} \quad (18)$$

where V_B is the voltage at last charging state and P_d is the load

RESULTS AND DISCUSSION

The solar radiation data of Kano, Nigeria (between 1971 and 2016), obtained from the Nigerian Metrological Agency (NiMet) is used to compute sunshine hours of the next 20 years. The results are presented in Figure 3 (the histogram) and Figure 4 (the probability density function) respectively. It can be observed that the minimum sunshine hours will be 3 hours and the maximum sunshine hours will be 9 hours. The modal sunshine hours expected is 6 hours that may occur 8,278 times representing 49.3% and the distribution resembles normal distribution.

The average domestic load requirement in a typical Nigeria’s rural house

is about 3.5kWh/day while in the city it can reach up to 12.5kWh/day. In Figure 5 the daily load demand of a typical residential home in Kano city, Nigeria shows that the highest demand of 750W used to occur between 11:00am to 1pm and the lowest demand of 250W occurs at midnight.

The monthly total energy is presented in Figure 6 showing the months of March and May with the highest values of 345.53kWh and 345.52kWh respectively. These months are characterised with higher temperatures. The months of December and through February recorded the lowest energy consumptions.

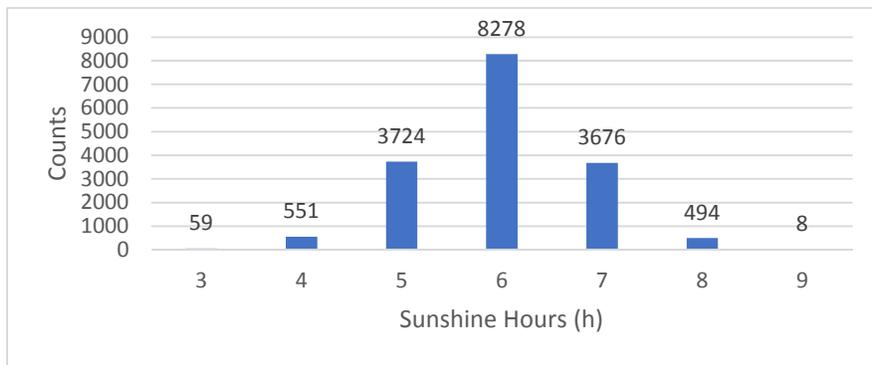


Figure 3: Histogram of Forecasted Sunshine Hours of Kano, Nigeria between 2017 and 2037

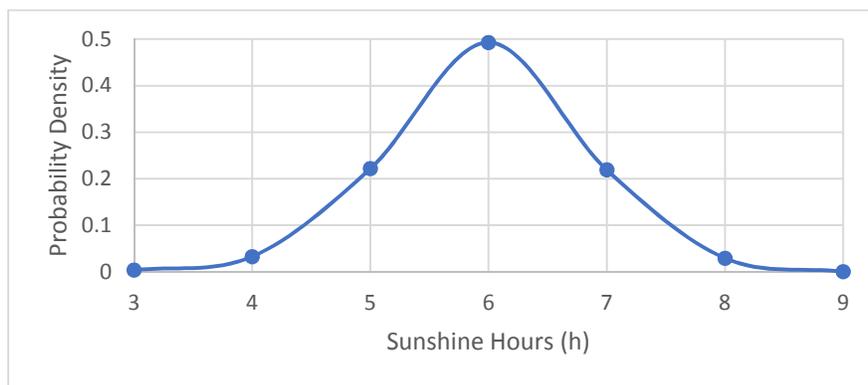


Figure 4: Probability Density Function of Sunshine Hours of Kano, Nigeria between 2017 and 2037

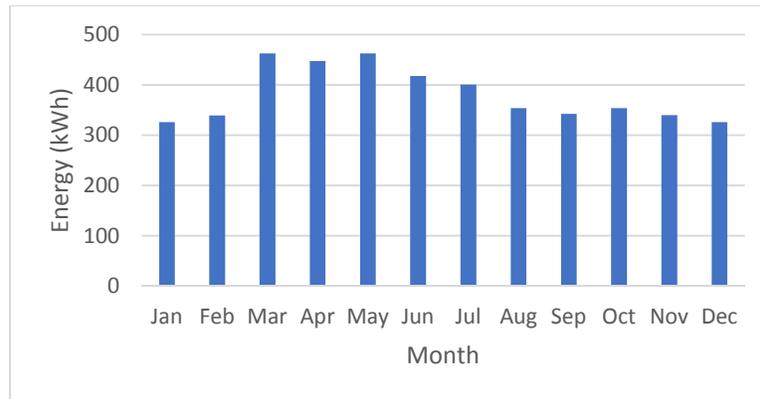


Figure 5: Monthly Total Energy Consumption

The developed models are tested using the predicted sunshine hours. Four different scenarios were considered that include: ideal, non-mitigated, panel-mitigated and battery-mitigated. However, some constraints are imposed on the simulations including battery limits as 25.8V and 23.4V for upper and lower limits respectively; a 50% depth of discharge and that the load demand must always be satisfied.

In the ideal scenario (Figure 7) it can be observed that the load demand is satisfied at all times and that the system is capable of generating enough solar-PV energy for the load and charges the battery during the 6-hours of sunshine and the batteries discharged to within the boundaries (upper and lower limits). This scenario is hardly achievable throughout the year as about 50.7% of the sunshine hours in the area are above or below 6 hours. The non-mitigated scenario used the minimum sunshine hours (3 hours) for the

design and as shown in Figure 8 it can be seen that the battery is drained below the lower limit after just 8 hours.

The panel-mitigated scenario (shown in Figure 9) is a situation where the panel size is increased to be able to charge the battery quickly at such period when the sunshine is very minimal (3 hours). Also, the battery capacity constant is reduced from 0.0048 (for reference and non-mitigated scenarios) to 0.000125, indicating that the charging side is almost doubled.

In Figure 10 the battery size is increased such that the amount of discharge will decrease. In this scenario, additional battery storage bank (secondary) is created as a backup only when the primary bank approaches the lower voltage limit. The major drawback could be that the battery can sometimes be redundant in some periods of the year, especially during longer sunshine hours.

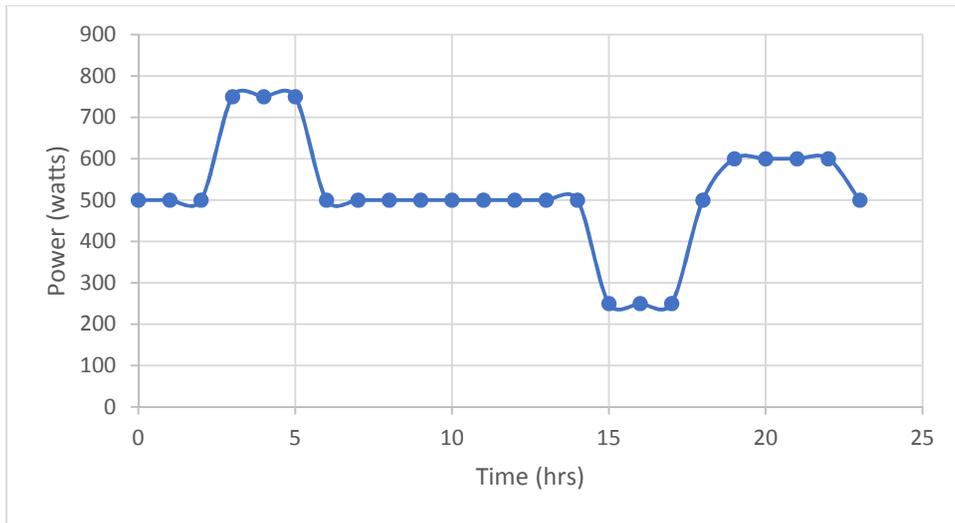


Figure 6: Daily Hourly Load Profile

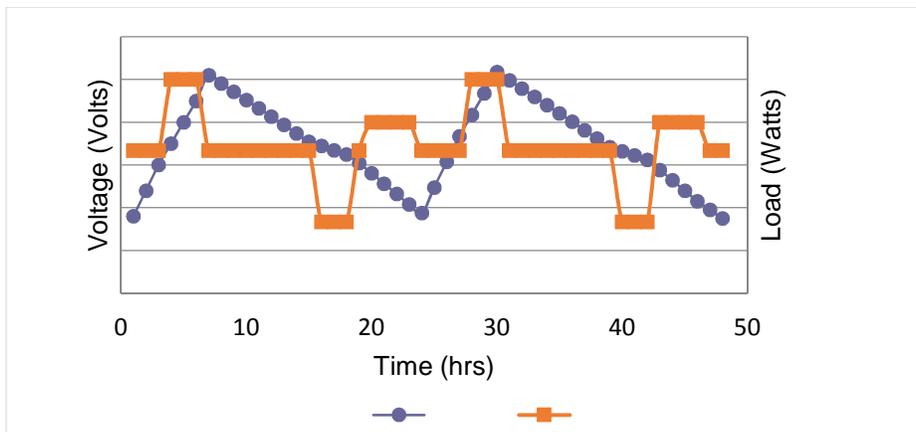


Figure 7: Load Demand and Battery Charging and Discharging of a Reference Installation

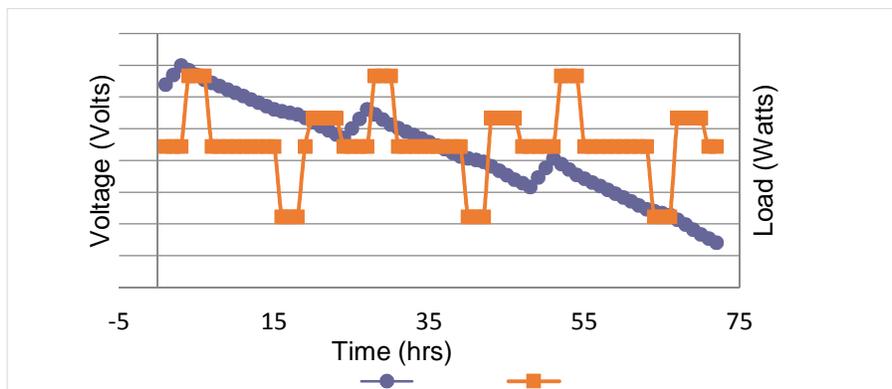


Figure 8: Non-mitigated Battery Voltage and Load Simulation of Minimal Sunshine Hours

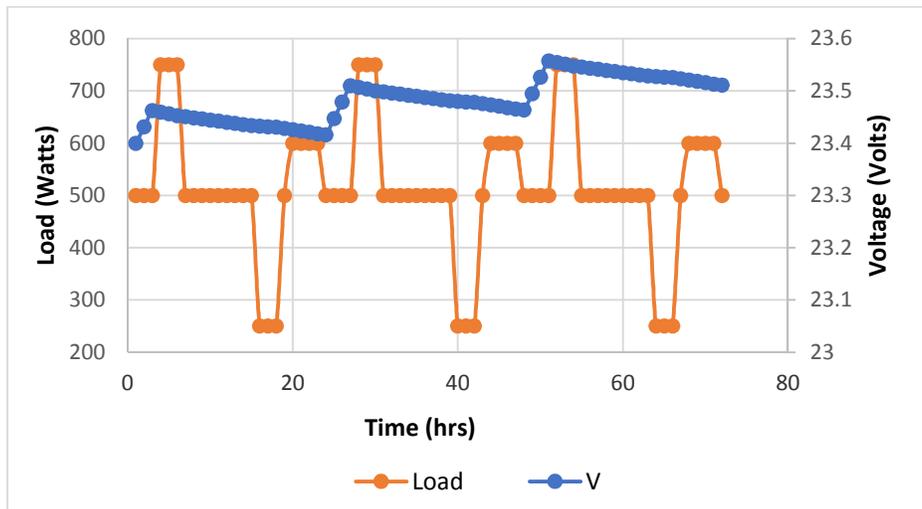


Figure 9: Panel-Mitigated Battery Voltage and Load for 3-Consecutive Days of Minimal Sunshine Hours

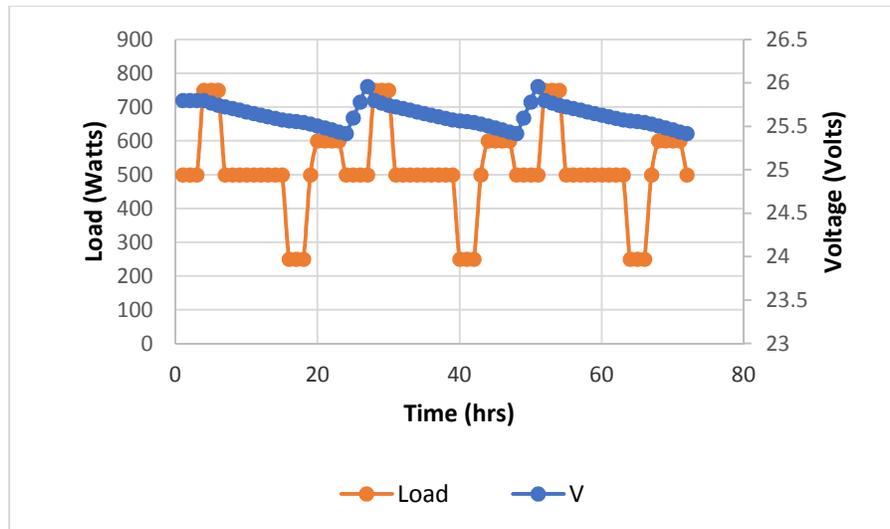


Figure 10: Battery-Mitigated Battery Voltage and Load for 3 Consecutive Days of Minimal Sunshine Hours

Table 1 presented the panel and battery sizes of the 3 scenarios using different capacities of the solar PV and batteries. It can be seen that the reference scenario has the least number of panels and

batteries with 8 number of 200Ah-battery and 20 number of 300W-panel. The battery-mitigated scenario has the highest number of components for all the different combinations.



Table 1: Solar PV Panel and Battery Sizing of Different Scenarios

Scenario	Batteries					PV Panels	
	Battery Capacity (Ah)	Parallel	Series	Total No. of Batteries	Panel Generated Current (A)	Panel Capacity (W)	No. of Panels
Reference	100	6	2	12	83.33	200	30
Reference	200	4	2	8	83.33	300	20
Panel-mitigated	100	16	2	32	166.67	200	75
Panel-mitigated	200	6	2	12	166.67	300	50
Battery-mitigated	100	20	2	40	83.33	200	67
Battery-mitigated	200	10	2	20	83.33	300	44

CONCLUSION

This paper shows the implementation of standalone autonomous solar-PV and battery system for the generation of electricity to a residential homes. Considering various scenarios of installations of the battery and solar panels, the reference and minimal sunshine hours were used to design a robust system. These approach helped to quantify the capacities and components sizes for the 4 different scenarios for the generation and required storage that satisfies the power demand. Among the scenarios, an ideal scenario provided the best design but is hardly feasible due to the fact that it is not possible for the sunshine hours to be unique throughout the year. The panel-mitigated scenario provided the best design with 12 number of batteries and 50 panels.

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